

# Control of the Modified Chaotic Chua's Circuit Using Threshold Method

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**Introduction.** General scientific fields where can be used circuits that realize chaotic behavior and generate chaotic oscillations are presented. Methods for control of chaotic oscillations are also presented. For modelling, analysis and demonstrate results was selected MultiSim software environment.

**Modelling and Analysis of Non-Linear Element.** This modified Chua's circuit has a simple non-linear element, designed to have a piecewise-linear characteristic, that is, a combination of an opamp with two diodes that are mutually inline. For realization of nonlinearity, for two diodes do not need a separate power source, only one bipolar power source for the opamp is enough. The scheme for modelling of the nonlinear element and the results of computer simulation, i.e. the volt-ampere characteristic (VAC) at certain values of the components of the scheme's nominal values, is presented. This modified Chua's circuit, which generates a chaotic and controlled attractor with a fixed period, can be used in modern transmission and reception systems of information.

**Modeling and Analysis of the Modified Chaotic Chua's Generator.** System's behavior is investigated through numerical simulations, by using well known tools of nonlinear theory, such as chaotic attractor and time distributions of the chaotic coordinates.

**Threshold Method for Control of Chaotic Oscillations.** System of equations that realize chaotic oscillations of Chua's circuit is presented. Using threshold method was practical realization of the control of chaotic attractor. This modified Chua's circuit that generate a chaotic and controlled attractor with a fixed period can be used in modern systemstransmitting and receiving information. Number of periodic (controlled) attractor can be used as a keys formasking of information carrier.

**Conclusions.** For the first time was used threshold method forcontrol of chaotic oscillations for modified Chua'schaotic generator. This modified Chua's circuit thatgenerate a chaotic and controlled attractor with afixed period can be used in modern systems transmittingand receiving information. Number of periodic(controlled) attractor can be used as a keys for maskingof information carrier.

*Key words:* chaos; Chua; control; threshold method

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## Introduction

Chaos theory have in different areas for application, such as biology [1], economy [2–4], plasmas [5], magnetism [6], memristor [7–19], electronics schemes [20,21], etc. There are many different circuit realizations of the chaotic Chua's generator.

For chaos control have been proposed many different approaches or techniques, such as linear feedback control, OGY, inverse optimal control, etc [22–30]. The theoretical basis of most known methods for control chaos is stabilizing the unstable periodic orbits via parameter perturbation.

For modelling, analysis and demonstrate results was selected software MultiSim.

## 1 Modelling and Analysis of Non-Linear Element

Nonlinear elements – these are elements in which the relation between voltage and current is a nonlinear function. An example is a diode, in which the current is an exponential function of the voltage. Circuits with nonlinear elements are harder to analyze and design, often requiring circuit simulation computer programs such as SPICE.

The circuit realization for modelling and analysis of the non-linear element is displayed in Fig. 1, with component: one operational amplifier TL082; resistors  $R_1 = R_2 = 220 \Omega$ ,  $R_3 = 1,2 \text{ k}\Omega$ ,  $R_4 = 6 \text{ k}\Omega$ ,  $R_5 = 800 \Omega$ ; two diodes 1N4148; voltage –  $\pm 9 \text{ V}$ .

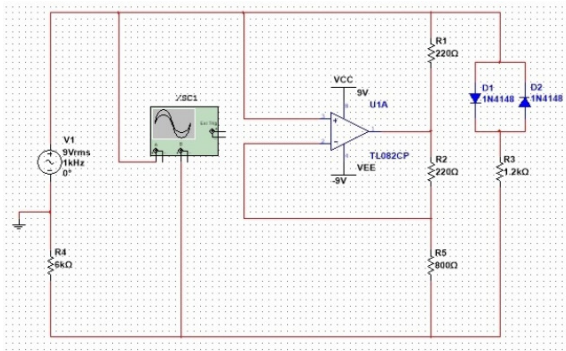


Fig. 1. Circuit realization for modelling and analysis of nonlinear characteristic

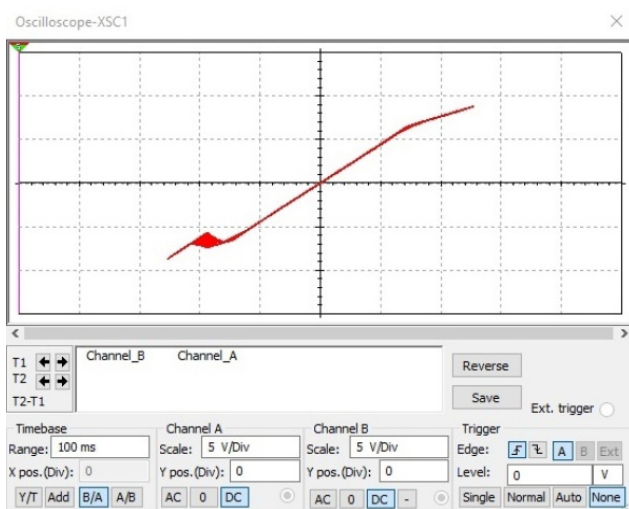


Fig. 2. V/I characteristic of nonlinear element

The nonlinear characteristic was modelled by the following parameters:  $E = 9\text{ V}$ ,  $f = 1\text{ kHz}$ ,  $R = 6\text{ k}\Omega$ . Fig. 2 shows result of modelling of nonlinear element using MultiSim. The simulation parameters:  $U_1 = 5\text{ V/div}$ ,  $U_2 = 5\text{ V/div}$ .

## 2 Modelling and Analysis of the Modified Chaotic Chua's Generator

Fig. 3 shows simulated scheme of the modified chaotic Chua's generator by using MultiSim. Circuit was realized on the one operational amplifier TL082, powered by a 9 V, two diodes 1N4148, resistors  $R_1 = R_2 = 220\ \Omega$ ,  $R_3 = 1.2\text{ k}\Omega$ ,  $R_4 = 800\ \Omega$ , potentiometer  $R_5 = 2\text{ k}\Omega$  (1.7 k $\Omega$ ), two capacitors  $C_1 = 10\text{ nF}$ ,  $C_2 = 100\text{ nF}$ , inductor  $L_1 = 18\text{ mH}$ .

Fig. 4 shows the result of circuit simulation. Generated chaotic signal in the plane XY presented on the virtual oscilloscope. Coordinate X in the circuit correspond voltage  $U_{C2}$ , coordinate Y – voltage  $U_{C1}$ . The simulation parameters:  $U_1 = 1\text{ V/div}$ ,  $U_2 = 2\text{ V/div}$ .

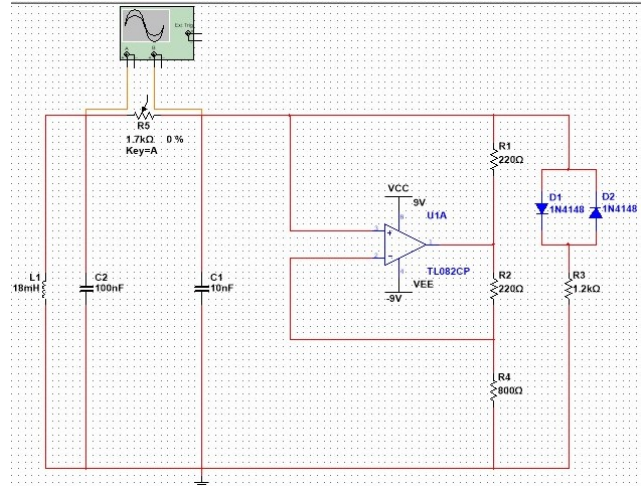


Fig. 3. The simulated circuit of the modified chaotic Chua's generator

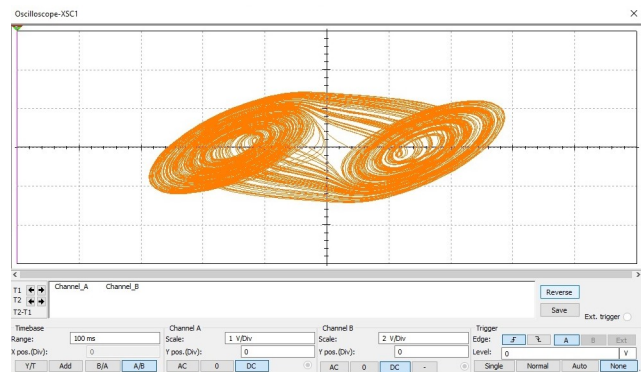


Fig. 4. Chaotic attractor

In Fig. 5 shows time dependences of the coordinates X and Y. The simulation parameters for Fig. 5:  $U_1 = 2\text{ V/div}$ ,  $U_2 = 5\text{ V/div}$ , time scale 2 ms/div.

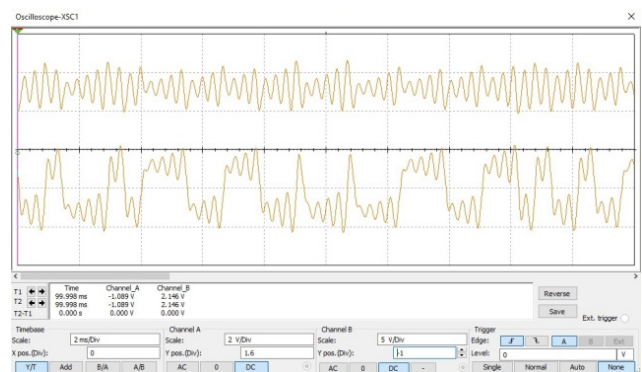


Fig. 5. Time dependences of the coordinate X and Y

## 3 Threshold method for control of chaotic oscillations

Consider a general N-dimensional dynamical system, described by the evolution equation  $\dot{x} = F(x, t)$  where  $x \equiv (x_1, x_2, \dots, x_N)$  are the state variables, and variable  $x_i$  is chosen to be monitored and threshold controlled. The prescription for threshold

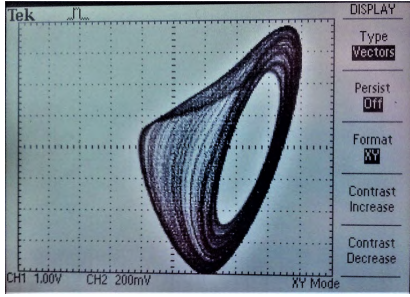


Fig. 6. Uncontrolled chaotic attractor in the  $V_1 - V_2$  plane

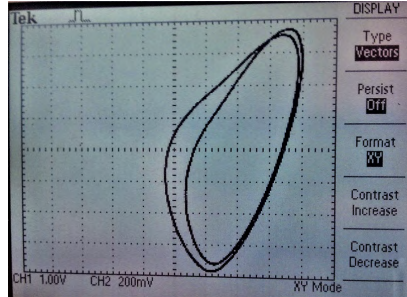


Fig. 7. 2-period controlled attractor obtained when  $x^*=2.7$  V in the  $V_1 - V_2$  plane

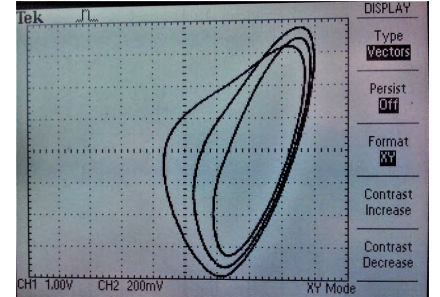


Fig. 8. 3-period controlled attractor obtained when  $x^*=2.71$  V in the  $V_1 - V_2$  plane

control in this system is as follows: control will be triggered whenever the value of the monitored variable exceeds a critical threshold  $x^*$  (i.e., when  $x_i > x^*$ ) and the variable  $x_i$  will then be reset to  $x^*$ . The dynamics continues till the next occurrence of  $x_i$  exceeding the threshold, when control resets its value to  $x^*$  again. No run-time knowledge of  $F(x)$  is involved, and no computation is needed to obtain the necessary control. The method only involves monitoring a single variable and no parameters are perturbed in the original system. The theoretical basis of the method does not involve stabilizing unstable periodic orbits, but rather involves clipping desired time sequences (symbol sequences in maps) and enforcing a periodicity on the sequence through the thresholding action which acts as a resetting of initial conditions. The effect of this scheme is to limit the dynamic range slightly, i.e., “snip” off small portions of the available phase space, and this small controlling action is effective in yielding a range of stable behaviors. In fact, chaos is advantageous here, as it possesses a rich range of temporal patterns which can be clipped to different behaviors. This immense variety is not available from thresholding regular systems. It can be shown analytically for one-dimensional maps and numerically for multidimensional systems that the threshold mechanism yields stable orbits of all orders by simply varying the threshold level. But so far there had been no direct experimental verification of this control scheme [31]. Now to experimentally demonstrate the range and efficacy of the method, we implement it on the modified chaotic Chua's circuit. We consider a realization of the double scroll chaotic Chua's attractor given by the following set of (rescaled) three coupled ODEs:

$$\frac{dx}{dt} = \alpha[y - x - g(x)], \quad (1)$$

$$\frac{dy}{dt} = x - y + z, \quad (2)$$

$$\frac{dz}{dt} = -\beta y, \quad (3)$$

where  $\alpha = 10, \beta = 14.87, g(x)$  — piecewise linear function. Chaotic oscillations were if system parameters  $a = 2, b = 6.7$ , and dynamic variables  $x = 1.2, y = 0.8, z = 1.4$ .

The circuit realization of the above is displayed in Fig. 9, with component values: capacitors  $C1 = 100$  nF,  $C2 = 10$  nF, DA1-DA4 — operational amplifier TL082, powered by a 9 V, GB1 — threshold reference voltage, inductor  $L1 = 18$  mH, resistors  $R1 = R2 = 1.71$  k $\Omega$ ,  $R3 = R4 = 220$   $\Omega$ ,  $R5 = 800$   $\Omega$ ,  $R6 = 1.2$  k $\Omega$ ,  $R7 = 1$  k $\Omega$ , potentiometer  $R8 = 100$  k $\Omega$ , diodes VD1-VD3 — 1N4148.

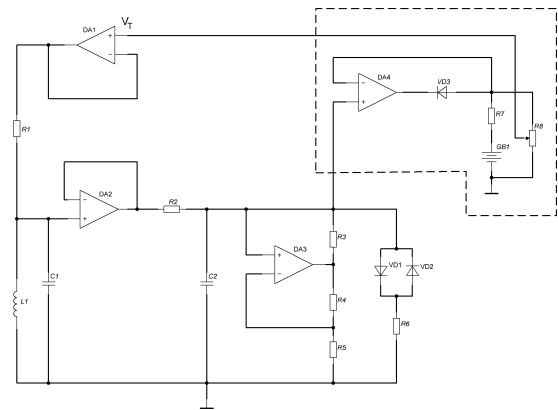


Fig. 9. Modified chaotic Chua's circuit with threshold level controlling circuit (shown in the dotted box).  $V_T$  is the threshold controlled signal

We implement an even more minimal thresholding. Instead of demanding that the  $x$  variable be reset to  $x^*$  if it exceeds  $x^*$ , we only demand this in Eq. (2). This has very easy implementation, as it avoids modifying the value of  $x$  in the nonlinear element  $g(x)$ , which is harder to do. So then all we do is to implement  $dy/dt = x^* - y + z$  instead of Eq. (2), when  $x > x^*$ , and there is no controlling action if  $x < x^*$ . In the circuit, the voltage  $V_T$  corresponds to  $x^*$ .

Fig. 6 – Fig. 8 shows experimental results of the control of chaotic oscillations.

## Conclusions

For the first time was used threshold method for control of chaotic oscillations for modified Chua's chaotic generator. This modified Chua's circuit that generate a chaotic and controlled attractor with a fixed period can be used in modern systems transmitting and receiving information. Number of periodic (controlled) attractor can be used as a keys for masking of information carrier.

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## Управління модифікованою хаотичною схемою Чуа пороговим методом

*Русин В., Прибылова Л., Дмитріу Д.-Г.*

В роботі представлена модифікована хаотична схема Чуа, яка реалізує хаотичну поведінку. Приведена система диференціальних рівнянь, що описує дану схему при певних значеннях системних параметрів та початкових умов, генерує хаотичний аттрактор. Ця схема має простий нелінійний елемент, спроектований так, щоб мати кусково-лінійну характеристику, тобто комбінацію операційного підсилювача з двома діодами, що ввімкнені назустріч один одному. Для реалізації нелінійності, для двох діодів не потрібно окремого джерела живлення, достатньо лише одного двополярного живлення для операційного підсилювача. Приведено схему для дослідження нелінійного елемента та результати комп'ютерного моделювання, тобто вольт-амперну характеристику (ВАХ) при певних значеннях номіналів компонентів схеми. Ця модифікована схема Чуа, яка генерує хаотичний та контрольований аттрактор з фіксованим періодом, може використовуватися в сучасних системах передавання та приймання інформації. Управління хаотичними коливаннями є досить важливим процесом в радіотехніці та телекомунікаціях. Число виділених окремих періодичних (контрольованих) аттракторів може використовуватися як ключі для маскуванню інформаційного носія. За допомогою програмного середовища MultiSim проведено схемотехнічний аналіз і представлено результати моделювання нелінійного елемента та

модифікованої хаотичної схеми Чуа. Досліджена поведінка системи за допомогою чисельного моделювання, використовуючи відомі інструменти нелінійної теорії, такі як хаотичний аттрактор і часові розподіли хаотичних координат. Приведено опис порогового методу для здійснення управління хаотичними коливаннями та представлено результати практичного застосування даного методу до модифікованої хаотичної схеми Чуа. Практичними результатами є виділені 2- та 3-періодні контрольовані орбіти із хаотичного аттрактора.

*Ключові слова:* хаос; Чуа; управління; пороговий метод

## Управление модифицированной хаотичной схемой Чуа пороговым методом

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В работе представлена модифицированная хаотичная схема Чуа, которая реализует хаотическое поведение. Эта схема имеет простой нелинейный элемент, спроектированный так, чтобы иметь кусочно-линейную характеристику. Эта модифицированная схема Чуа, которая генерирует хаотический и контролируемый аттрактор с фиксированным периодом, может использоваться в современных системах передачи и получения информации. Количество периодических (контролируемых) аттракторов может использоваться как ключи для маскировки информационного носителя. С помощью программной среды MultiSim проведен схемотехнический анализ и представлены результаты моделирования нелинейного элемента и модифицированной хаотической схемы Чуа. Исследовано поведение системы с помощью численного моделирования, используя известные инструменты нелинейной теории, такие как хаотичный аттрактор и временные распределения хаотических координат. Приведено описание порогового метода для осуществления управления хаотическими колебаниями и представлены результаты практического применения данного метода к модифицированной хаотической схеме Чуа. Практическими результатами являются выделенные 2- и 3-периодные контролируемые орбиты с хаотического аттрактора.

*Ключевые слова:* хаос; Чуа; управление; пороговый метод