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PHENOMENOLOGICAL DAMAGE MODELS OF ANISOTROPIC STRUCTURAL MATERIALS

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ФЕНОМЕНОЛОГИЧЕСКИЕ МОДЕЛИ ПОВРЕЖДЕННОСТИ АНИЗОТРОПНЫХ КОНСТРУКЦИОННЫХ МАТЕРИАЛОВ

Damage in metals is mainly the process of the initiation and growth of voids. A formulation for anisotropic damage is established in the framework of the principle of strain equivalence, principle of increment complementary energy equivalence and principle of elastic energy equivalence. This paper presents the development of an anisotropic damage theory. This work is focused on the development of evolution anisotropic damage models which is based on a Young's modulus/Poisson's ratio change of the initial isotropic material. Anisotropic damage account is as important as accounting of the loading history and the type of stress state. Therefore, validation of the existing damage accumulation models with anisotropy account and the development of new ones is an important and promising direction in the solid mechanics. Today more widely for engineering applications the phenomenological approach, which is based on the continuum damage mechanics (CDM) and the thermodynamics of irreversible processes are used. The main idea of all damage models consists in replacing the conventional stress with the effective stress in the constitutive equation.
Keywords: anisotropic material, damage, effective stress, strain equivalence, increment complementary energy equivalence, elastic energy equivalence.

1. Introduction

In practice along with isotropic materials used anisotropic material, i.e. materials which properties vary in different directions. This concerns to the characteristics of elasticity (elastic moduli of the first and second order) and to the characterization of the limiting state (yield stress, ultimate stress). Exists anisotropy of two types: primary (initial), existing before the loading and the secondary (deformative), which may also occur in the initial isotropic materials under elastoplastic deformation. There are three types of mechanical properties anisotropy: crystallographic, technological and compositional [Friedman].

In engineering calculations the account of the anisotropic plastic deformation is same important as well is the loading history and the type of stress state. Anisotropy ignoring in the calculations of plastic deformation leads to significant (up to 50 %) deviation of calculated values of critical stress from the real ones.

Engineering, technology and design development and their production processes, as well as process tool manufacturing often performed without taking into account the anisotropy. This is due to the fact that so far there is no unified system for calculating the forming process parameters and not systematized data on the anisotropy for different metals and alloys. Recent experimental evidence indicates that structure failures are often associated with the development of anisotropic material damage, even if the initial material properties are isotropic [2]. The difference in the values of physical and mechanical properties of the same material in different directions can be up to 35%.

First introduction of the CDM was mesioned in works of Kachanov [3] and Rabotnov [4] and has been successfully used to describe processes of brittle failure of metal materials under uniaxial creep. These works have been expanded further within frameworks of thermodynamics of irreversible processes for the description of the complex stress state [5]. The main idea of all damage models consists in replacing the conventional stress with the effective stress in the constitutive equation. Practicing engineers concerned with three concepts: equivalence of strain [6], equivalence of incremental complementary energy [7] and equivalence of elastic energy [2].

The object of this paper is to estimate the reliability of these approaches. For this, were considered all three approaches to the description of anisotropic damage for aluminum alloy 5052, which is used in particular in centrifuges dynamic parts.

At manufacturing of centrifuge cups by drawing, properties of this material changed considerably [8]. The study of the manufacturing process influence on the fibers direction of the initial material is an important task by determining the parameters of the damage accumulation equation and the lifetime estimation of the cup and centrifuge.

2. Thermodynamic approach

The damage theories are considered in their rate-independent form. In this case, we limit ourselves to the case of the elastic damaged material under small strains and isothermal conditions [9]. From a thermodynamic point of view the damage equations can be presented through the following values: the thermodynamic potential and the dissipative potential [10].

Thermodynamic potential defines the present (damage) state of the material. Under isothermal conditions, the free energy density can be written [11]:

$$\Psi = \Psi\left(\varepsilon_{ij}^{(e)},\ D\right) = \frac{1}{2}\varepsilon_{ij}^{(e)} E_{ijkl}\left(D\right)\varepsilon_{kl}^{(e)} \tag{1}$$

where $\varepsilon_{ij}^{(e)}$ - the elastic strain tensor, D - damage variable. $E_{ijkl}\left(D\right)$ - fourth-order symmetric tensor secant stiffness, function of damage. In the case of active damage (open microcracks) can be written

$$\sigma_{ij} = \frac{\partial\Psi}{\partial\varepsilon_{ij}} = E_{ijkl}\left(D\right)\varepsilon_{kl}^{(e)} \quad . \tag{2}$$

Thus, Y is the strain energy density release rate:

$$Y = -\frac{\partial\Psi}{\partial D} = -\frac{1}{2}\varepsilon_{ij}^{(e)}\frac{\partial E_{ijkl}}{\partial D}\varepsilon_{kl}^{(e)} \quad . \tag{3}$$

It should be noted that the thermodynamic potential contains all the information about the damage effect on the material stress-strain behavior.

The dissipative potential describes the damage evolution and the corresponding irreversible processes. The second thermodynamic principle can be reduced to the following inequality [9]:

$$\Phi = Y\dot{D} \geq 0 \tag{4}$$

If Y is a positive quadratic function, the damage energy release rate is always positive.

2.1 Hypothesis of strain equivalence

The extension of isotropic damage theory to anisotropy is not a straight forward task in the coupling between elasticity and damage [6]. In the case of isotropic damage represented by scalar variable D, the effective stress concept associated to the principle of strain equivalence for elasticity $\tilde{\sigma}_{ij} = \sigma_{ij} \big/ \left(1-D\right) = E_{jkl}\varepsilon_{kl}^{(e)}$. In the case of a general anisotropy, the damage variable is represented by a fourth order tensor [12-14]. In our case we consider a second order tensor [15], which corresponds to orthotropy.

For a uniaxial tension damage can be obtained from the changes of the elastic characteristics, considering a representative volume element in an orthotropic frame (Fig. 1):

$$D = \begin{bmatrix} D_1 & 0 & 0 \\ 0 & D_2 & 0 \\ 0 & 0 & D_3 \end{bmatrix}. \tag{5}$$

The elastic strains in this frame are:

$$\begin{bmatrix} \varepsilon_1^{(e)} & 0 & 0 \\ 0 & \varepsilon_2^{(e)} & 0 \\ 0 & 0 & \varepsilon_3^{(e)} \end{bmatrix} = \frac{1-2\nu}{3E}\frac{\sigma_1}{1-\eta D_0}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} +$$
$$+\frac{1+\nu}{E}\left(\begin{bmatrix} \frac{1}{\sqrt{1-D_1}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-D_2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-D_3}} \end{bmatrix}\begin{bmatrix} \frac{2\sigma_1}{3} & 0 & 0 \\ 0 & \frac{-\sigma_1}{3} & 0 \\ 0 & 0 & \frac{-\sigma_1}{3} \end{bmatrix}\begin{bmatrix} \frac{1}{\sqrt{1-D_1}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-D_2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-D_3}} \end{bmatrix}\right)^D, \tag{6}$$

where η - is a necessary parameter for a correct representation of experiments concerning the variations in the Poisson's ratio with damage., D_0 - the mean damage, ν .- the Poisson's ratio;

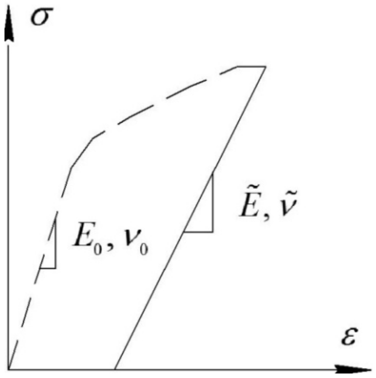

Fig. 1. Elastic properties in the orthotropic frame

Damaged elastic modulus in direction 1 and the associated contraction ratios are defined by:

$$\tilde{E}_1 = \frac{\sigma_1}{\varepsilon_1^{(e)}},\ \tilde{\nu}_{12} = -\frac{\varepsilon_2^{(e)}}{\varepsilon_1^{(e)}},\ \tilde{\nu}_{13} = -\frac{\varepsilon_3^{(e)}}{\varepsilon_1^{(e)}}. \tag{7}$$

Then

$$\frac{E}{\tilde{E}_1} = \frac{1+\nu}{9}\left(\frac{4}{1-D_1}+\frac{1}{1-D_2}+\frac{1}{1-D_3}\right)+\frac{1-2\nu}{3(1-\eta D_H)}, \tag{8}$$

$$\tilde{\nu}_{12}\frac{E}{\tilde{E}_1} = \frac{1+\nu}{9}\left(\frac{2}{1-D_1}+\frac{2}{1-D_2}-\frac{1}{1-D_3}\right)-\frac{1-2\nu}{3(1-\eta D_H)}, \tag{9}$$

$$\tilde{\nu}_{13}\frac{E}{\tilde{E}_1} = \frac{1+\nu}{9}\left(\frac{2}{1-D_1}-\frac{1}{1-D_2}+\frac{2}{1-D_3}\right)-\frac{1-2\nu}{3(1-\eta D_H)}. \tag{10}$$

The same operation for directions 2 and 3 gives nine equations to determine the three components of the damage D_1, D_2, D_3 and the coefficient η.

In the case of the damaged plane sheet

$$D_1 = 1-\frac{\tilde{E}_1}{E}(1+\nu)\left[2+\tilde{\nu}_{12}-\frac{\tilde{E}_1}{\tilde{E}_2}\right]^{-1}, \tag{11}$$

$$D_2 = 1-\frac{\tilde{E}_2}{E}(1+\nu)\left[2-\left(1-\tilde{\nu}_{12}\right)\frac{\tilde{E}_2}{\tilde{E}_1}\right]^{-1}, \tag{12}$$

$$\eta D_0 = 1-\frac{\tilde{E}_1}{E}\frac{1-2\nu}{1-2\tilde{\nu}_{12}}. \tag{13}$$

If tension is applied in direction 1, D_1 and D_2 are determined by equations (11) and (12), $D_3 = D_2$ for a material initially isotropic and $D_0 = (D_1 + 2D_2)/3$. Then η we obtained from equation (13).

2.2 Hypothesis of increment complementary energy equivalence

For an anisotropic damage, the relationship between conventional stress and effective stress can be written as [7]:

$$\tilde{\sigma} = M(D)\bullet\sigma, \tag{14}$$

where $M(D)$ is a damage effect tensor of the fourth order.

The elastic tensor for an isotropic material is given by:

$$E = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix}. \tag{15}$$

A large damage process is assumed that it can be modeled as a series of small incremental damage. Small damage refers to when damage variables $D << 1$. In the case of a large isotropic damage the damage variable is defined as:

$$D = \ln\frac{A}{\tilde{A}}, \tag{16}$$

where A and $\tilde{A}$ - the original and effective cross sectional areas, respectively.

Thus, the effective stress can be written as:

$$\tilde{\sigma} = e^D\sigma. \tag{17}$$

The effective elastic tensor for large damage [7]

$$\left[\tilde{E}_{ij}\right]_D^{-1}=\frac{1}{E}\begin{bmatrix} \frac{1}{e^{-2D_1}} & \frac{-\nu}{e^{-(D_1+D_2)}} & \frac{-\nu}{e^{-(D_1+D_3)}} & 0 & 0 & 0 \\ \frac{-\nu}{e^{-(D_1+D_2)}} & \frac{1}{e^{-2D_2}} & \frac{-\nu}{e^{-(D_2+D_3)}} & 0 & 0 & 0 \\ \frac{-\nu}{e^{-(D_1+D_3)}} & \frac{-\nu}{e^{-(D_2+D_3)}} & \frac{-\nu}{e^{-2D_3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{e^{-(D_1+D_2)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{e^{-(D_2+D_3)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{e^{-(D_3+D_1)}} \end{bmatrix}. \quad (18)$$

Using this tensor, the constitutive equation for large damage is (uniaxial tension):

$$\varepsilon=\tilde{E}_D^{-1}\cdot\sigma. \quad (19)$$

The damage effect tensor for large anisotropic damage tensor is given by [7]:

$$\left[M\left(D\right)\right]=\begin{bmatrix} \frac{1}{e^{-D_1}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{e^{-D_2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{e^{-D_3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{e^{-1/2(D_1+D_2)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{e^{-1/2(D_2+D_3)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{e^{-1/2(D_3+D_1)}} \end{bmatrix}. \quad (20)$$

Note that the $M\left(D\right)$ allows the large effective tensor in (18) to be recovered by carrying out the following algebraic manipulation:

$$\tilde{E}_D^{-1}=M\left(D\right)^T\cdot E^{-1}\cdot M\left(D\right). \quad (21)$$

In accordance with the large damage theory for the anisotropic damage, the constitutive equations under tension are [7]:

$$\varepsilon_1=\frac{1}{E}e^{2D_1}\sigma_1=\frac{1}{\tilde{E}}\sigma_1, \quad (22)$$

$$\varepsilon_2=\frac{-\nu_{12}}{Ee^{-(D_1+D_3)}}\sigma_1=\frac{-\tilde{\nu}_{12}}{\tilde{E}}\sigma_1, \quad (23)$$

$$\varepsilon_3=\frac{-\nu_{13}}{Ee^{-(D_1+D_3)}}\sigma_1=\frac{-\tilde{\nu}_{13}}{\tilde{E}}\sigma_1, \quad (24)$$

where $\tilde{E}=e^{-2D_1}E$, $\tilde{\nu}_{12}=e^{(D_2-D_1)}\nu_{12}$, $\tilde{\nu}_{13}=e^{(D_3-D_1)}\nu_{13}$ are the effective Young's modulus and Poisson's ratios, respectively. Thus, the damage variables can be determined from:

$$D_1=-\frac{1}{2}\ln\frac{\tilde{E}}{E}, \quad (25)$$

$$D_2=D_1-\ln\frac{\nu_{12}}{\tilde{\nu}_{12}}=-\ln\left(\frac{\nu_{12}}{\tilde{\nu}_{12}}\sqrt{\frac{\tilde{E}}{E}}\right), \quad (26)$$

$$D_3 = D_1 - \ln\frac{\nu_{13}}{\tilde{\nu}_{13}} = -\ln\left(\frac{\nu_{13}}{\tilde{\nu}_{13}}\sqrt{\frac{\tilde{E}}{E}}\right). \qquad (27)$$

2.3 Hypothesis of elastic energy equivalence

Lemaitre [6] proposed a hypothesis of strain equivalence for isotropic damage by replacing the conventional stress with the effective stress in the constitutive equations. This leads to asymmetry of the stiffness matrix for anisotropic damage. To avoid this, Sidoroff [16] has postulated that the complementary elastic energy for a damaged material is the same form as that of an undamaged material, except that the stress is replaced by the effective stress in the energy formulation [2].

The damage effect tensor in the principal coordinate system [2]:

$$\left|M_{ij}(D)\right| = \begin{bmatrix} \frac{1}{1-D_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{1-D_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1-D_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{(1-D_2)(1-D_3)}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{(1-D_3)(1-D_1)}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{(1-D_1)(1-D_2)}} \end{bmatrix} \qquad (28)$$

Under uniaxial tension constitutive equations are:

$$\varepsilon_1^{(e)} = \frac{\sigma_1}{E(1-D_1)^2} = \frac{\sigma_1}{\tilde{E}}, \qquad (29)$$

$$\varepsilon_2^{(e)} = \frac{\nu\sigma_1}{E(1-D_1)(1-D_2)} = -\frac{\tilde{\nu}_{12}}{\tilde{E}}\sigma_1, \qquad (30)$$

$$\varepsilon_3^{(e)} = \frac{\nu\sigma_1}{E(1-D_1)(1-D_3)} = -\frac{\tilde{\nu}_{13}}{\tilde{E}}\sigma_1, \qquad (31)$$

where

$$\tilde{E} = E(1-D),\ \tilde{\nu} = \nu(1-D)/(1-D),\ \tilde{\nu} = \nu(1-D)/(1-D).$$

Accordingly, the damage variables can be defined as:

$$D_1 = 1-\sqrt{\frac{\tilde{E}}{E}},\ \ D_2 = 1-\frac{\nu}{\tilde{\nu}_{12}}(1-D_1),\ \ D_3 = 1-\frac{\nu}{\tilde{\nu}_{13}}(1-D_1). \qquad (32)$$

3. Experimental investigation

3.1 Material and Equipment

Experimental studies were carried out at the I stitute of Mechanics, Otto-von-Guericke University Magdeburg on a hydraulic test system MTS 810 (Fig. 2). Were used extensometers to measure the axial and cross sectional strain (Fig.3). The measurements were obtained from tensile specimens of aluminum alloy 5052 whose dimensions are shown on Fig.4. Specimens were produced by laser cutting sheet material at angles of 0, 45 and 90 degrees to the direction of rolling (Fig. 5). The tests were performed at room temperature.

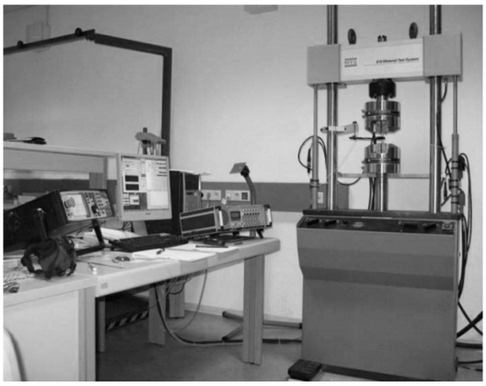

Fig. 2. Test system MTS 810

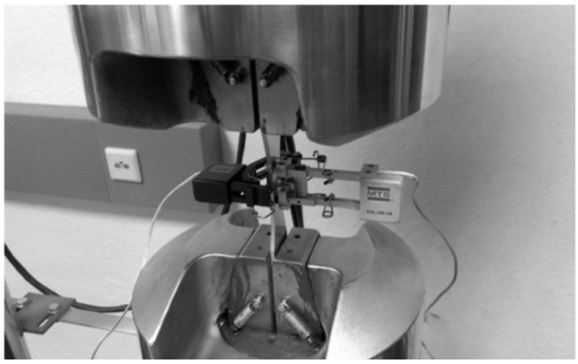

Fig. 3. Axial and cross sectional strain extensometers

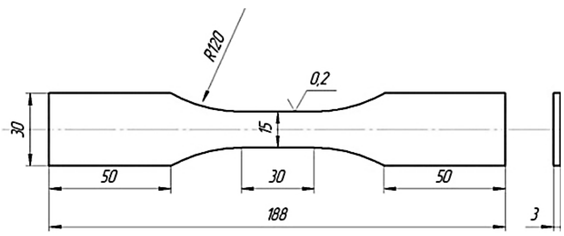

Fig. 4. Tensile specimen

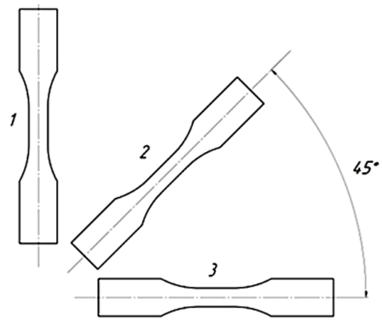

Fig. 5. Specimens cut in 3 directions

3.2 Application

In this work a set of uniaxial tests was carried out to measure E, ν from which the damage variable D can be computed. Tests (9 specimens) were conducted on uniaxial tensile strain (Fig. 6). Mechanical properties of the tested specimens (yield stress σ_T , ultimate stress σ_B and strain to rupture ε_R) are shown in Table 1.

Table 1

Mechanical properties of aluminum alloy 5052

# - Cutting direction degree	σ_T , MPa	σ_B , MPa	ε_R
1-0	79,3	226,7	0,2805
2-0	80,1	215,1	0,2362
3-0	86,0	222,9	0,2666
1-45	80,5	209,8	0,3108
2-45	84,4	172,6	0,2442
3-45	93,4	216,2	0,3055
1-90	86,9	172,9	0,1419
2-90	90,6	209,3	0,2808
3-90	87,0	209,6	0,2848

A plot of the Young's modulus versus strain depending on the direction of cut is presented in Fig. 7. Maximum value of the Young's modulus degradation is 41% compared to the undamaged value. The change in the elastic modulus versus cut angle (0^0, 45^0 and 90^0) is not strongly pronounced.

Change in the transverse strain versus axial, depending on the direction of cut are depicted in Fig. 8. Based on experimental data, we can conclude, that the transvers stain on specimens which were cut at 45^0 has higher value (differs up to 19%) than on specimens that were cut at 0^0 and 90^0.

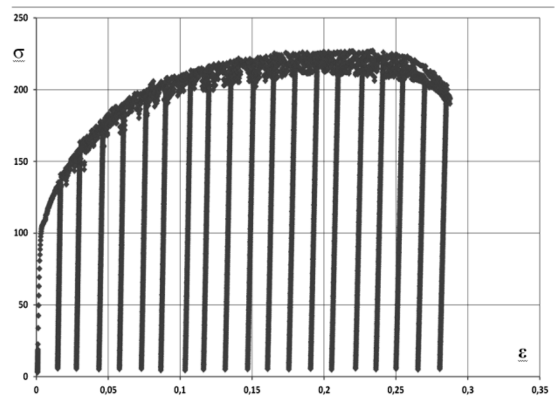

Fig. 6. Tensile test

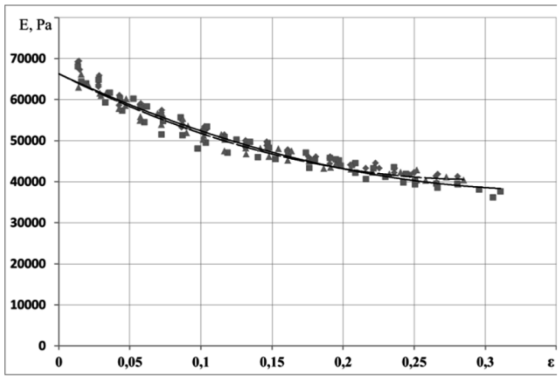

Fig. 7. Elastic modulus versus strain (cut direction: 0^0, 45^0 and 90^0)

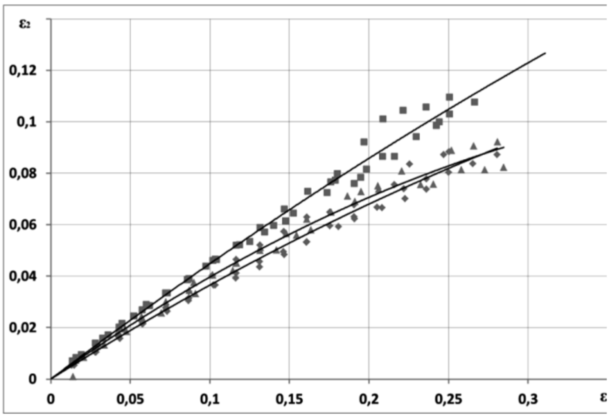

Fig. 8. Transvers strain versus axial strain (cut direction: 0^0, 45^0 and 90^0)

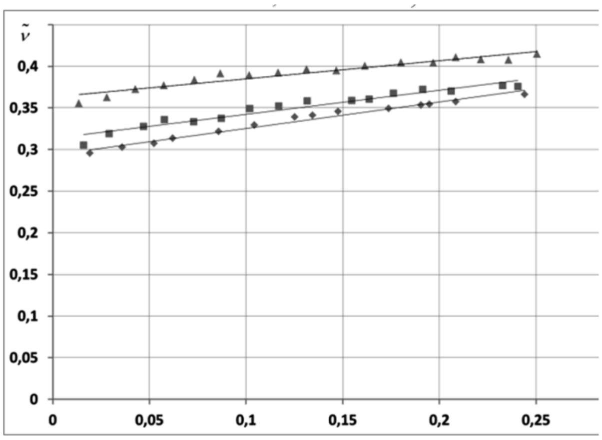

Fig. 9. Poisson's ratio versus strain (cut direction: 0^0, 45^0 and 90^0)

In Fig.10-12 are depicted the damage variable versus strain for three cut directions. Lemaitre, Chow and Luo [6, 2, 7] model were used.

The graphs show, that Lemaitre model gives higher values of the damage variable in comparison with Chow and Luo model, which are based on an energy approach and gives very similar numerical values.

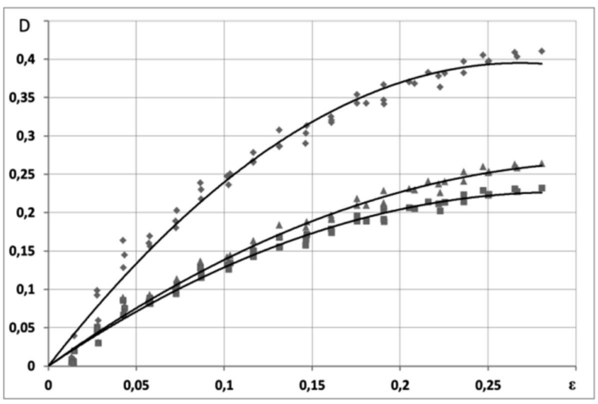

Fig. 10. Damage variable versus strain (■ –Chow, ▲ – Luo, ◊ - Lemaitre) (0^0)

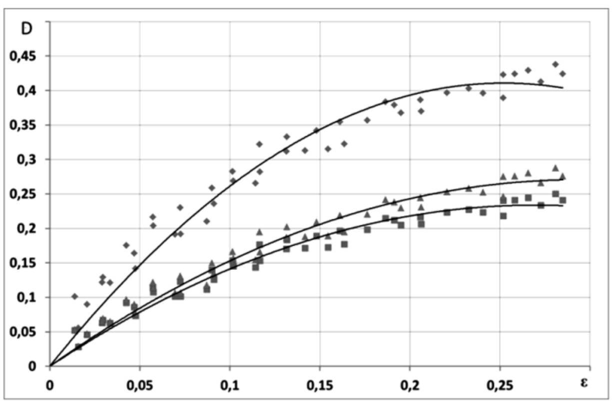

Fig. 11. Damage variable versus strain (■ –Chow, ▲ – Luo, ◊ - Lemaitre) (90^0)

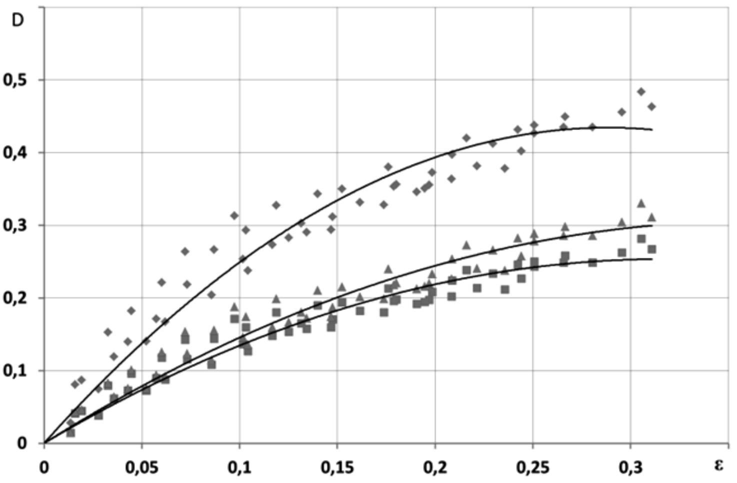

Fig. 12. Damage variable versus strain (■ –Chow, ▲ – Luo, ◊ - Lemaitre) (45^0)

In the [17] is shown, that the modified energy approach is more accurate in describing processes of damage accumulation in comparison with Lemaitre's model. As actual value was taken damage variable obtained from the change in electrical resistivity [11].

4. Conclusions

1. In experimental studies it was found that the ultimate degradation of the Young's modulus can reach 41% in comparison to the initial value for the aluminum alloy 5052.

2. For the specimens that were cut out at 45^0 transvers strain up to 19% higher than for specimens that were cut out in other directions.

3. Lemaitre model gives higher values of the damage variable in comparison with Chow and Luo model, which are based on an energy approach and gives very similar numerical values.

4. Further research is needed to obtain the anisotropy coefficient and formulate adequate damage model under static and under cyclic loading.

Анотація. *У статті наведено результати експериментальних та теоретичних досліджень впливу анізотропії механічних властивостей на кінетику накопичення пошкоджень при пружнопластичного деформуванні. Отримано залежності зміни модуля пружності від рівня деформації в залежності від напрямку прокату матеріалу. Показано, що гранична деградація модуля пружності досягає 41% в порівнянні з початковим значенням. При показано цьому, що залежність зміни модуля пружності від кута явно не виражена .*
Ключевые слова: анизотропный материал, повреждаемость, эффективное напряжение, эквивалентность деформаций, прирост дополнительной энергии, эквивалентность упругой энергии

Аннотация. *В статье приведены результаты экспериментальных и теоретических исследований влияния анизотропии механических свойств на кинетику накопление повреждений при упругопластическом деформировании. Получены графики изменения модуля упругости от уровня деформации в зависимости от направления проката материала . Показано, что предельная деградация модуля упругости достигает 41% в сравнении с начальным значением. При показано этом, что зависимость изменения модуля упругости от угла вырезания ярко не выражена.*
Ключові слова: анізотропний матеріал, пошкоджуваність, ефективне напруження, еквівалентність деформацій, приріст додаткової енергії, еквівалентність пружної енергії

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