УДК 621.831.7 MATHEMATHICAL MODELLING OF METAMATERIALS AND ANALYSIS OF THEIR OPTICAL PROPERTIES

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In this work were studied fishnet metamaterials with proposed meshy structure. The spectral bands with negative refractive index were discovered in near infrared and visible regions. The linear red shift of spectral features was observed with the increasing partition order of fractal. These optical features of discussed metamaterials could find possible applications in optical devices (filters, lenses, modulators etc.), high-sensitive pressure sensor meters and other devices. Obtained in math modeling linear dependences of the peak positions of negative refractive index (NRI) from geometric parameters of metamaterials can be used to adjust the spectral position of optical features in the production of meshy metamaterials with given functional characteristics. In the work were discussed prospects of use of the considered metamaterials in optical devices such as filters, lenses, modulators, etc., as well as highly sensitive pressure sensors, biological and chemical sensors with anti-interference remote control.

Keywords: metamaterials, negative refractive index, fractals, optical metamaterial, fishnet meshy metamaterial, fractal partition.

Introduction

Due to the development of technologies of the synthesis and design of nanoscale structures, now it is possible to create unique metamaterials with a negative index of refraction(fisnet metamaterials) in the near infrared and visible spectral regions and a lot of work has been done in this area by Shelby R. A.,Smith D.R. and others. In this context, the relevant tasks are geometry optimization and the search for new types of metamaterials, which allow with the existing technology base to provide left-handed properties of the optical field in the shortwave spectrum, to minimize losses at the stage of radiation transmission and provide flexibility to alter the spectral range of negative refraction.

The aim of this work was to develop new structures of fractal metamaterials with negative refractive index and to analyse their optical properties in the near infrared and visible region of the spectrum.

The scientific importance of research performed associated with the development of ideas about the influence of the new geometrical parameter - order fractal partition - on electro- magnetic properties of metamaterials in the visible and near-infrared regions spectrum.

The main part

As a basis for the construction of the geometric structure of the metamaterial was choosen a plane fractal. Figure1 illustrates the process of constructing fractal. Structure with layers of "metal -insulator-metal " with a unit cell in the form of the first-order fractal is a traditional mesh metamaterial.



Fig.1. Structure of the unit cell of traditional and fractal mesh metamaterial at different orders of partition k

In this work we created a model of mesh fractal metamaterial. Metamaterial unit cell for fourth-order decomposition and fragment fractal metamaterial structures for k = 3 shown in figure 2. Materials (Ag-MgF2-Ag) and layer thickness (40-17-40 nm) were selected according to the model described in [1], for which the negatively - refractive index in the visible spectrum was previously obtained. Shown metamaterial structure with k-th order partition from now on is denoted as Ωk .



Fig.2. Single cell mesh fractal metamaterial (k = 4) and a fragment of the structure of the metamaterial with a period d (k = 3)

Modeling of metamaterials with Ωk structure with period d = 300 nm and partition order k = 1 ÷ 4 was carried out using CST Microwave studio software package. In this work we analyzed the interaction of radiation with a wavelength in the range of 375 ÷ 1600 nm with metamaterials. Complex range of permittivity $\epsilon Ag(\lambda)$ for the silver layer has been set in accordance with the experimental data [2] with a threefold increase in the imaginary part, according to [3]; refractive index for fluoride layer was taken equal 1,38.

We established the area of modeling (figure 3). At its edges perpendicular to the Z-axis were set up opened boundary conditions, which simulate the spread of electromagnetic waves along the Z axis in unlimited free space. Also on these edges are arranged waveguide ports: the top port is used to enter plane electromagnetic wave as

a model signal with a Gaussian envelope, which propagate perpendicular to the plane



Fig. 3. The boundary conditions on the edges of the modeling area: 1 – an ideal electrical conductor, 2 – an ideal magnetic conductor, 3 – an open border ; a waveguide port

of the cell. And also this port is used for the reception of the reflected radiation; through the lower port – the reception of the transmitted radiation. The distance between the ports was selected equal 8 mm.

On the adges of the modeling area perpendicular to axis X, were set up conditions of ideal electrical conductivity, to axis Y were set up conditions of ideal magnetic conductivity. Therefore we simulate normal incidence of plane electromagnetic wave to an endless periodic metamaterial.

Simulation results

The result of this calculation is the spectral dependence of the matrix disscattering elements, of the form [4]:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (1)$$

 a_1, a_2 - complex amplitudes of the incident waves, b_1, b_2 - complex amplitudes of the

reflected waves, S_{ii} - amplitude reflection coefficients for the i-th input, S_{ij} - the amplitude transmission coefficients from the j-th input to the i-th output.

In the case of normal incidence of a plane wave on the surface of effectively homogeneous plate from a source, which is located on the first (left) plate surface, S_{11} is the amplitude reflection coefficient, and S_{21} associated with transmission coefficient for amplitude T through the expression [5]:

$$S_{21} = T \cdot e^{ik_o h}, \tag{2}$$

 k_0 - the wave number of the incident wave in free space.

Calculation of the refractive index on the basis of previously obtained spectral dependences was based on the method described in [5]. S-parameters associated with exponent index n and impedance z through expressions:

$$S_{11} = \frac{R_{01} \cdot \left(1 - e^{i \cdot 2nk_0 h}\right)}{1 - R_{01}^2 e^{i \cdot 2nk_0 h}}$$
(3)

$$S_{21} = \frac{\left(1 - R_{01}^2\right)e^{ink_0h}}{1 - R_{01}^2e^{ink_0h}}$$
(4)

h - uniform thickness of the plate; n- complex refractive index of the plate; k_0 - the wave number of the incident wave in free space; $R_{01} = \frac{z-1}{z+1}$ - reflection coefficient of the air-metamaterial bound.

Taking into consideration that the considered metamaterial is a passive medium outside of the resonance peaks, and using the conditions of continuity of functions $\varepsilon''(\lambda)$ and $\mu''(\lambda)$, we obtain a uniquely defined function n' (λ) in the wavelength range, wherein for metamaterial is performed approximation of effective medium.

We found that for the considered structures of fractal mesh metamaterials with a period d = 300 nm and order fractal partition k = 1 ÷ 4, in the visible and near-infrared region of the spectrum there are two bands of negative refraction. Parameters of the peaks are presented in table 1, an example of the spectral dependence n'(λ) – in figure 4.



Fig.4. Spectral dependence of the real part of the refractive index for the structure with a period of d = 300 nm and the order fractal partition k = 3

Analysis of the results

Two-peak configuration spectral dependences $n'(\lambda)$ have been observed earlier for traditional structures of mesh metamaterials [6], and the ratio of amplitudes of NRI peaks depend on the ratio of the lengths of sides of the unit cell [6]. With an increasing in order of fractal partition there is a decreases of the amplitude and halfwidth of the NRI peaks, and also observable shift to longer wavelengths. The amplitude of the second peak located at shorter wavelength region is more sensitive to the change in k.

As a result of analysis of all the data obtained we built charts of dependencies of the minimum peaks positions of NRI from the fractal partition of the structure (figure 5).

Table 1. Parameters of peaks of NRI for metamaterials with	structure Ωk
and $d = 300 \text{ nm}$	

Partition order,	Position of the		Peak half-width		The amplitude of	
k	peak's	minimum,	FWHM, nm		the Peak,	
	λ,nm				$-n_{\gamma}$	nin
	1peak	2 peak	1 peak	2peak	1 peak	2 peak
1	898,3	634,6	26,4	11,0	3,98	1,68
2	902,2	656,8	24,0	8,5	3,64	1,11
3	930,1	673,7	20,6	9,1	3,31	0,80
4	951,2	695,3	18,9	9,1	3,23	0,60



Fig.5. Dependence of the peak positions of the minima of the negative refractive index of theorder of partition k Ω k structure with period d = 300 nm

As shown in figure 5, the shift of peaks of negative refraction in the long-wave region of the spectrum when you change the order of the fractal structure of the partition Ωk is close to linear in nature. Angle of slope of approximating line for peak 1 is about 27 nm / order, for peak 2 -21 nm / order. The amplitude of the second peak decreases with the increase of k, that's why it's impractical to use fractal mesh metamaterials with large k. In independent literature both experimentally [7] and theoretically [8] have been shown that for different types of metamaterials in the range of wavelengths under study with increasing period of the structure we can observe close to linear peaks' shifts of negative refractive index.

Thus, the change in the period and the order of the partition of the fractal structure can be used for discrete and continuous adjustment of the position of the peaks of NRI. Changing the size of the holes in the variation of k will also adjust the sensitivity of chemical sensors based on fractal metamaterials to the desired level. As was mentioned in chapter 3, for the development of computational models in this work used a threefold increased experimental values $\varepsilon''(\lambda)$ obtained for macroscopically large samples of silver [2]. According to [3] for silver strips with the width less than 100 nm free path of electrons on the metal surface becomes comparable with the size of the strips , which leads to a significant increase in the frequency of collisions, growth of ε'' and consequently entails more losses. Selected threefold increase ε' , according to [3], valid for silver bars from 50 to120 nm wide, for smaller widths of silver bars ε'' can increase 5 times or more compared with experimental data for a macroscopically large samples. Additional contribution to the loss makes the roughness of silver surface and metamaterial fabrication defects. For some models transverse sizes of the elements were found to be less than 50 nm, so the losses in manufactured metamaterials with a similar structure may exceed theoretical estimates, in particular for small periods of the structure and large values of the partition k.

So in the production of metamaterials in order to increase the accuracy of calculation of their characteristics it is necessary to adjust $\varepsilon''(\lambda)$ according to experimental data, which was obtained for test samples with the corresponding sizes of the structure.

Difference of real $\varepsilon''(\lambda)$ and $\varepsilon''(\lambda)$ used in the calculation for the model will cause nonconformance of amplitudes of NRI peaks, but will not change their position. Therefore, the positions of the peaks' dependences of the negative refractive index from the fractal partition obtained in this work remain valid.

Conclusions

In this work we developed the fractal structure of meshy metamaterial. For model structures with fractal partition order $k = 1 \div 4$ and period d = 300 nm in the wavelength range $\lambda = 375 \div 1500$ nm were calculated elements of scattering matrix, which quantitatively describe the processes of interaction of optical radiation with condensed matter.

We observed two peaks of the negative refractive index for all analyzed metamaterials in the visible and infrared spectral range.

The smallest value of NRI: $n_{min} = -3,96$. We found that for these structures with the increase of fractal partition (k) we observe linear shift of optical features to the long- wave region.

Future work on this subject should include the issues of metamaterials' design and manufacturing for the devices of optical frequency.

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УДК 528.7, 629.78 МЕТОДИ ПОКРАЩЕННЯ МЕТРОЛОГІЧНИХ ХАРАКТЕРИСТИК ДИФУЗНОГО ВИПРОМІНЮВАЧА ЗМІННОЇ ЯСКРАВОСТІ

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В статті розглянуто проблему покращення метрологічних характеристик дифузного випромінювача змінної яскравості на законі зворотних квадратів. Запропоновано схему дифузного випромінювача, де в ролі джерела випромінювання використовується матриця з галогенних ламп, а в якості розсіювача – матоване кварцове скло. Представлена математична модель дифузного випромінювача з матричним джерелом випромінювання. По приведеній математичній моделі зроблені розрахунки основних вихідних характеристик дифузного випромінювача з матричним джерелом випромінювання та проведений аналіз результатів.

Ключові слова: дифузний випромінювач змінної яскравості, матричне джерело випромінювання.

Введення

Калібрувальні дифузні випромінювачі змінної яскравості (ДВЗЯ) є одними з основних елементів сучасної прецизійної радіометрії. Вони широко використовуються при вимірюванні енергетичних характеристик приймачів випромінювання, калібруванні фотометричного обладнання, атестації оптико-електронних вимірювальних приладів [1-3]. Однак, незважаючи на останні досягнення в цьому напрямку, існуючі ДВЗЯ не повною мірою задовольняють вимогам сучасної оптичної метрології. У першу чергу це стосується завдань радіометричного калібрування багатоелементних приймачів випромінювання і пристроїв на їх основі [4,5]. Водночас, основними проблемами відомих ДВЗЯ є недостатня інтегральна яскравість, малий динамічний діапазон її зміни, недостатня ефективна апертура, вузький спектральний діапазон і ряд інших [6,7].

Постановка задачі

Метою цієї роботи є поліпшення метрологічних характеристик одного з кращих приладів розглянутого класу - ДВЗЯ, заснованого на законі зворотних квадратів (ЗЗК), шляхом використання матричних джерел випромінювання (ДВ) і нових типів розсіювачів на базі кварцового матованого скла.