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Resonant modes in the system of two wide interacting Josephson junctions

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Zero-field steps and Fiske steps in IV-characteristics of the stack of two inductively coupled wide Josephson junctions with normal edges were investigated numerically. It was found that inductive coupling of junctions led to splitting of each of the step in two steps. Dependences of frequencies of steps on the parameter which characterizes inductive coupling of junctions obeyed the relation obtained for frequencies of inductively coupled resonance contours. Dependences of amplitudes of split Fiske steps on applied external magnetic field were calculated and compared with the theory.

Keywords: Josephson junctions, high-temperature superconductors, Fiske steps, resonant modes.

Сходинок нульового поля та сходинок Фіске на вольт-амперних характеристиках пачки з двох індуктивно з'єднаних між собою широких джозефсонівських контактів з нормальними краями були досліджені чисельно. Знайдено, що індуктивний зв'язок привів до розщеплення кожної сходинок на дві сходинок. Залежності частот сходинок від параметру, який характеризує індуктивний зв'язок між контактами, підкоряються співвідношенню, що було отримано для частот індуктивно з'єднаних резонансних контурів. Розраховані та порівняні з теорією залежності амплітуд розщеплених сходинок Фіске від прикладеного зовнішнього магнітного поля.

Ключові слова: джозефсонівські контакти, високотемпературні надпровідники, сходинок Фіске, резонансні моди.

Были исследованы численно ступеньки нулевого поля и ступеньки Фиске на вольт-амперных характеристиках пачки, состоящей из двух индуктивно связанных между собой широких джозефсоновских переходов с нормальными краями. Было найдено, что индуктивная связь переходов привела к расщеплению каждой ступеньки на две. Зависимости частот ступенек от параметра, который характеризует индуктивную связь переходов, подчиняются соотношению, которое было получено для частот индуктивно связанных резонансных контуров. Рассчитаны и сравнены с теорией зависимости амплитуд расщеплённых ступенек Фиске от приложенного внешнего магнитного поля.

Ключевые слова: джозефсоновские переходы, высокотемпературные сверхпроводники, ступеньки Фиске, резонансные моды.

Introduction

The problem of the adequate description of IV-characteristics of intrinsic Josephson junctions in high-temperature superconductors includes the development of different types of the interaction between junctions such as the inductive coupling of intrinsic junctions [1, 2] and resistive shunting of neighbor junctions [3, 4]. On the other hand, these inductive and resistive couplings produce new effects which can be responsible for the appearance of coherent emission in the stack of intrinsic Josephson junctions without applied external magnetic field [5]. It was found [5] that coherent emission appeared in the vicinity of steps in IV-characteristic. The origin of this effect is the subject of the contemporary investigations [6]. We found recently that the IV-characteristic of the wide Josephson junction with the normal edges revealed so-called zero-field steps [7]. These zero-field steps appear in the IV-curve when the edges of the stack are in the normal state due

to diffusion of the oxygen out of the stack. The physical origin of the formation of the zero-field step consists in the interaction of the ac Josephson current with excitations of the electromagnetic field which are introduced into the junction through normal edges [8]. Voltages which correspond to these steps are equal to

$$V_s = \frac{\Phi_0 \bar{c} s}{D}, \quad (1)$$

where D is the width of the junction, Φ_0 is the quantum of magnetic flux, $\bar{c}$ is the velocity of light in the junction and s is an integer. These voltages correspond to even Fiske steps:

$$V_p = \frac{\Phi_0 \bar{c} p}{2D}, \quad (2)$$

where p is an integer, so $p = 2s$. Fiske steps appear when external magnetic field is applied in parallel to the junction [9]. Their origin is connected with the interaction of the

ac Josephson current and the standing-wave electromagnetic fields in the junction. However, steps which appear due to the process of the propagation and the reflection of electromagnetic excitations are observed without applied external magnetic field. Therefore they are called as zero-field steps.

In the present paper zero-field steps in IV-characteristics of the stack of two Josephson junctions with normal edges are investigated numerically at different values of the parameter which characterizes inductive coupling between junctions. Inductive coupling is introduced similarly to the model developed in Refs. [1, 2]. We show that the coupling leads to the split of zero-field step. We also investigate the influence of coupling on the first and the second Fiske steps.

The model

The stack of wide Josephson junctions is presented in Fig. 1a, and the high-frequency scheme of the stack is shown in Fig. 1b. Each of the wide junctions is divided into n 'elementary junctions' with critical currents $I_{ci,j}$, where $i=1,2$ is the number of the wide junction in the stack, $j=1\dots n$ is the number of the 'elementary junction'. It is supposed that the dc bias current I_b feeds each of the 'elementary junctions' independently. For the use of visual aids, two wide junctions in the high-frequency scheme are fully separated, though in the equivalent high-frequency scheme one can connect each pair of inductances L_f in 'elementary cells' into one common mutual inductance and connect also each 'elementary junction' in the upper wide junction with the corresponding 'elementary junction' in the lower wide junction by the current line. According to the resistively and capacitively shunted model [10], each of the 'elementary junctions' has the resistance $R_{ei,j}$, the capacitance C and the source of the Josephson current $I_{Ji,j} = I_{ci,j} \sin \varphi_{i,j}$ with $\varphi_{i,j}$ is the difference of the phase of the order parameter across the junction (Fig. 1b). We assume that the critical voltage is equal to $V_c = I_{ci,j} R_{ei,j} = \text{const}$ for all 'elementary junctions'. 'Elementary junctions' are divided by the distance $\zeta = \sqrt{cCL}$, where L is the inductance of the 'elementary cell' between junctions. To model normal edges of the stack, each edge of the wide junctions is connected to the contour which contains the resistance $R_{ei,j}$, the capacitance $C_{ei,j}$ and the inductance $L_{ei,j}$ (Fig. 1b). These additional contours introduce electromagnetic excitations in the stack. To show main features of our model, we assume the simplest case when $R_{ei,j} = \text{const.}$, $C_{ei,j} = \text{const.}$ and $L_{ei,j} = \text{const.}$, i.e. all resistances $R_{ei,j}$ are the same for all edges etc. Then the full system of equations which describe the high-frequency scheme of the whole stack of junctions with boundary conditions include current conservation conditions for junctions, Kirchhoff rules and flux quantization conditions:

$$\frac{\Phi_0 C_{ei,j}}{2\pi} \frac{d^2 \varphi_{i,j}}{dt^2} + \frac{\Phi_0}{2\pi R_{ei,j}} \frac{d \varphi_{i,j}}{dt} + I_{ci,j} \sin \varphi_{i,j} = I_b - I_{i,j}^R + I_{i,j+1}^R, \quad i=1,2, \quad j=2\dots n-1, \quad (3)$$

$$\frac{\Phi_0 C_{ei,n}}{2\pi} \frac{d^2 \varphi_{i,n}}{dt^2} + \frac{\Phi_0}{2\pi R} \frac{d \varphi_{i,n}}{dt} + I_{ci,n} \sin \varphi_{i,n} = I_b - I_{i,n}^R + \frac{dq_{i,n+1}}{dt}, \quad i=1,2, \quad j=1, \quad (4)$$

$$\frac{\Phi_0 C_{ei,n}}{2\pi} \frac{d^2 \varphi_{i,n}}{dt^2} + \frac{\Phi_0}{2\pi R} \frac{d \varphi_{i,n}}{dt} + I_{ci,n} \sin \varphi_{i,n} = I_b - I_{i,n}^R + \frac{dq_{i,n+1}}{dt}, \quad i=1,2, \quad j=n, \quad (5)$$

$$L_{ei,j} \frac{d^2 q_{i,j}}{dt^2} + R_{ei,j} \frac{dq_{i,j}}{dt} + \frac{q_{i,j}}{C_{ei,j}} = \mp \frac{\Phi_0}{2\pi} \frac{d \varphi_{i,j \pm 1}}{dt}, \quad i=1,2, \quad j=0, n+1, \quad (6)$$

$$LI_{1,j}^R - L_f I_{2,j}^R + \left[\Phi_e + \frac{\Phi_0}{2\pi} (\varphi_{1,j-1} - \varphi_{1,j}) \right] = 0, \quad j=2\dots n, \quad (7)$$

$$-L_f I_{1,j}^R + LI_{2,j}^R + \left[\Phi_e + \frac{\Phi_0}{2\pi} (\varphi_{2,j-1} - \varphi_{2,j}) \right] = 0, \quad j=2\dots n, \quad (8)$$

where $I_{i,j}^R$ is the current in the loop between two 'elementary junctions', Φ_e is magnetic flux created by the external applied magnetic field, L_f is the mutual inductance between two adjacent cells of the 'elementary stack'. Note that additional contours at edges have numbers $j=0$ and $j=n+1$ for each wide junction (see Eq. (6)).

Now we describe the set of parameters which was defined for the solution of the described above system of equations. At first, the width D , the length W , density of the critical current J_c , critical voltage V_c and inductance of the wide junction per unit of length L_{ul} were set the same for all wide junctions. The length of the junction W was introduced formally because our model was one-dimensional. Then wide junctions were divided into n cells with the length $\zeta = D/n$, so the

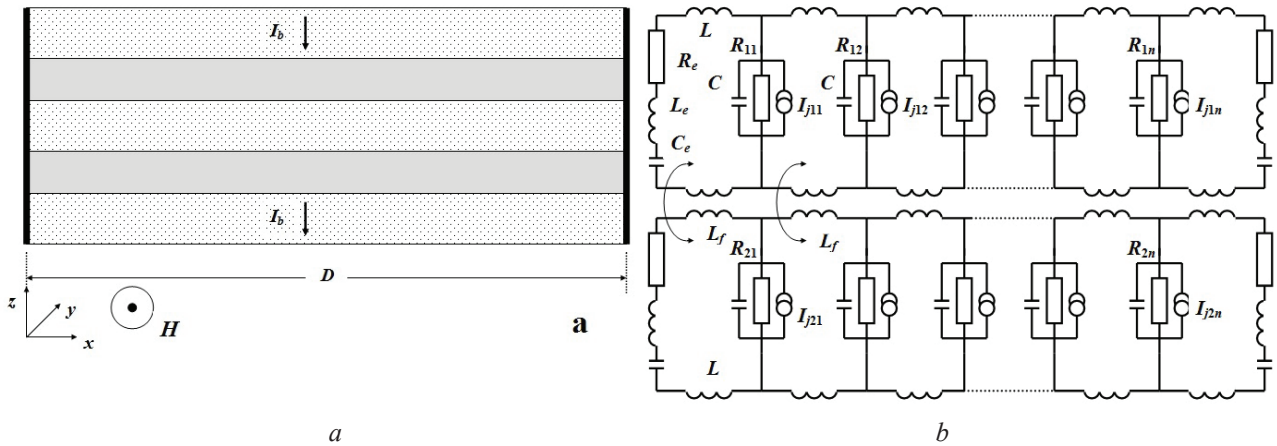


Fig. 1. (a)- the stack of two wide Josephson junctions. Insulator is shaded. Black solid lines at ends of the stack symbolize the $R_e L_e C_e$ -shunting. (b)- the electrical scheme of the stack of two inductively coupled junctions.

critical current and the resistance of each ‘elementary junction’ were calculated as $I_{c,ij} = I_c = J_c \cdot W \cdot \zeta$ and $R = V_c / I_c$. The inductance of each ‘elementary cell’ and the capacitance of the ‘elementary junction’ were calculated as $L = L_{ul} \cdot \zeta$ and $C = \zeta^2 / (\bar{c}^2 L)$ [11, 12]. The value of mutual inductance between ‘elementary junctions’ in the stack was defined as $L_f = \alpha L$, where α is dimensionless parameter. Then Eqs. (3)-(8) were solved for different bias currents. IV-characteristics were obtained in calculations. The normalized to the quantity of wide junctions

voltage over the stack was calculated as

$$V = \frac{\Phi_0}{2\pi} \frac{1}{2n} \left\langle \sum_{j=1}^n \left(\sum_{i=1}^2 \frac{d\varphi_{i,j}}{dt} \right) \right\rangle, \text{ where the sign } \langle \dots \rangle$$

means full averaging over the large interval of time [7, 10]. Values of parameters were chosen as follows: $D = 70 \cdot 10^{-6}$ m, $W = 300 \cdot 10^{-6}$ m, $J_{ctot} = 10^5$ A/m², $V_c = 4.74$ mV, $L_{ul} = 7 \cdot 10^{-9}$ H/m, $\bar{c} = 6.708 \cdot 10^7$ m/sec [5], $\beta_c = 68.4$, $n = 20$. The Josephson depth of penetration of magnetic field in the single junction was equal to $\lambda_J = 55 \cdot 10^{-6}$ m. For boundary conditions we chose values of $R_{eij} = 600$ Ohm, $L_{eij} = 0.23$ pH, $C_{eji} = 0.011$ pF for both junctions with $i = 1, 2$ and both boundaries with $j = 0$ and $j = n+1$.

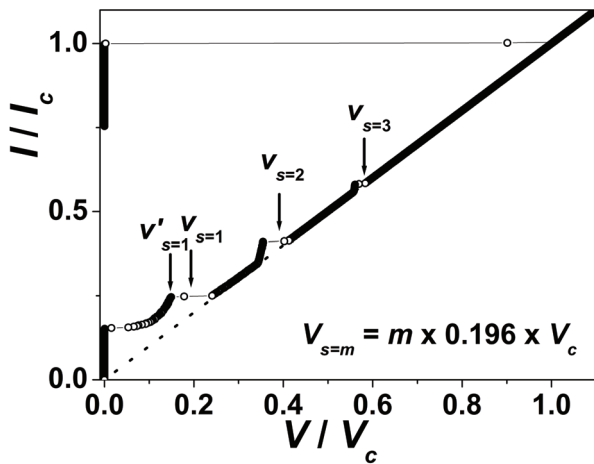


Fig. 2. Zero-field steps in the normalized IV-curve of the single wide Josephson junction. The interval between steps is $V_s = 0.199 \cdot V_c$. Positions of voltages $v_s = V_s / V_c$ of zero-field steps are marked by arrows. The voltage $v'_{s=1} = V'_{s=1} / V_c$ corresponds to the highest visible value of the current in the first step. The dotted line $I_b/I_c = V/V_c$ is the Ohm law which determines the hysteretic branch. Parameters of the junction (see the section 2): $D = 150 \cdot 10^{-6}$ m, $\lambda_J = 36 \cdot 10^{-6}$ m, $W = 300 \cdot 10^{-6}$ m, $L_{ul} = 8 \cdot 10^{-9}$ H/m, $n = 30$, $J_c = 10^5$ A/m², $V_c = 4.736$ mV, $L_{e0} = L_{en+1} = 3.7 \cdot 10^{-13}$ H, $R_{e0} = R_{en+1} = 600$ Ohm, $C_{e0} = C_{en+1} = 1.5 \cdot 10^{-14}$ F.

Results and Discussion

Steps in IV-characteristics obtained in the range of the resistively shunted model have some particularities. As an example, let us consider the IV-curve of single wide Josephson junction which contains zero-field steps (Fig. 2) [7]. All of them are situated in intervals of jumps of voltages which appear due to the instability of the IV-curve (voltages at the step and at the hysteretic branch correspond to the same value of the current). Experiments on junctions with the overlap geometry [13] prove this result. Therefore, instead of V_s , we consider the abscissa V'_s of the highest point of the visible part of the step (see Fig. 2). It is possible if we are interesting not in the exact value of V_s but in the change of V_s in some process (like the dependence of V_s on α) supposing that the difference $V_s - V'_s$ remains constant.

IV-characteristics of the stack of junctions are shown in Fig. 3a,b for $\alpha = 0$ and $\alpha = 0.3$, correspondingly. The step is seen at $V'_{s=1} = 0.392 \cdot V_c$ in Fig. 3a, whereas in Fig. 3b there are the zero-field step at $V'_{s=1,d} = 0.335 \cdot V_c$ and the zero-field step at $V'_{s=1,u} = 0.474 \cdot V_c$. Two wide junctions in the stack do not interact with each other at $\alpha = 0$.

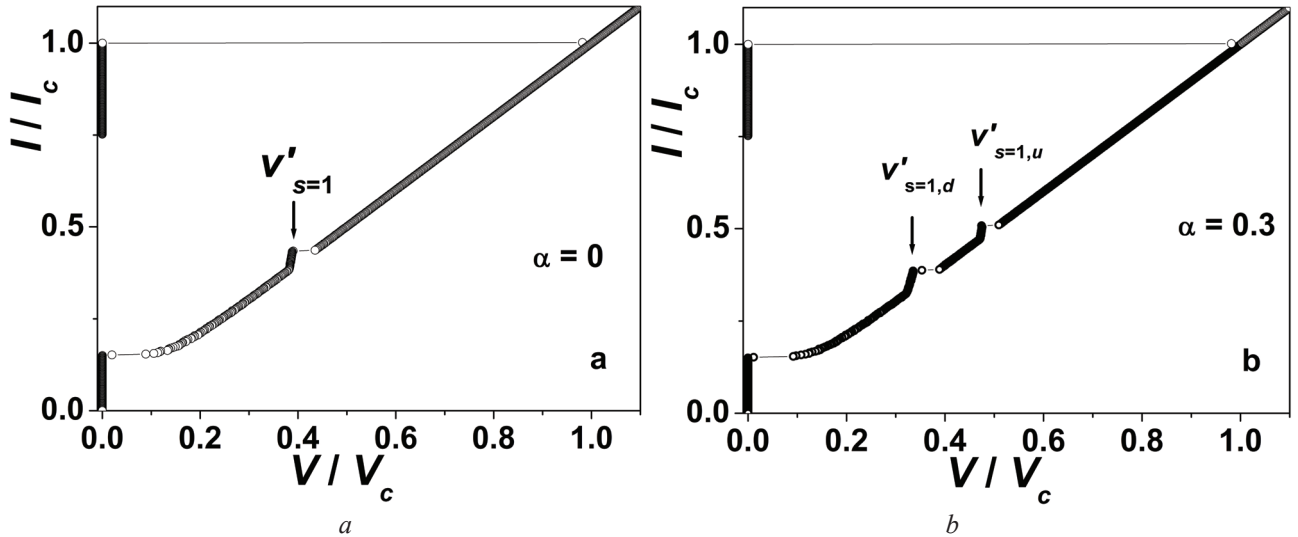


Fig.3. IV-characteristics of the stack of two wide junctions without the inductive coupling of junctions (a) and with inductive coupling of junctions (b). The parameter of coupling is equal to $\alpha = 0$ for (a) and $\alpha = 0.3$ for (b). Arrows mark the value of $v'_s = V'_s/V_c$ in (a) and $v'_{s,d} = V'_{s,d}/V_c$, $v'_{s,u} = V'_{s,u}/V_c$ in (b).

However, at $\alpha = 0.3$ there is inductive coupling between junctions. We can conclude that the inductive interaction leads to the split of the zero-field step into two steps. This conclusion is in agreement with the well-known split of resonant frequencies of two inductively coupled resonant contours. In Fig. 4 positions of zero-field maxima are plotted as a function of the parameter α . These plots can be approximated with the good accuracy (solid lines in Fig. 4) by functions

$$V'_{s=1,d} = \frac{V'_{s=1}}{\sqrt{1+\alpha}} \text{ and } V'_{s=1,u} = \frac{V'_{s=1}}{\sqrt{1-\alpha}}, \quad (9)$$

which are well known from the theory of inductively coupled resonance contours. We would like to emphasize that voltages $V'_{s=1,d}$ and $V'_{s=1,u}$ correspond to some

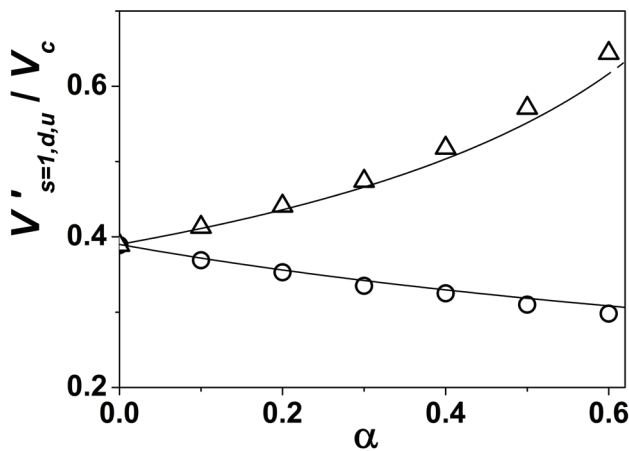


Fig. 4. Dependences of $V'_{s=1,d}/V_c$ (circles) and $V'_{s=1,u}/V_c$ (triangles) on the parameter α . Solid lines are approximations by Eqs. (9).

resonant frequencies of the stack of two interacting wide junctions but they do not correspond to any resonant frequencies of the separate wide junctions. This effect is quite similar to the formation of frequencies of normal vibrations in the system of interacting oscillators and, as far as we know, it is obtained here for the first time.

The found effect of the split of the zero-field step in the stack of two interacting wide junctions can be generalized to the stack which consists of many junctions. In this case the interval of voltages of zero-field steps forms a band, and voltages inside this band correspond to frequencies of vibration modes in the whole stack. Bands can overlap each other. To investigate the formation of bands one should apply another model for 'elementary junctions', because in the ranges of the resistively and capacitively shunted model the band is situated in the interval of the jump of voltage. However, in the present paper we investigate the interval $0 \leq \alpha \leq 0.6$ at which the overlap is absent (Fig. 4).

The example of the influence of applied external magnetic field on IV-characteristics of the stack of two non-interacting junctions ($\alpha = 0$) and interacting junctions ($\alpha = 0.3$) is shown in Fig. 5a, b. The external applied field $B = 44.2$ G corresponds to the value of $\Phi/\Phi_0 = 1.3$ in the stack of two intrinsic junctions of the high-temperature superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ mesa with the width 70 micrometers. In the following consideration we will use values of the normalized magnetic flux $\phi = \Phi/\Phi_0$ instead of magnetic field. Four Fiske steps at $V'_p = p \cdot 0.199 \cdot V_c$ are clearly seen in the IV-curve of the stack with $\alpha = 0$ (Fig. 5a). The position of the second Fiske step coincides with the position of the zero-field step

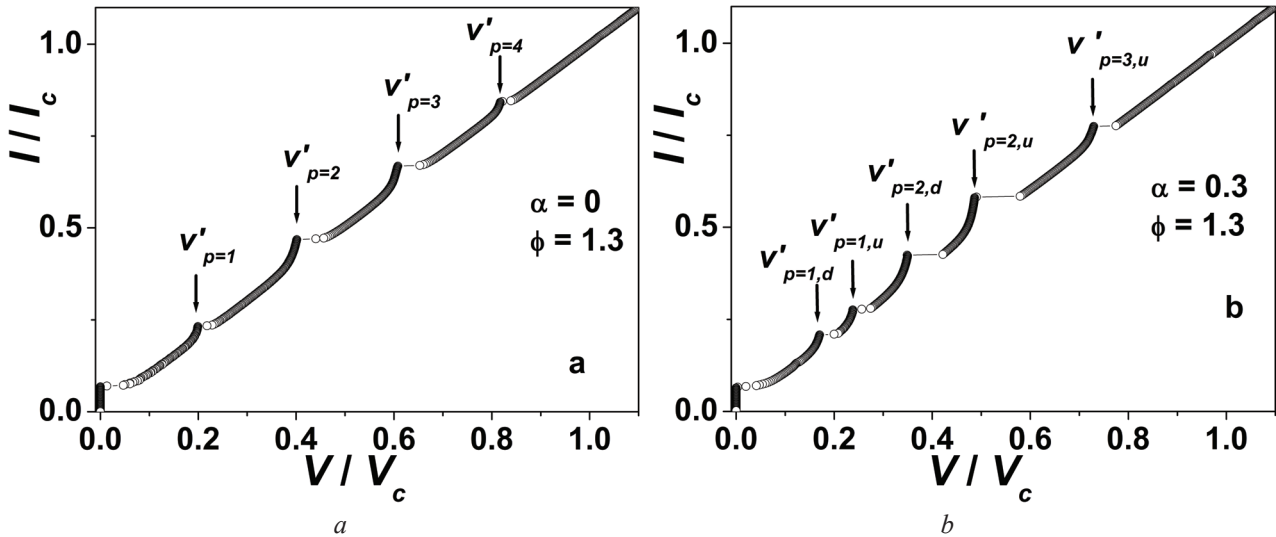


Fig. 5. IV-characteristics of the stack of two wide junctions in magnetic field which corresponds to $\phi = 1.3$ for $\alpha = 0$ (a) and $\alpha = 0.3$ (b). Arrows mark values of $v'_p = V'_p / V_c$ in (a) and $v'_{p,d} = V'_{p,d} / V_c$, $v'_{p,u} = V'_{p,u} / V_c$ in (b) for different p .

in Fig. 3a. In the IV-characteristic of the stack with inductive coupling between junctions ($\alpha = 0.3$) each of the Fiske steps V'_p is split in two steps at voltages $V'_{p,d}$ and $V'_{p,u}$ like the zero-field step considered above. The first Fiske step $V'_{p=1} = 0.199 \cdot V_c$ splits to the lower step at $V'_{p=1,d} = 0.170 \cdot V_c$ and the upper step $V'_{p=1,u} = 0.238 \cdot V_c$ etc. We can conclude that the inductive interaction between junctions leads to the split of the Fiske steps too.

It is necessary to determine if amplitudes of these split steps obey the usual dependences of Fiske steps on applied external magnetic field. Therefore we calculated IV-characteristics at different values of the normalized magnetic flux and determined the dependence of the maximum Josephson current I_m of the stack on the value of ϕ . The dependence $I_m(\phi)$ is shown in Fig. 6a by circles. This dependence has the usual Fraunhofer form (solid curve in Fig. 6a). We obtained also dependences of amplitudes of Fiske steps on ϕ . As we noted above, the application of the resistively shunted model for the description of wide Josephson junctions gives the possibility to determine only bottom parts of Fiske steps, so the height of the step can not be determined exactly. However, we can expect that the behaviour of remaining parts of steps obey the physical processes which take place when external magnetic field is applied to the system. Taking into account this consideration, we determined amplitudes of the first and the second Fiske steps $I^{F1,2} / I_c$ straightforward from IV-characteristics, plotted these amplitudes as functions of normalized magnetic flux and

fitted obtained plots in ranges of the theory of Fiske steps by standard equations [9,14]:

$$I^{Fp}(\phi) = I_c J_0\left(\frac{a}{2}\right) J_1\left(\frac{a}{2}\right) F_p(\phi), \quad (10)$$

$$F_p(\phi) = \frac{2}{\pi} \frac{\phi \left| \sin(\pi\phi - \pi(p/2)) \right|}{\left| \phi^2 - p^2/4 \right|}, \quad (11)$$

where p is the number of the Fiske step and $J_p(a/2)$ are Bessel functions of the p -th order. The parameter a is the root of the following equation [9]:

$$J_0\left(\frac{a}{2}\right) = \frac{a}{Z_p F_p(\phi)}, \quad (12)$$

where $Z_p = \left(\frac{D}{\lambda_J}\right)^2 \frac{Q_p}{\pi^2 p^2}$, Q_p is the quality factor for the p -th resonance. The plot of dependences $I_d^{F1}(\phi) / I_c$ and $I_u^{F1}(\phi) / I_c$ for both split steps is shown in Fig. 6b. The values of the amplitude of the upper step I_u^{F1} / I_c can not be determined in some interval of ϕ around the first maximum of $I_u^{F1}(\phi) / I_c$ (see Fig. 6b, triangles) because voltage jumps from the lower step to the hysteretic IV-curve. It is seen from Fig. 6b that the plot of the amplitude of the lower step I_d^{F1} on ϕ is satisfactory described by theoretical equations (10)-(12) (solid line in Fig. 6b). Because we can approximate only the part of the step, the found value of $Q_1 = 37$ can be treated only as a fitting parameter.

Amplitudes of split second Fiske steps on normalized

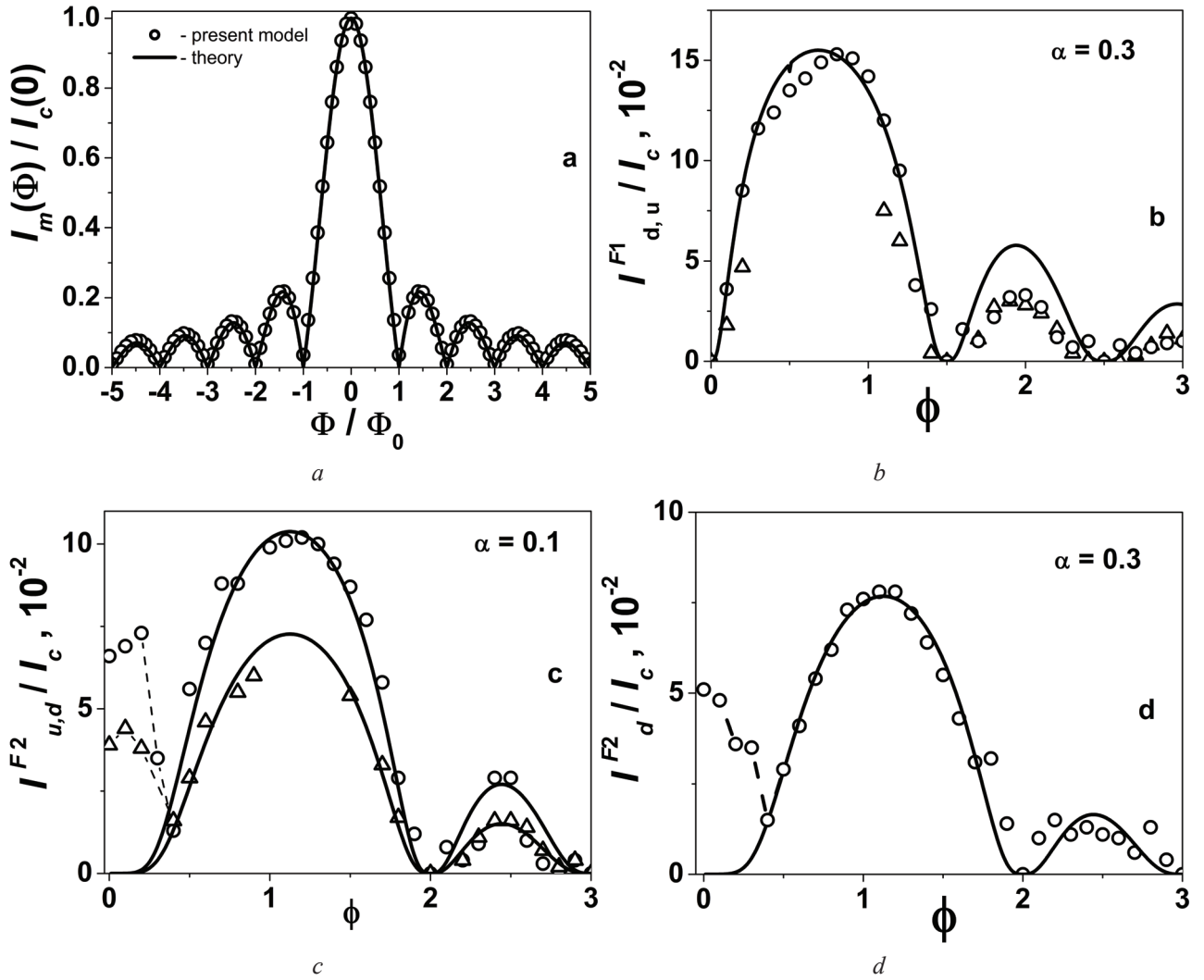


Fig. 6. (a) - the dependence of the normalized maximal Josephson current I_m/I_c on the normalized magnetic flux Φ/Φ_0 for the whole stack of two wide junctions; (b) - dependences $I_d^{F1}(\phi)/I_c$ and $I_u^{F1}(\phi)/I_c$ for the stack with $\alpha = 0.3$. Circles and triangles correspond to the lower and the upper step, correspondingly; (c) - dependences $I_d^{F2}(\phi)/I_c$ and $I_u^{F2}(\phi)/I_c$ for the stack with $\alpha = 0.1$. Circles and triangles correspond to the lower and the upper step, correspondingly; (d) - the dependence of $I_d^{F2}(\phi)/I_c$ for the stack with $\alpha = 0.3$. Solid lines in plots (b)-(d) are approximations by Eqs. (10)-(12).

magnetic flux are plotted in Fig. 6c for the system with $\alpha = 0.1$ and in Fig. 6d for the system with $\alpha = 0.3$ (in this plot only data for the lower step are shown). It is seen that in all cases maxima of dependences $I_d^{F2}(\phi)/I_c$ and $I_u^{F2}(\phi)/I_c$ for both lower and upper steps are satisfactory described by Eqs. (10)-(12) with $Q_2 = 97$ for the upper steps and $Q_2 = 110$ for the lower steps besides the interval $0 \leq \phi \leq 1/2$ (see Figs. 6c, d). The obtained in our model dependences $I_d^{F2}(\phi)/I_c$ and $I_u^{F2}(\phi)/I_c$ deviates drastically from the dependence predicted by Eqs. (10)-(12) in this interval of ϕ . The amplitude of the step

decreases to some non-zero value and then at $\phi \geq 0.5$ it increases again according to the predicted behaviour of the second Fiske step. This deviation was found in the experiment with the single Josephson junction which revealed the zero-field step in the IV-characteristic [9,15]. Theoretical treatment of zero-field steps in the presence of multimode oscillations proved also such a form of the dependence $I^{F2}(\phi)/I_c$ for single junction [16]. We can conclude that the found form of the second Fiske step is characteristic for junctions which reveal zero-field steps. It is necessary to check this supposition experimentally and to investigate the physical origin of this effect in high-

temperature superconductors.

Conclusions

In the present paper we investigated zero-field steps and Fiske steps in the stack of two interacting wide Josephson junctions with normal edges. Each of the wide junctions in the stack was modeled as a multijunction interferometer consisted of twenty ‘elementary junctions’. These ‘elementary junctions’ were described in the range of the resistively shunted model. In our model wide junctions in the stack can inductively interact with each other. Zero-field steps appear in IV-curves of the stack as a result of the interaction of Josephson generation with electromagnetic excitations which are introduced through normal edges. We found that due to the interaction, the zero-field step was split in two zero-field steps. Frequencies of split steps obeyed relations for inductively interacting resonance contours. We investigated also the behaviour of amplitudes of the first and the second Fiske steps in applied external magnetic field. Due to the inductive interaction between junctions, Fiske steps were also split. Dependences of amplitudes of the split first Fiske steps on the normalized magnetic flux were approximated by the theory with the satisfactory agreement, whereas the behaviour of the split second Fiske step at $\phi \leq 0.5$ deviated from predictions of the theory.

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