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The interaction of the gravitational field and laser radiation as a basis for precision measurements

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In this article we discuss conditions of the precision frequency measurements, which are based on highly stable lasers. In this paper we analyze conditions of the measurement of the gravitational field in spatial domain of the Lagrange points.

Keywords: optical measurements, gravitational field, laser radiation, heterodyne method, reference frequency.

В работе обсуждаются условия прецизионных частотных измерений, в основе которых используются высокостабильные по частоте лазеры. В настоящей статье на основе модели взаимодействия гравитационного поля с оптическим излучением, анализируются условия измерения неоднородности гравитационного поля в пространственной области точек Лагранжа.

Ключевые слова: оптические измерения, гравитационное поле, лазерное излучение, гетеродинный метод, эталонная частота.

У роботі обговорюються умови прецизійних частотних вимірювань, в основі яких використовуються високостабільні по частоті лазери. У цій статті на основі моделі взаємодії гравітаційного поля з оптичним випромінюванням, аналізуються умови виміру неоднорідності гравітаційного поля в просторовій області точок Лагранжа.

Ключові слова: оптичні вимірювання, гравітаційне поле, лазерне випромінювання, гетеродинний метод, еталонна частота.

Introduction

Well-known heterodyne methods allow to determine the optical frequencies, their shifts and their temporal characteristics of one laser relative to other laser. Technical realization of the heterodyne method is possible when we have reference laser with high stability of frequency radiation.

Till now the single field of practical utilization of optical frequencies existed. It is in metrological provision of optical frequency measurements. These measurements are based on a set of reference lasers, which are included in the recommended list (26 optical frequencies) of defined frequency characteristics. These characteristics are a relative standard uncertainty and long-term frequency instability. Further development of the heterodyne methods is related with the expansion of areas of use for the frequency measurement of physical quantities. Therefore, the aim of this paper is to search conditions of measurement of the gravitational field in the interplanetary space.

Features of frequency standards for precision heterodyne systems

The development of optical frequency measurements is persistent. Basing on the development of the reference lasers, which allow to improve the accuracy of measurements, when optical frequency difference is decreasing. The frequency standard, which builds on basis of frequency reference, may be an example of this assertion. This frequency reference is formed from cooled strontium ion 88 [1]. The updated list of laser frequency standards was presented in the final decision of CIPM (2009) [2]. It is characterized by significant decrease of relative standard uncertainty of frequency emission 10^{-15}. But the frequency standard with relative standard uncertainty for 1000 sec of averaging 10^{-18} on the same wavelength was introduced a few years later. However, this result was not included in the CIPM list as certified and tested frequency standards. Therefore, standards from this list (CIPM 2009) can be used in modern circuits of heterodyne measurements. Thus, high precision and high stability standards of optical frequencies can be used for special heterodyne methods of physical quantities measurements at present time.

The heterodyne methods of measurement gravitational fields

The heterodyne method of measurement laser frequency consists in registering of the beat frequency between two lasers, the frequency of the one of laser is the reference or known, and frequency of a second laser will be determined.

In the case where the frequency stability of the reference laser is characterized by a very low relative frequency uncertainty, the heterodyne method allows us to investigate and monitor change in the frequency of the second laser under the influence of the physical phenomena. Examples of using optical frequency measurements to solve practical problems are not so many. One of such effects of the change frequency is used in the two-frequency interferometers, in which not only the distance but also the speed of the object with reflector can be measured and to determine direction of object's motion.

Measurements, which are performed in order to solve the fundamental problems of physics, should be used on the basis of new techniques, which are based on the developing of physical theories.

Basing on the change of frequency radiation under the influence of the gravitational field in the space the research of the process in the interplanetary space may be performed by the difference between gravitational potential in the space point and the potential in the reference point.

Using of frequency measurements as the most high-precision makes it possible to measure very little changes in the magnitude of the gravitational potential. The ability to receive the signal of the beat frequencies between the two lasers in the space depends on the location of the lasers in the Lagrange points.

Placement of lasers in these points is determined by the solving problem, namely, we study the structure of the gravitational field in the area of saddle point (L1) or in the area of unstable point (L4). It should be noted that the theoretical description of characteristics of gravitational field is related with the amount accounted celestial bodies and their weight. From this it follows that the structure of the gravitational field is quite difficult to determine theoretically in the planetary system and experimental determination of relative distribution of the gravitational potential field is possible with using of a laser heterodyne system described in [3].

If the frequency difference between two lasers for a long time has been monitored and has been measured with an uncertainty of $10^{-16} - 10^{-17}$, this value allows to fix the effect of the gravity offset of the optical frequency. When one of the lasers is placed on the artificial satellite of the Earth, the frequency difference will be influence by Doppler Effect of 1st and 2nd order.

For the measurement of the gravitational frequency shift it is necessary to eliminate influence of the Doppler effect of the 1st order. For this it is necessary to reduce the distance between the lasers to ten meters, but the stability of the frequency radiation must be several orders of magnitude greater than in the case of laser arrangement in Earth orbit. In this case, the influence of the Doppler effect of the 1st-order is negligible. This condition allows only by measurement of the optical frequency difference to determine the basic characteristics of inhomogeneity of the gravitational field. It should be noted that such examination is possible in the linear approximation to Einstein's theory, because only in this approximation remains valid concept of Newton's potential.

Due to the principle of equivalence, which is based by A. Einstein in 1911 [3], the increase of electromagnetic radiation by passing difference of gravitational potentials, leads to increase on value of the photon energy ${hf}/{c^2 \Delta\varphi}$. On the other hand, the time of the flight radiation from the source to the observer is ${h}/{c}$, where h - the distance between spatial points. In equivalent coordinate system, if ${gh}/{c}$ is small (g is the acceleration of the gravity), the first-order Doppler effect leads to increase the frequency by an amount equal ${gh}/{c^2}$ and the observer records frequency equal to:

$$f_2 = f_1(1 + gh / c^2) \qquad (1)$$

The coefficient ${g}/{c^2}$ is 10^{-16}, that is why changes of the frequency can be registered at the instability of frequency of lasers 10^{-17} and less [4]. Because the necessary highly stable lasers have been developed during last 10 – 15 years, the opportunity of work in the optical range did not exist in the 60s years. In connection with the search for possible ways of experimental verification of the gravitational shift of optical frequencies Pound and Rebka used Mossbauer Effect.

Conditions of the interaction of electromagnetic radiation with gravitational field

The effect of changing in the frequency of radiation is linked with the time of radiation propagation in the nonuniform gravitational field massive object.

The influence of Earth gravity on the frequency of electromagnetic radiation, which manifests in gravitational violet shift [5] can be recorded and evaluated by measuring the shift of optical frequency of laser relative to the other lasers, which are located at a predetermined distance between themselves. The reference laser is located on the surface of Earth. Both the frequency of radiation of lasers are stabilized by frequency reference, between which a predetermined frequency is separation $\Delta\varphi$, which can have value from Hz to hundreds of MHz. It is possible to use either a differential or integral measurement method to determine the influence of gravitational field on frequency of optical radiation. In this paper the differential method is used, which allows to determine acceleration of free fall, based on gravitational potentials.

The equation of measurements which are based on differential method, obtained in the conditions of decomposition of gravitational potential u near the observation point u_0. Points with gravitational potentials u and u_0 are spaced apart in height above the Earth at a sufficiently small distance ΔH, at which the variable changes linearly. Then, by limiting to simplify the analysis, linear by ΔH terms of the expansion, we obtain an expression for the potential near the observation point:

$$u_1 = u_0 + \frac{\partial u}{\partial H}\Delta H + ... \qquad (2)$$

Because the vertical gradient of the potential is not more than the acceleration of gravity $g = \frac{\partial u}{\partial H}$, then in the view of (2) we obtain the relation between the frequency shift and value *g*, which is described by the equation:

$$\frac{f_0 - f_1}{f_0} = g\frac{\Delta H}{c^2}. \qquad (3)$$

The limit of the optical frequency increment to the value of the height increment is a value proportional to the acceleration of the free fall. It is possible to determine the value of *g* by measuring the difference of optical frequency $\Delta f = f_0 - f_1$ with registration of the distance ΔH:

$$g = \frac{\Delta f}{f}\frac{c^2}{\Delta H} \qquad (4)$$

It is possible to use a laser interferometer with an uncertainty of measurement distance of $1.5\,\mu m / m$ (measuring interferometer type LSP-30-Compact [6]) to accurately measure the distance ΔH. Lasers, which are used in the experiment, must have a long-term stability of the frequency of radiation. This stability should allow to measure the change in frequency of radiation caused by influence of the gravitational field.

When you measure difference between optical frequencies, it should be noted that absolute value of difference optical frequencies may be registered and the sign of difference frequency remain constant and unchanged. While investigating this problem, the information of sign of optical frequency difference does not play a fundamental role, but it is very important in clarifying the provisions of general relativity and fundamental provisions in cosmology [7]. To make an important experiment the long-term stability of laser radiation should be characterized by magnitude of less than 10^{-16}.

It should be noted, that only natural optical radiation sources have been used until recently. But artificial sources, which include various types of lasers, have advantage in its time frequency and spatial characteristics of radiation. At first this is high stability of frequency radiation.

These advantages allow essentially improve accuracy of frequency measurements. Depending on conditions for realization of measurements circuit two extreme cases can be focused. In the first case, a small base between lasers is used. It's range from 1 to 10 meters, and in the second realization is at high base (100 m to 400 km) between the lasers.

The accuracy, which is required for measuring of acceleration of the gravity $\sigma_g = 5\ \mu g$,i.e the quantity, which corresponds to modern absolute gravimeters (type GABL and FG-5).

The distance between lasers is ΔH selected small – *10 m*, that allows make measurement of the acceleration of free fall quickly and efficiently. The distance before main measurements is refined by measuring laser interferometer, such as LSP-30-Compact. The error of interferometer for linear measurements is $1.5\,\mu m / m$. Therefore, in this case:

$$\sigma_{\Delta H} = 1{,}5\times 10^{-6} m/m \cdot 10m = 1{,}5\times 10^{-5} m \qquad (5)$$

The phase range finders must be used in measurements with long distances. The measurement error in these rangefinders depends on methods of precision phase measurements.

The difference of frequencies between two lasers $\Delta f = |f_1 - f_2|$ is measured by an optical heterodyne system.

The reference frequency f_1 of the laser, which is located on the reference plane, is determined by the type of the selected laser, its system of frequency stabilization and natural frequency reference, which is used. The frequency f_2 of the second laser is also stabilized by natural frequency reference and is separated from the frequency f_1 of a small fixed value $f_2 = f_1 + \Delta\varphi$. On the other hand, the change of frequency in the propagation by the vertical is described:

$$f_2 = (f_1 + \Delta\varphi)(1 + \alpha h) \qquad (6)$$

Where $\alpha = g/c^2$ – the linear coefficient of increasing the frequency on each meter of radiation, it is estimated as 10^{-16}.

The difference of frequencies $\left|f_1 - f_2(h)\right|$, except values of frequency reference, which stabilizes frequency of laser radiation can ranging from a few tens of kHz to MHz:

$$\Delta f = f_1 - f_2\left(h\right) = \Delta\varphi + (f_1 + \Delta\varphi)\alpha h \qquad (7)$$

The frequency offset Δf , which is caused by influence of gravity, in cause, that $\Delta\varphi$ eliminated by heterodyning in the radio engineering range of frequencies, is like this:

$$\Delta f = f_1\alpha h = 10^{-16}\cdot 10\cdot 5,0\cdot 10^{14}\,Hz = 5\cdot 10^{-1}\,Hz \quad (8)$$

In order to register this value of the frequency shift, the optical frequency stability should allow measure this frequency shift. Therefore, this problem is solved by applying a high-precision and the high stability laser technique.

Conclusions

As a result of this article, conditions of heterodyne measurement of frequency shifts between two lasers in the conditions of the interplanetary space are determined. The main technical conditions for realization of such a system for study of the influence gravitational field on measurement of absolute values of optical frequencies are defined as well.

It was found that to register the influence the gravitational potential on system of heterodyne frequencies measurements it is necessary to use highly stable frequency and high precision laser technology. The development of such laser technology has started recently.

These studies make it possible to measure the value of gravity acceleration and may find application in the fields of geodesy, geophysics, optical transmission of time, distance measurement, etc.

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