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COMPARATIVE KINETICS OF IONS FOR DIFFERENT MODES OF PARAMETRIC INSTABILITY OF INTENSE LANGMUIR WAVES IN PLASMA

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Nonlinear regimes of one-dimensional parametric instabilities of long-wave plasma waves are considered for the cases when the average plasma field energy density is less (Zakharov's model) or greater (Silin's model) than the plasma thermal energy. The evolution of ion energy distribution is studied.After saturation of the instability, the ion kinetic energy density normalized to the initial field energy density is found to be of the order of the ratio of linear growth rate to the plasma frequency, for the case when the initial field energy far exceeds the plasma thermal energy. In this case, the ion energy distributionis differentfrom the Maxwellian. In the opposite case of hot plasma, the ions acquire a part of initial field energy, which is approximately equal to the half of ratio of initial Langmuir field energy to the plasma thermal energy. At this, the ion kinetic energy distribution is close to the Maxwellian, and it is reasonably to speak about ion temperature. The crossing of ion trajectories in the surrounding of density cavities is a reason of instability quenching in both cases.

KEYWORDS: parametric instability of plasma waves, plasma, Zakharov's model, Silin's model, ion heating

ПОРІВНЯЛЬНА КІНЕТИКА ІОНІВ У РІЗНИХ РЕЖИМАХ ПАРАМЕТРИЧНОЇ НЕСТІЙКОСТІ ІНТЕНСИВНИХ ЛЕНГМЮРОВСКИХ ХВИЛЬ У ПЛАЗМІ

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Аналізується розвиток 1D параметричних нестійкостей довгохвильових ленгмюровских хвиль у випадках, коли середня енергія поля менше (модель Захарова) і більше (модель Силіна) за теплову енергію плазми. Обговорюється характер спектру іонів за енергіями. Показано, що енергія іонів при насиченні нестійкостей виявляється порядку відношення лінійного інкремента до частоти у випадку, коли початкова енергія поля помітно перевищує теплову енергію плазми. Розподіл іонів за енергіями відрізняється від максвеллівського. В умовах гарячої плазми іонам передається частка енергії, рівна половині відношення початкової енергії поля до теплової енергії плазми. При цьому, так як розподіл іонів близький до нормального, можна говорити про температуру іонів. Перетин траєкторій іонів поблизу каверн щільності є причиною зриву нестійкості в обох випадках.

КЛЮЧОВІ СЛОВА:параметрична нестійкість ленгмюрівских хвиль, плазма, модель Захарова, модель Силіна, нагрів іонів

СРАВНИТЕЛЬНАЯ КИНЕТИКА ИОНОВ В РАЗЛИЧНЫХ РЕЖИМАХ ПАРАМЕТРИЧЕСКОЙ НЕУСТОЙЧИВОСТИ ИНТЕНСИВНЫХ ЛЕНГМЮРОВСКИХ ВОЛН В ПЛАЗМЕ

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Анализируется развитие 1D параметрических неустойчивостей длинноволновых ленгмюровских волн в случаях, когда средняя энергия поля меньше (модель Захарова) и больше (модель Силина) тепловой энергии плазмы. Обсуждается характер спектра ионов по энергиям. Показано, что энергия ионов при насыщении неустойчивостей оказывается порядка отношения линейного инкремента к частоте в случае, когда начальная энергия поля заметно превышает тепловую энергию плазмы. Распределение ионов по энергиям отличается от максвелловского. В условиях горячей плазмы ионам передается доля энергии, равная половине отношения начальной энергии поля к тепловой энергии плазмы. При этом, так как распределение ионов близко к нормальному, можно говорить о температуре ионов. Пересечение траекторий ионов вблизи каверн плотности является причиной срыва неустойчивости в обоих случаях.

КЛЮЧЕВЫЕ СЛОВА:параметрическая неустойчивость ленгмюровских волн, плазма, модель Захарова, модель Силина, нагрев ионов

The interest in parametric instability of intensive Langmuir waves, which can be easily excited in the plasma by various sources [1-9], was stipulated, in particular, by the new possibilities in heating of electrons and ions in plasma. The correct methods for description of parametric instability of long-wave plasma waves were developed in the pioneering works of V.P. Silin [10] and V.E. Zakharov [11]. The theoretical concepts of [10] were confirmed by the early numerical experiments on the one-dimensional modelling of parametric decay of plasma oscillations [12] (see also [13, 14] and review [15]).However, the greatest interest has been expressed by experimenters in the mechanism of dissipation of wave energy discovered by V.E. Zakharov. The analytical studies, laboratory-based experiments and numerical simulations performed at an early stage of studying these phenomena have confirmed [16-18] the fact that in some cases during the instability development a significant part of the pump field energy transfers into the energy of short-wave Langmuir oscillations attended with bursts of fast particles [16-27].

In this paper, we compare the models of Silin and Zakharov by the example of one-dimensional description. The choice of one-dimensional approach, as was noted by J. Dawson [28], «…often keeps the main features of the processes, but simplifies their description and leads to a fuller understanding of what the important phenomena are». Of particular interest is the ion heating, so we use in this work the super-particle (or finite-sized particle) description for ions because the account of inertial effects can be significant just at the nonlinear stage of the process [29].

It was observed in [29, 30] that simulation with using of the so-called hybrid model (incorporating one of the Zakharov equations for the high-frequency waves and using particles description for simulation of ion dynamics) demonstrates that fluctuations of ion density are rather significant and accelerate the development of parametric instability. The non-resonant interaction between super-particles-ions and high-frequency plasma oscillations, along with the trapping of ions into the potential wells, formed by these oscillations, leads to instability of the density cavities resulting from the modulation instability.

In paper [30], the hybrid model was compared with Zakharov's hydrodynamic model. Due to higher level of ion density fluctuation, the number of cavities in the hybrid model appears to be significantly greater than in the Zaharov model and their depth is less. Integral characteristics of both models are essentially identical. Note that both the hydrodynamic description within framework of the Zaharov model [30] and description based on the kinetic equations [31], in which there is no non-resonant interactions such as "particle– finite amplitude wave", don't consider the trapping of particles by the wave. As a sequence, the resulting cavities remain stable until the moment when the high frequency plasma field is "burned out" due to the Landau damping – the process that can be better described with using the method of finite-sized particles, as was reasonably pointed out in [30].

Below, we discuss the integral characteristics of the modulation instability developing in cold and hot non-isothermal plasma within framework of hybrid models.

THE MODELS OF PARAMETRIC INSTABILITY

The Silin hybrid model

When the intensity of external electric field is much greater than the specific thermal energy of plasma electrons $W = |E_0|^2/4\pi >> n_0 T_e$, it is reasonable to explore the approach presented by V.P. Silin [32] .

Let consider a one-dimensional plasma system, where an intense plasma wave with the wavelength λ_0 and frequency ω_0 is excited by an external source. This intense wave will be referred to as the pumping wave. Since the parametric instability results in the growth of oscillations with rather small wavelength $\lambda << \lambda_0$, the pumping wave can be considered as spatially uniform within the region of interaction:

$$E = -\frac{i}{2}\left(|E_0|\exp(i\omega_0 t + i\phi) - |E_0|\exp(-i\omega_0 t - i\phi)\right), \quad (1)$$

where $|E_0|$ and ϕ are the slowly varying wave amplitude and phase correspondingly, ω_0 is external wave frequency, n_0 and T_e are the density and temperature of plasma electrons. Charged particles of plasma oscillate under the action of the electric field and their velocities can be written as $u_{0\alpha} = -e_\alpha |E_0|/m_\alpha \cdot \omega_0 \cos\phi = -\omega_0 a \cdot \cos\phi$, where $a = e_\alpha |E_0|/m_\alpha$ is the oscillation amplitude.

The equations, governing the nonlinear dynamics of the parametric instability of intensive plasma wave, were derived in [33]. The equations for high-frequency plasma field spectrum modes $E = \sum_n E_n(t)\cdot\exp(ink_0 x)$ (plasma electrons are considered as fluid and described by hydrodynamic equations) have a form

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0^2}{2\omega_0}E_n - \frac{4\pi\omega_{pe}\nu_{i,n}}{k_0 n}J_1(a_n)\cdot\exp(i\phi) - i\frac{\omega_0}{2en_0}\sum_m \nu_{i,n-m}\cdot\left[E_{-m}^* J_2(a_{n-m})e^{2i\phi} + E_m\cdot J_0(a_{n-m})\right] = 0 . \quad (2)$$

Here $\omega_{pe} = \sqrt{4\pi e^2 n_0 / m_e}$ is the background electron plasma frequency, e and m_e are the mass and the magnitude of the charge of an electron, M is the mass of an ion, $E_n = |E_n|\cdot\exp(i\psi_n)$ is a slowly varying complex amplitude of the electric field of electron plasma oscillations, which wave number is $k_n = nk_0$, $k_0 = 2\pi/L$, where L is a characteristic dimension of the plasma system, $\nu_i = \sum_n \nu_{i,n}(t)\cdot\exp(ink_0 x)$ is the ion charge density, $a_n = a\cdot n$, n,m are integers which are not equal to zero and ± 1.

The motion equations for ion super-particles can be written as follows

$$\frac{d^2 x_s}{dt^2} = \frac{e}{M}\sum_n \overline{E}_n \cdot \exp(ink_0 x_s) , \quad (3)$$

and the ion density can be determined from

$$n_{i,n} = \nu_{i,n}/e = n_0 \cdot \frac{k_0}{2\pi}\int_{-\pi/k_0}^{\pi/k_0} \exp[-ink_0 \cdot x_s(x_{s0},t)]\cdot dx_{s0} . \quad (4)$$

The slowly varying electric field strength $\bar{E}_n$, acting on the ions, is equal

$$\bar{E}_n = -\frac{4\pi i v_{i,n}}{k_0 n}\left[1 - J_0^2(a_n) + \frac{2}{3}J_2{}^2(a_n)\right] + \frac{1}{2}J_1(a_n)[E_n \cdot e^{-i\phi} - E_{-n}^* \cdot e^{i\phi}] - \\ -\frac{ink_0}{16\pi e n_0}J_0(a_n)\sum_m E_{n-m}E_{-m}^* - \frac{ik_0 J_2(a_n)}{16\pi e n_0}\sum_m (n-m)\left[E_{n-m}E_m \cdot e^{-2i\phi} + E_{m-n}^* E_{-m}^* \cdot e^{2i\phi}\right]. \quad (5)$$

The equation for uniform component of the electric field $E_0 = |E_0| \cdot \exp(i\phi)$ can be written as

$$\frac{\partial E_0}{\partial t} = -\frac{\omega_0}{2en_0}\sum_m v_{i,-m} \cdot \left[E_{-m}^* J_2(a_m) \cdot e^{2i\phi} + E_m J_0(a_m)\right]. \quad (6)$$

Note, that the values with subscripts of different signs are independent. In Eqs.(2)-(6), we used the formula [34]

$$\exp(ia \cdot \sin\Phi) = \sum_{m=-\infty}^{\infty} J_m(a) \cdot \exp(im\Phi), \quad (7)$$

where $J_m(x)$ is the Bessel function.

The normalized frequency shift $\Delta = (\omega_{pe}^2 - \omega_0{}^2)/2\delta\omega_{pe}$ reaches the value of $(m_e/2M)^{1/3} J_1{}^{2/3}(a_n)$ for a mode with the maximal growth rate of parametric instability δ [32]

$$\delta/\omega_{pe} = \frac{i}{\sqrt[3]{2}}\left(\frac{m_e}{M}\right)^{1/3} J_1^{2/3}(a_n). \quad (8)$$

The Zakharov hybrid model

As shown in [33], Eqs. (2)-(6) are in agree with equations, obtained in [35] after following substitutions: $(\omega_{pe}^2 - \omega_0{}^2)/2\omega_0 \to (\omega_{pe}^2 - \omega_0{}^2 + k^2{}_0 n^2 v_{Te}^2)/2\omega_0$ and $E_0 \to -iE_0$, $E_0^* \to iE_0^*$ under condition $a_n << 1$, when $J_1(a_n) \approx a_n/2$, $J_0(a_n) \approx 1$, $J_2(a_n) \approx a_n^2/8$

$$\frac{\partial E_n}{\partial t} - i\frac{\omega_{pe}^2 - \omega_0{}^2 + k^2{}_0 n^2 v_{Te}^2}{2\omega_0}E_n - i\frac{\omega_0}{2n_0} \cdot \left[n_{i,n}E_0 + \sum_{m\neq 0} n_{i,n-m}E_m\right] = 0. \quad (9)$$

The slowly varying electric field amplitude in this case takes the form

$$\bar{E}_n = -\frac{ik_0 ne}{4m\omega_{pe}^2}\left(E_n E_0^* + E_0 E_{-n}^* + \sum_{m\neq 0,n} E_{n-m}E_{-m}^*\right), \quad (10)$$

that enables description of ions using the super-particle method with use of Eqs.(3)-(4). The pump wave amplitude E_0 is governed by equation

$$\frac{\partial E_0}{\partial t} - i\frac{\omega_0}{2n_0} \cdot \sum_m n_{i,-m}E_m = 0. \quad (11)$$

The growth rate of parametric instability in this case is [35]

$$\delta/\omega_{pe} = \sqrt{\frac{1}{2}\frac{|E_0|^2}{4\pi n_0 T_e}\frac{m_e}{M}} = \sqrt{\frac{1}{2}\frac{W}{n_0 T_e}\frac{m_e}{M}}. \quad (12)$$

The growth rates (8) and (12) are defined as usually from the dynamics of small field perturbations $E(\mathrm{t}) = E(0)\exp(\delta \cdot t)$ at the initial (linear) stage of the parametric instability.

NUMERICAL SIMULATION. PROBLEM FORMULATION

The purpose of this paper is to clarify the characteristics of the dynamics of modulation instability both for the cases of non-isothermal hot and cold plasma by using of the hybrid models.

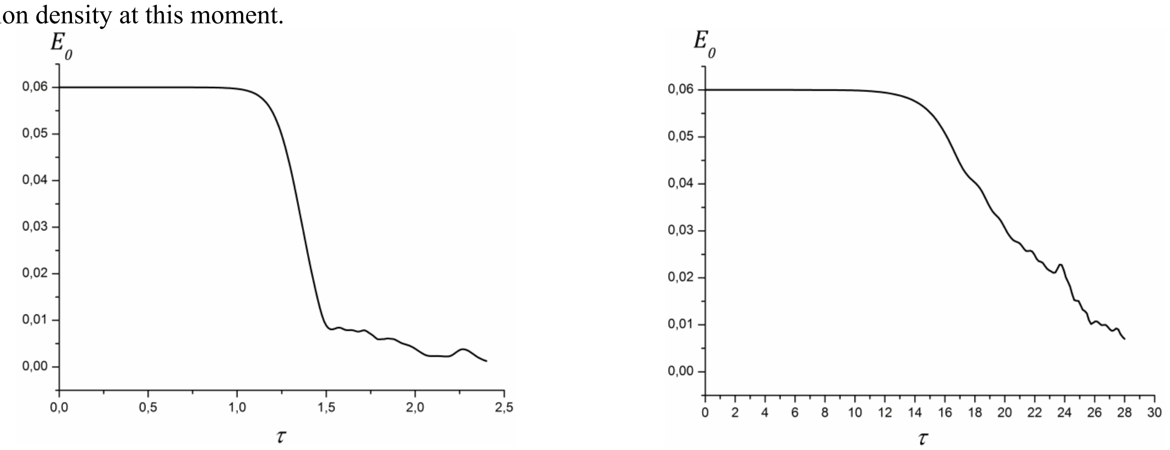
Below, we have used the following initial conditions and parameters unless otherwise specified in the text. The number of super-particles, simulating the dynamics of ions is $0 < s \le S = 20000$, the number of spectrum modes is $-N < n < N$, $N = S/100 = 200$, $a_0(0) = ek_0E_0(0)/m_e\omega_{pe}^2 = 0.06$, $d\xi_s/d\tau|_{\tau=0} = v_s|_{\tau=0} = 0$, $\xi_s = k_0 x_s$, $\tau = \delta \cdot t$. Note that the linear growth rates for the Silin and Zakharov models are essentially different (see Eq. (8) and (12)). So the time scales for these two models are different too.The initial amplitude of HF field is determined from $e_n|_{\tau=0} = e_{n0} = (0.5 + g) \cdot 10^{-4}$, where $g \in [0;1]$ is a random value, the initial phases of spectral modes $\psi_n|_{\tau=0}$ are also randomly distributed in the interval $0 \div 2\pi$, $\Delta = 1$, $m_e/M = 10^{-3}$. The ions are supposed to be uniformly distributed over the interval $-1/2 < \xi < 1/2$.

The program, which implements a mathematical model of the problem under consideration, was developed with the use of JCUDA technology. JCUDA technology provides interface between CUDA (Compute Unified Device

Architecture) and Java application. CUDA is a parallel computing platform and programming model created by NVIDIA. CUDA enables scientists to utilize the extreme computational power available on modern GPUs.

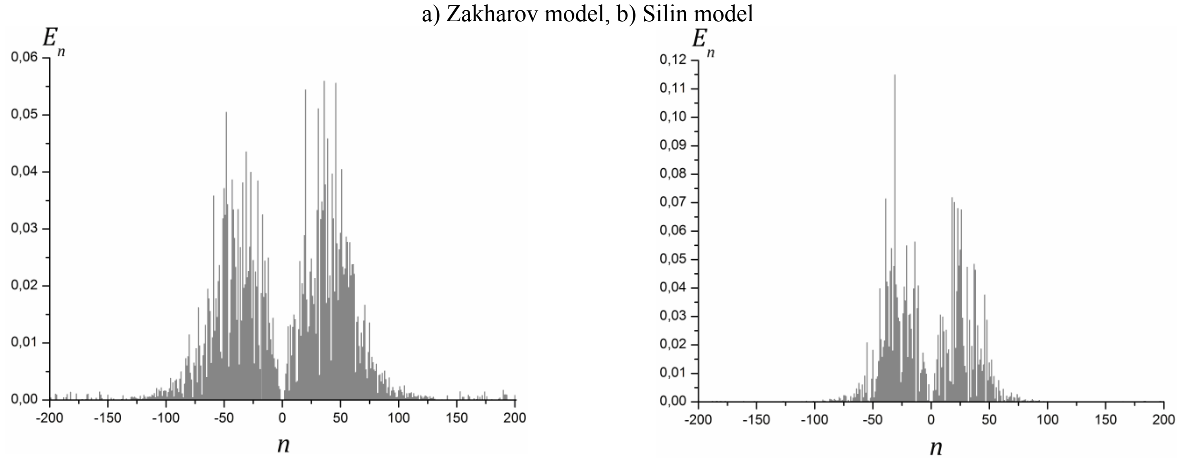
NUMERICAL SIMULATION. RESULTS

As shown in Fig. 1, the pump wave amplitude decreases with development of the parametric instability. Denote the time, when the total kinetic energy of ions reaches a maximum, as $\tau_{\max}$. Fig. 2 demonstrates the spectrum of HF plasma oscillations at the moment $\tau_{\max}$ calculated for both models. Correspondingly, Fig. 3 presents the spatial spectrum of ion density at this moment.



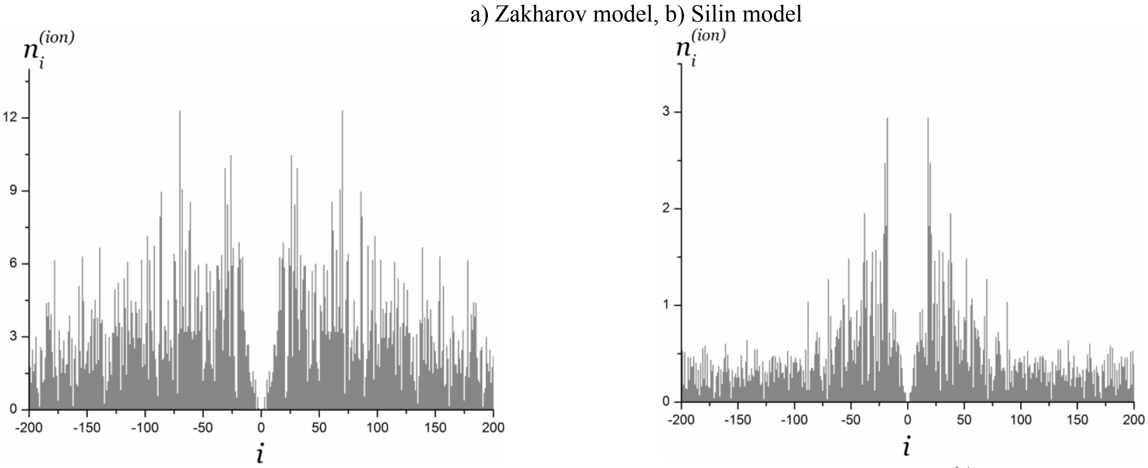
a) b)

Fig.1. Evolution of pump wave amplitude
a) Zakharov model, b) Silin model



a) b)

Fig. 2. The spectrum of HF plasma oscillations at the moment $\tau_{\max}$
a) Zakharov model, b) Silin model



a) b)

Fig. 3. The spatial spectrum of ion density at the moment $\tau_{\max}$
a) Zakharov model, b) Silin model

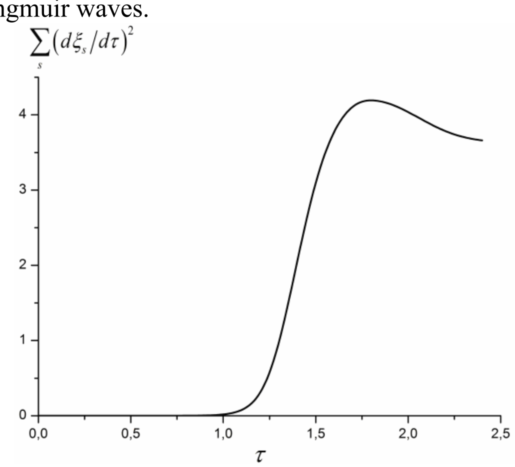
The total kinetic energy of the ions, located on the length of the pump wave, can be expressed as the sum of squared dimensionless velocities $I=\sum_s\left(d\xi_s/d\tau\right)^2$ (evolution dynamics of this value is shown in Fig. 4) and the number of super-particles S as

$$\frac{2\pi}{k_0}\left[\frac{1}{2}n_0 M\left\langle\left(dx_s/dt\right)^2\right\rangle\right]=\frac{4\pi^2\delta^2 M n_0}{2k_0{}^2 S}\frac{2\pi}{k_0}I\ , \tag{13}$$

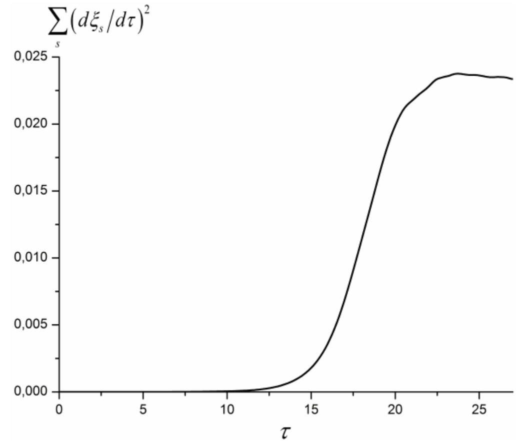
where $\langle(dx_s/dt)^2\rangle$ is the ensemble average. The ratio of the ion kinetic energy to the initial energy of intense long-wavelength Langmuir wave scan be written as

$$\frac{\mathrm{E}_{kin}}{W_0}=\frac{2\pi}{k_0}\left(\frac{1}{2}n_0 M\left\langle\left(dx_s/dt\right)^2\right\rangle\right)\Big/\left(\frac{2\pi}{k_0}\frac{|E_0|^2}{4\pi}\right)=2\pi^2\frac{I}{a_0{}^2 S}\cdot\frac{M}{m}\cdot\frac{\delta^2}{\omega_{pe}^2}\ , \tag{14}$$

where E_{kin} is the density of the ion kinetic energy, $W_0=|E_0|^2/4\pi$ is the initial energy density of long-wavelength Langmuir waves.



a)



b)

Fig.4. Evolution of the quadratic sum $\sum_s(d\xi_s/dt)^2$

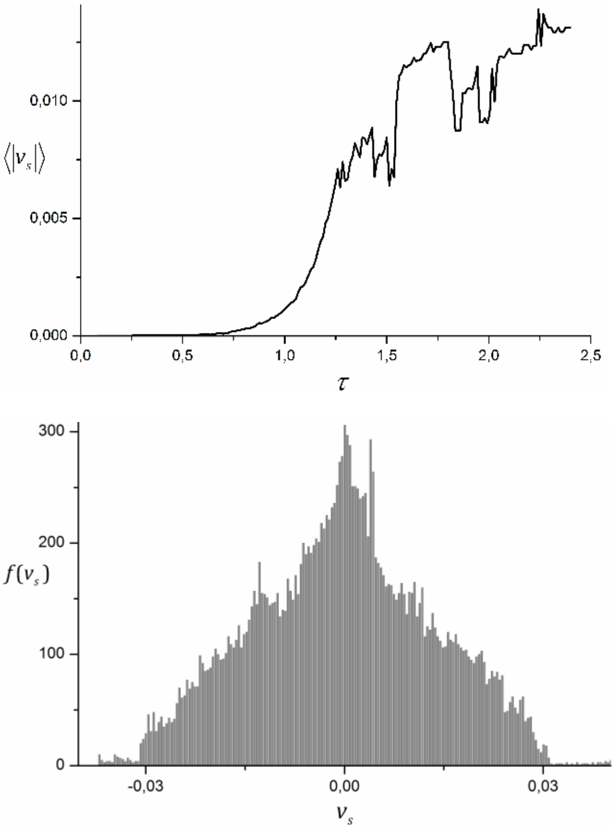
a) Zakharov model, b) Silin model

The ratio of time scales for these two models is of the order of $(m_e/M)^{1/6}(W/n_0T_e)^{1/2}$. Considering this, it was found that the kinetic energy of ions to be of the same order in both models. The ratio of ion kinetic energy to the initial energy of long-wave oscillations occurs equal to $\mathrm{E}_{kin}/W_0\sim\delta/\omega_{pe}$ for Silin's model and $\mathrm{E}_{kin}/W_0\simeq 0.5\cdot W_0/n_0T_e$ for Zakharov's model. This means that in the Silin model, the ions derive a portion of field energy of the order of δ/ω_{pe} (Fig. 4b). This effect was predicted in [19] and confirmed in [29]. A portion of transferred energy in Zakharov's model is of the order of W_0/n_0T_e (Fig. 4a).

A normal relation between half-width of the Maxwellian velocity distribution function and thermal velocity is $\bar{v}=1.18v_T$. It can be shown that the root-mean-square value of ion velocity at the moment when the instability is saturated is equal $\sqrt{<v_s^2>}=\sqrt{I_s/S}$ (in relative units) (see Fig. 5). If this value is of the order of $0.85\cdot\bar{v}$, i.e. $\sqrt{I_s/S}\approx 0.85\cdot\bar{v}$, then we can suppose that ion velocity distribution function is close to the Maxwellian and one can say about ion temperature. If $\sqrt{I_s/S}>0.85\cdot\bar{v}$, than the distribution function has a non-Maxwellian "tail" of fast particles. It is easy to verify that the ion velocity distribution function in hot plasma (the Zakharov model) is close to the Maxwellian and ion temperature is well defined an can be estimated as $T_i\sim W_0^2/n_0^2T_e$. For the case of cold plasma the ion velocity distribution function contains a significant part of fast particles as was observed in experiments [36]. Actually, the ratio of ion kinetic energy to the initial energy of the plasma wave field (14) is

$$\frac{\mathrm{E}_{kin}}{W_0}\approx 0.27\cdot I\cdot\left(\frac{M}{m}\right)\cdot\delta^2/\omega_{pe}^2\,. \tag{15}$$

For the case of hot electrons (the Zakharov model), where $m_e/M=2000^{-1}$, $\delta/\omega_{pe}=3.5\cdot10^{-3}$, $I\approx 4.584$ one can obtain $\mathrm{E}_{kin}/W_0\approx 3\cdot10^{-2}$. For the case of cold plasma (the Silin model), where $m_e/M=2000^{-1}$, $\delta/\omega_{pe}=0.034$, $I\approx 0.08$, this ratio is equal $\mathrm{E}_{kin}/W_0\approx 5\cdot10^{-2}$.



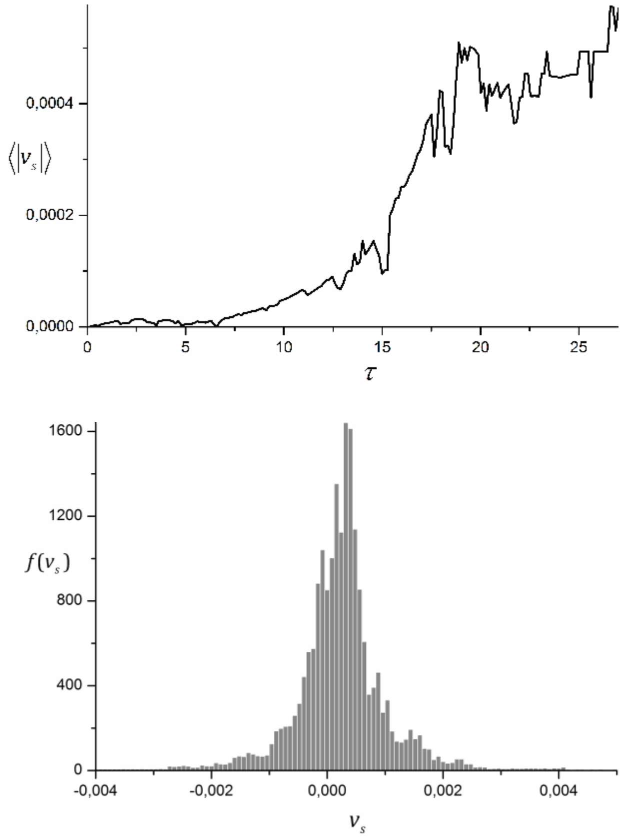


Fig. 5. The evolution of normalized mean-square velocity of ions (top) and ion velocity distribution function (bottom) at the moment $\tau_{\max}$
a) Zakharov model, b) Silin model

We can adjust parameters of the normal distribution in such a way as to minimize the number of particles outside of this distribution. The ratio of total kinetic energy of these "non-Maxwellian" particles to the total kinetic energy of ions is of the order of 0.14 for the case of hot plasma (the Zakharov model) and of the order of 0.58 for the case of hot plasma (the Silin model). Thus, theionkineticenergydistributionforthehotplasmaisclosetoMaxwellianand the total kinetic energy of fast ions is comparable with the total kinetic energy of the normally distributed ions. The origin of fast particles is stipulated not only by the destruction of cavities due to absorption of short wave RF field (nucleation), but also by the earlier process of exclusion of ions from cavities by RF field.

CONCLUSIONS

The hybrid models describe the nonlinear stage of the parametric instability in plasma more accurately than the models based on hydro dynamic or kinetic description due to considering the effects of ions trapping and trajectories crossing resulting from large inertia of the heavy particles.

The instability in the Zakharov model develops faster that in the Silin model, since all unstable modes of the spectrum have approximately the same growth rate (see Eq. (12)). In the Silinmodel, the maximum growth rate displaces to the short wavelength domain with the development of the instability.

In hot plasma a fraction of the field energy, which is transferred to the ions, is proportional to the ratio of the field energy density to the plasma thermal energy density W_0/n_0T_e. In cold plasma, a fraction of the field energy, which is transferred to the ions, is of the order of δ/ω_{pe}, or that the same, is proportional to $(m_e/M)^{1/3}$.

The kinetic energy distribution of ions for the Zakharov model is close to the Maxwellian, and we can estimate the ion temperature $T_i \sim W_0^2 / n_0^2 T_e$. The kinetic energy distribution of ions for the Silin model is significantly different from Maxwellian and contains a large fraction of fast particles.

Let us note in conclusion, that the dynamics of parametric instability for Silin's and Zakharov's models are similar in appearance first of all due to the similarity of the input equations. Note also, that the number of particles $S = 20000$, used for one-dimensional simulation, corresponds to the number of particles near $10^{12} - 10^{13}$ for three-dimensional case, that is in agree with the condition of most of the experiments.

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