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## The characteristic parameters of charge carriers in the p-type $\text{Si}_{0.2}\text{Ge}_{0.8}$ quantum well with two subbands occupied

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The kinetic characteristics of the two-dimensional system of charge carriers in the p-type heterostructure  $\text{Si}_{0.7}\text{Ge}_{0.3}/\text{Si}_{0.2}\text{Ge}_{0.8}/\text{Si}_{0.7}\text{Ge}_{0.3}$  in condition of two subbands occupied has been calculated. The density, mobility and the effective mass of the charge carriers in each subbands have been estimated. The effects of weak localization of charge carriers under the condition of strong spin-orbit influence have been analyzed. The phase relaxation time and the spin - orbit interaction of the charge carriers, as well as the value of spin splitting have been found. The results are in good agreement with available theoretical models.

**Keywords:** magnetoresistance, weak localization.

Подано розрахунок кінетичних характеристик двовимірної системи носіїв заряду в дірковій гетероструктурі  $\text{Si}_{0.7}\text{Ge}_{0.3}/\text{Si}_{0.2}\text{Ge}_{0.8}/\text{Si}_{0.7}\text{Ge}_{0.3}$  за умов заселення двох квантових рівнів. Отримано значення концентрації, рухливості і ефективної маси носіїв заряду на кожному з квантових рівнів. Проведено аналіз ефектів слабкої локалізації носіїв заряду в умовах сильних спин-орбітальних ефектів. Отримано значення часів фазової релаксації і спин - орбітальної взаємодії носіїв заряду, а також величини спінового розщеплення. Отримані результати добре відповідають наявним теоретичним моделям.

**Ключові слова:** магнітоопір, слабка локалізація.

Представлен расчет кинетических характеристик двумерной системы носителей заряда в дырочной гетероструктуре  $\text{Si}_{0.7}\text{Ge}_{0.3}/\text{Si}_{0.2}\text{Ge}_{0.8}/\text{Si}_{0.7}\text{Ge}_{0.3}$  в условиях заселения двух квантовых уровней. Получены значения концентрации, подвижности и эффективной массы носителей заряда на каждом из квантовых уровней. Проведен анализ эффектов слабой локализации носителей заряда в условиях сильных спин-орбитальных эффектов. Получены значения времен фазовой релаксации и спин – орбитального взаимодействия носителей заряда, а также величины спинового расщепления. Полученные результаты хорошо соответствуют имеющимся теоретическим моделям.

**Ключевые слова:** магнитосопротивление, слабая локализация.

### Introduction

The electron properties of two-dimensional conducting systems have been studied extensively for several decades. In modulation-doped heterostructures the motion of charge carriers in the direction perpendicular to the interface is quantized and forms a sequence of quantum levels ( $E_1, E_2, \dots$ ). At low temperatures the mobility of charge carriers is strongly dependent on their density. When the number of carriers is low, only the lowest subband ( $E_1$ ) is occupied. In such systems the mechanisms of electron scattering are mainly determined by the scattering at

ionized impurities and the interface roughness, as well as by the intrasubband scattering [1]. As the number of charge carriers increases, the Fermi level shifts too. When it exceeds the energy of the second subband ( $E_2$ ), the charge carriers start to occupy it. In this case the intersubband scattering should also be taken into consideration [2]. The occupation of the second subband in a two-dimensional conducting system affects its electrical conduction. The effect has been studied for a long time with a regard on the influence of the intersubband scattering upon electric transport [3-5]. Recently much researcher' attention has

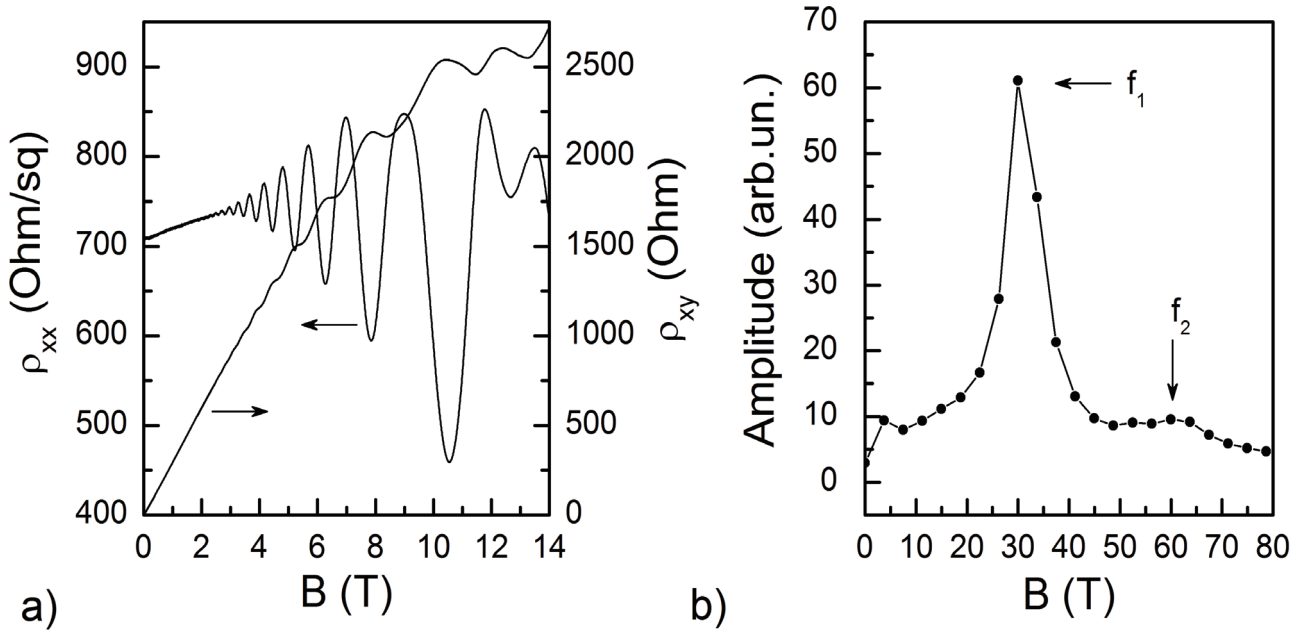


Fig. 1. The dependences of the longitudinal and transverse components of resistance on the magnetic field at  $T=1.5K$  a) and the Fourier spectrum of SdH oscillations b).

been given to systems with two subbands occupied [6,7] in which the intersubband scattering is rather weak [8].

The Rashba effect of spin-orbit interaction has attracted much interest in the context of its possible use in spintronics [9,10]. In particular, the Rashba effect can produce significant influence to the transport properties of two-dimensional conducting systems [11, 12].

### Results and discussion

In this study we report the calculation of the kinetic and quantum interference characteristics of the p-type heterostructure  $Si_{0.7}Ge_{0.3}/Si_{0.2}Ge_{0.8}/Si_{0.7}Ge_{0.3}$  under the condition when two subbands are occupied.

The width of the quantum well  $Si_{0.2}Ge_{0.8}$  was 14 nm. The heterostructure configuration and the preparation technology are considered in Ref. [13]. The conducting region was shaped as a Hall bars  $\sim 0.55$  mm wide and  $\sim 2.25$  mm long, the distance between two pairs of narrow potential contacts being  $\sim 1.22$  mm. The investigations were performed at the International Laboratory of High Magnetic Fields and Low Temperatures (Wroclaw, Poland) in magnetic fields up to 14 T at temperatures down to 1.5 K using the standard lock-in technology.

The measured dependences of the longitudinal  $\rho_{xx}(B)$  and the transverse  $\rho_{xy}(B)$  components of magnetoresistance are illustrated in Fig.1a). The curves demonstrate distinct Shubnikov-de Haas (ShdH) oscillations (longitudinal component) and the corresponding quantum plateaus of the Hall effect (transverse component). However, the Hall component of magnetoresistance is essentially nonlinear in high magnetic fields (see Fig. 1a), which suggests that two subbands are occupied in the

system investigated. This is supported by the presence of two maxima with frequencies  $f_1$  and  $f_2$  on the Fourier spectrum of the magnetoresistance dependence (Fig. 1b)).

In this case the simple formulas  $\mu = \sigma R_H$  and  $p = 1/R_H e$  ( $R_H$  is the Hall coefficient,  $e$  is the electron charge) are not fully correct for calculation of the charge carrier density  $p$  and carrier mobility  $\mu$ . The charge carrier density at each subband can be calculated using relation  $p_i = 2ef_i/h$  where  $f_i$  is the frequency at which the maxima appear in the Fourier spectrum. The maximum at the frequency  $f_1$  corresponds to the contribution of the carriers at the lowest subband and the maximum at the frequency  $f_2$  corresponds to the contribution of a group of excited-state carriers. The broad maximum at frequency  $f_3 = 60 T$  (Fig. 1b) corresponds to the second harmonic of principal maximum ( $f_1$ ). The composite harmonics ( $f_1 \pm f_2$ ) are not visible at the steep slopes of maximum in  $f_1$ . Using the experimental values  $f_1 = 27.8 T$  and  $f_2 = 3 T$ , we were able to calculate the hole concentration at the first ( $p_1 = 1.5 \times 10^{12} cm^{-2}$ ) and the second ( $p_2 = 1.75 \times 10^{11} cm^{-2}$ ) subbands.

The charge carrier mobility at each subband was calculated using the theoretical model of [14]. It allowing us to estimate in addition to mobilities of  $\mu_1$  and  $\mu_2$  on first and second subband respectively, also the parameter of intersubband interaction  $r$ , which can be significant in

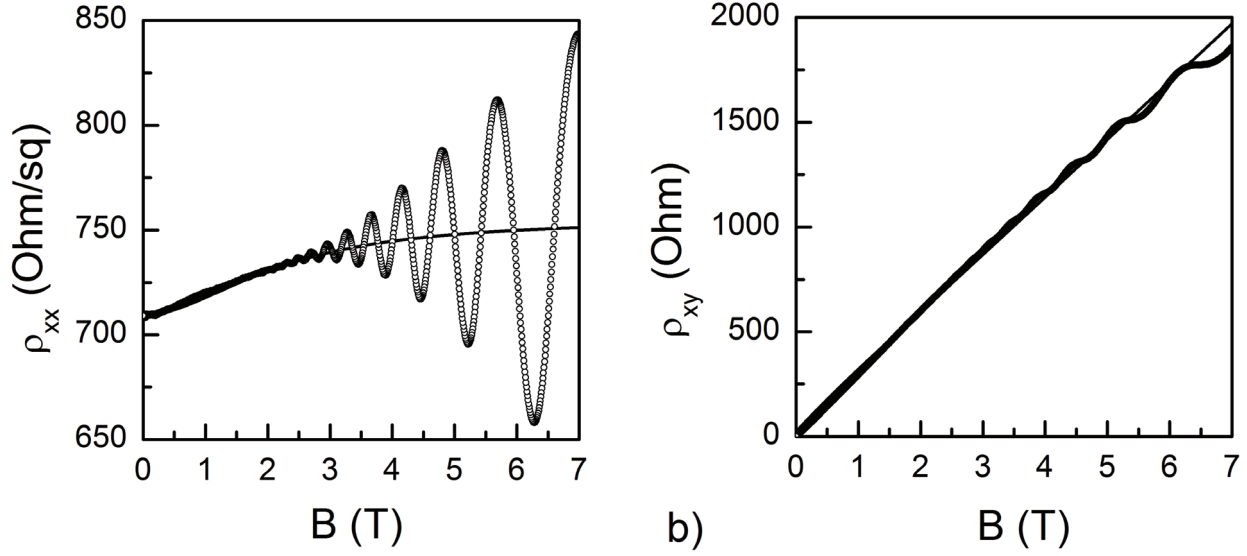


Fig. 2. The dependences of the longitudinal a) and transverse b) components of the resistance of the sample at  $T=1.5\text{K}$ . Solid lines are calculation in terms of theory [14].

such systems. In this theory the longitudinal and the transverse components of magnetoresistance can be described by the following expressions:

$$\rho_{xx}(B) = \rho_0 \left( 1 + \frac{rp_1p_2\mu_1\mu_2(\mu_1 - \mu_2)^2 B^2}{(p_1\mu_1 + p_2\mu_2)^2 + (rp_H\mu_1\mu_2)^2 B^2} \right), \quad (1)$$

and

$$\rho_{xy}(B) = -\frac{\langle \mu^2 \rangle + (r\mu_1\mu_2 B)^2}{\langle \mu \rangle^2 + (r\mu_1\mu_2 B)^2} \frac{B}{p_{Hall}e}, \quad (2)$$

where  $\rho_0 = \frac{1}{(p_1\mu_1 + p_2\mu_2)e}$  is the resistance in a zero

magnetic field,  $\langle \mu \rangle = \frac{p_1\mu_1 + p_2\mu_2}{p_1 + p_2}$  is the averaged

mobility. If  $r=1$ , Eqs. (1) and (2) turn to the expressions common for noninteracting conducting channels. The description of the measured magnetic field dependences of the longitudinal and Hall components of magnetoresistance in terms of Eqs. (1) and (2) is illustrated in Fig.2. Using this approach, we calculated  $\mu_1$ ,  $\mu_2$  and  $r$ . For example, at  $T=1.5\text{K}$  the mobilities at the first and second subbands were  $\mu_1 = 3000\text{ cm}^2/\text{Vs}$  and  $\mu_2 = 6600\text{ cm}^2/\text{Vs}$ . The parameter  $r$  was nearly equal 0.3. In this case, the quantum channels are weakly interacted one with other.

The effective masses of charge carriers can be found by analyzing the ShdH amplitude variations with temperature

and magnetic field in terms of the theoretical model [15] follow the procedure described in Ref. [16]. According to the theory, the resistance variation is described by the formula

$$\rho_{xx} = \rho_0 \left[ 1 + 4 \sum_{s=1}^{\infty} \left( \frac{\Psi_s}{\sinh \Psi_s} \right) \times \exp \left( -\frac{\pi s}{\omega_c \tau_q} \right) \cdot \cos \left( \frac{2\pi s \varepsilon_F}{\hbar \omega_c} - \Phi \right) \right], \quad (3)$$

where  $(\Psi = \frac{2\pi^2 k_B T}{\hbar \omega_c})$  determines the temperature and

the magnetic field dependences of the oscillation amplitude,

$\omega_c = \frac{eB}{m^*}$  is the cyclotron frequency,  $\tau_q$  is the quantum

(one-particle) relaxation time of charge carriers which characterized the collisional broadening of the Landau levels,  $\Phi$  is the phase. The calculated results are shown in Figs. 3a,b.

The experimental points in Fig.3a taken in different magnetic fields at different temperatures form a single line for the effective mass  $m^* = 0.18 m_0$  and the parameter  $\alpha = \tau/\tau_q = 3.3$  ( $\tau$  is the transport relaxation time). As follows from Eq. (3), in this arrangement the points corresponding to the extrema with different values of filling factor  $\nu$  are expected to fall onto a single straight line with a slope angle of  $\pi\alpha$  (the solid line in Fig.3a). In this case the effective mass  $m^*$  serves as a fitting parameter for aligning the points taken at different temperatures on a single curve. However, in high fields the lines formed with

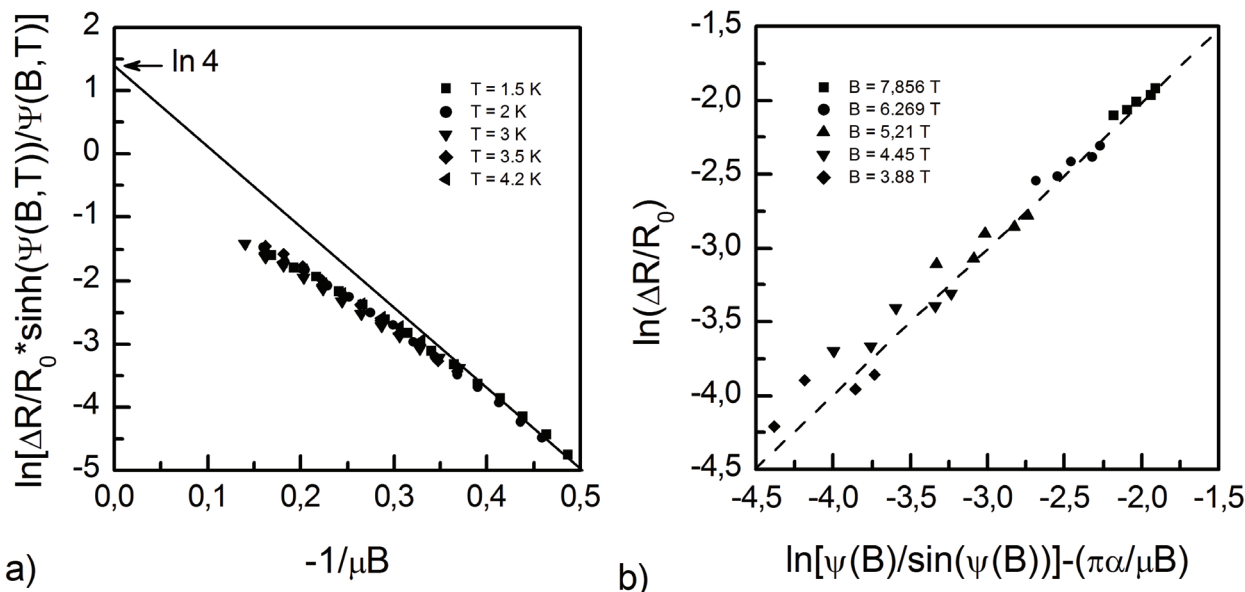


Fig. 3. The illustration of the numerical calculation of the parameters  $m^*$  and  $\alpha$ . Dash line -  $45^\circ$  slope.

this these points deviated from the linear dependence. The deviation may be attributed to the influence of the second subband on the amplitude of the resistance oscillations, which is evident in higher field. The possibility of fitting all the experimental points, even those taken in high fields, onto a single curve suggests that the effective masses of the charge carriers at the first and the second quantum levels are equal, being  $0.18m_0$ , which is close to  $0.16m_0$  obtained for a similar sample [17] with 10 nm wide quantum well and one subband occupied.

In the region of a zero magnetic field the initial part of the magnetic field dependences of resistance exhibits a positive magnetoresistance which changes then to a negative one with formation of maximum (Fig. 4a). This is an evidence of the influence to the sample magnetoresistance the weak localization (WL) effects under the condition of strong spin-orbit interaction. The experimental results were described in terms of the WL theory. The model used [18] is applicable for considering non-deformed and deformed bulk p-type semiconductors and semiconductor

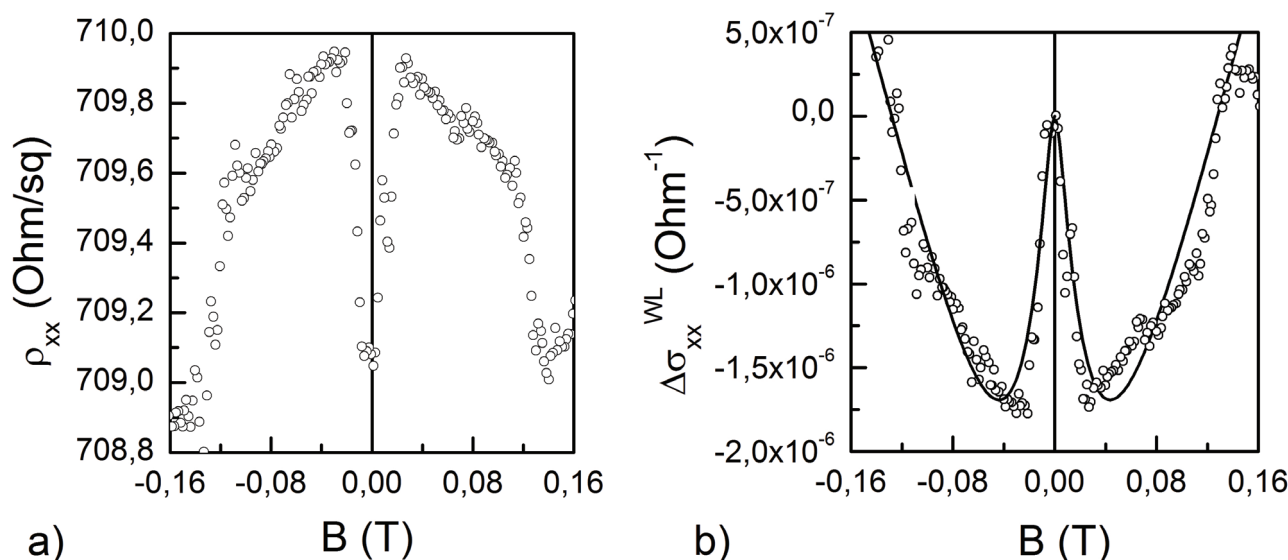


Fig. 4. The dependence of the longitudinal component of the resistance in low magnetic fields at  $T = 1.5 K$  a). The calculation (theory [18]) of the magnetic field – induced change in the localization correction to conduction  $\Delta\sigma_{xx}^{WL}(B)$  b).

- based structures with quantum wells. According to the model, the magnetic field dependence of the localization correction to conductivity can be described as

$$\Delta\sigma_{xx}^{WL}(B) = \frac{D_{ij}^0}{D_a^0} \cdot G_0 \left( f_2 \left( \frac{4eDB}{\hbar} \cdot \frac{\tau_\varphi \tau_\parallel}{\tau_\varphi + \tau_\parallel} \right) + \frac{1}{2} f_2 \left( \frac{4eDB}{\hbar} \cdot \frac{\tau_\varphi \tau_\perp}{\tau_\varphi + \tau_\perp} \right) - \frac{1}{2} f_2 \left( \frac{4eDB}{\hbar} \cdot \tau_\varphi \right) \right), \quad (4)$$

where  $G_0 = e^2 / (2\pi^2 \hbar)$ ,  $\tau_\varphi$  is the phase relaxation time,  $\tau_\parallel$  and  $\tau_\perp$  are the times of the longitudinal and transverse spin relaxation, and the normal to the quantum well plane is taken as a preferred axis, the ratio  $D_{ij}^0 / D_a^0$  characterizes the relative values of the components of the diffusion coefficient. The description of the experimental results in terms of the theory [18] is illustrated in Fig. 4b. The calculated  $\tau_\varphi$  values can be approximated by the dependence  $\tau_\varphi \propto T^{-0.75}$  close to the  $\propto T^{-1}$  - type dependence characteristic of the charge carriers interaction in a two-dimensional system [19].

The estimated time of the spin-orbit scattering  $\tau_\perp = 4 \times 10^{-12} \text{ s}$  makes it possible to find the value of spin splitting in terms of the Diakonov-Perel theory [20]:

$$\tau_{SO}^{-1} \approx \Omega^2 \tau, \quad (5)$$

where the frequency of spin precession is  $\Omega = \Delta / 2\hbar$ . The value of spin splitting  $\Delta = 1.02 \text{ meV}$  was obtained from Eq. (5). It is close to value  $\Delta = 1.81 \text{ meV}$  find in in [17].

### Conclusions

1. The periods of the Shubnikov-de Haas oscillations have been calculated on the basis of experimental results, which permitted is to find the hole density at the first ( $p_1 = 1.5 \times 10^{12} \text{ cm}^{-2}$ ) and the second ( $p_2 = 1.75 \times 10^{11} \text{ cm}^{-2}$ ) subbands .

2. The mobility of charge carriers at both the subbands ( $\mu_1 = 3000 \text{ cm}^2 / \text{Vs}$ ,  $\mu_2 = 6600 \text{ cm}^2 / \text{Vs}$ ) and the interaction parameter ( $r = 0.3$ ) of the carriers have been estimated.

3. The effective masses of the charge carriers occupying the first and second subbands have been obtained. They appear to be equal in value ( $m^* = 0.18 m_0$ ),  $m_0$  is the free electron mass.

4. The form of the magnetic field dependences of resistance in low magnetic fields (before the Landau

quantization reveals itself) points to the contribution of the weak localization (WL) effect under the condition of strong spin-orbit interaction.

5. The estimated time of spin-orbit scattering was used to find the value of spin-orbit splitting ( $\Delta = 1.02 \text{ meV}$ ).

We have thus calculated the kinetic characteristics of a two-dimensional charge carrier system in the p-type heterostructure  $\text{Si}_{0.7}\text{Ge}_{0.3} / \text{Si}_{0.2}\text{Ge}_{0.8} / \text{Si}_{0.7}\text{Ge}_{0.3}$  when two subbands are occupied. The effects of week localization of charge carriers have been analyzed under the condition of strong spin-orbit influence. The results obtained are in good agreement with the current theoretical models.

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