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Methods of determination of polytrophic effectiveness factor of the centrifugal supercharger

Yu.A. Oleynik, S.A. Saprykin

Ukrainian research institute of natural gases
Ukraine, 61010, Kharkov, Gimnazicheskaya Emb., 20
oleynik.juriy@ndigas.com.ua

Mathematical models of determination of the polytrophic effectiveness factor (EF) of the centrifugal supercharger (CS) of natural gas were analyzed. On this basis four methods of calculation of polytrophic EF of CS were investigated. According to these four investigated methods there was carried out the practical calculation and comparison of changes of values of polytrophic efficiency of CS. The simplest method was allocated, that helps to determine the polytrophic EF of CS with entrance pressure of 0,5-2,3 MPa very quickly.

Keywords: polytrophic process; adiabatic process; polytrophic effectiveness factor.

Проанализированы математические модели определения политропного коэффициента полезного действия (КПД) центробежного нагнетателя (ЦБН) природного газа, на основе чего описаны четыре метода расчёта политропного КПД ЦБН. По описанным четырем методам проведен практический расчет и сравнение изменений значений политропного КПД ЦБН. Выделен самый простой метод, позволяющий быстро определять политропный КПД ЦБН при входных давлениях 0,5-2,3 МПа.

Ключевые слова: политропный процесс; адиабатный процесс; политропный КПД.

Проведено аналіз математичних моделей визначення політропного коефіцієнта корисної дії (ККД) відцентрового нагнітача (ВЦН) природного газу, на основі чого було описано чотири методи розрахунку політропного ККД ВЦН. Згідно з описаними чотирма методами було проведено практичний розрахунок та зрівняння змін значень політропного ККД ВЦН. Виділено найпростіший метод, який дозволяє швидко визначати політропний ККД ВЦН при вхідних тисках 0,5-2,3 МПа.

Ключові слова: політропний процес; адіабатний процес; політропний ККД.

Introduction

For assessment of the technical condition of the centrifugal supercharger (CS) of the natural gas (NG) it is necessary to determine its polytrophic effectiveness factor (EF). The simple method of calculation is necessary to make it under operating conditions CS at compressor station. The existing mathematical models (MM) of determination of polytrophic EC CS [1-6] will be considered in this article. After their analysis the simplest mathematical methods will be allocated, which allow to determine the polytrophic EC CS under operating conditions.

Parameters of natural gas

Let us consider the NG parameters, which are necessary for calculation of polytrophic EC CS. In fig. 1 axes of temperatures, pressure and specific enthalpies of NG at its compression in CS are shown.

On an axis of temperatures of fig. 1 the following temperatures of NG are shown: T_1 – NG temperature on CS entrance, K; T_2 – NG temperature at CS exit at polytrophic process of compression, K; T_{2a} – NG temperature at CS exit at adiabatic process of compression, K.

On an axis of pressure of fig. 1 the following pressure of NG are shown: p_1 – NG pressure on CS entrance, Pa; p_2 – NG pressure at CS exit at polytrophic process of compression, Pa; p_{2a} – NG pressure at CS exit at adiabatic process of compression, Pa.

On an axis of specific enthalpies of fig. 1 specific enthalpies of real NG at adiabatic and polytrophic processes of compression of NG are shown. Also in the drawing specific enthalpies of NG are shown if it has properties of a perfect gas at temperatures $Ò_2$ and $Ò_{2a}$.

For specific enthalpies and specific works in fig. 1 the following designations are entered: h_1 – specific enthalpy of NG on CS entrance, J/kg; h_2 – specific enthalpy of NG at CS exit at polytrophic process of compression of NG, J/kg ; h_{2a} – specific enthalpy of NG at CS exit at adiabatic process of compression of NG, J/kg; $-h_{1i}$, h_{2i} – specific

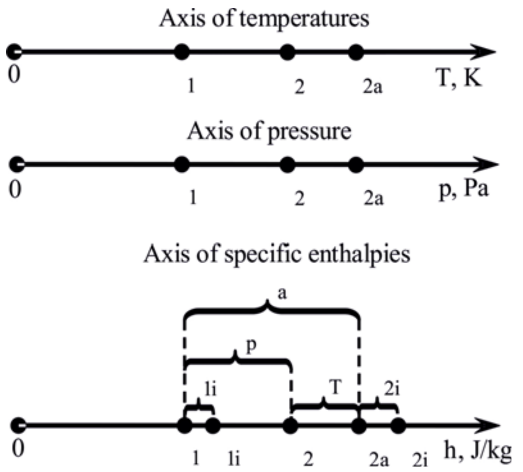


Fig.1. Temperatures, pressure and specific energy of NG on an entrance and an exit of CS.

enthalpies of NG on an entrance of CS and an exit of CS if the compressed NG has properties of a perfect gas, J/kg; e_T – losses of specific energy of NG due to return of heat from NG to the external environment, J/kg; e_{1i}, e_{2i} – difference between a specific enthalpy of NG in perfect condition and NG in a real state at $e_T = 0$, J/kg; A_a – the specific work made by CS over NG at adiabatic process of compression, J/kg; A_p – the specific work made by CS over NG at polytrophic process of compression, J/kg.

In the description of calculations of the NG parameters we won't consider coefficient of compressibility of NG for simplification of definition of η – polytrophic EF CS.

Mathematical models of calculation of polytrophic EF CS

For sizes i_{2a} and i_1 we will write down the equations according to standards [1, 2]:

$$h_{2a} = h_{2i} - e_{2i}; \tag{1}$$

$$h_1 = h_{1i} - e_{1i}; \tag{2}$$

$$h_{2i} = (2{,}6\,R + 149)\,T_2 + 1{,}225\,T_2^2;$$

$$h_{1i} = (2{,}6\,R + 149)\,T_1 + 1{,}225\,T_1^2;$$

$$e_{2i} = R\,T_c\,\delta_2;$$

$$e_{1i} = R\,T_c\,\delta_1;$$

$$\delta_2 = \tau_2\left[\left(\frac{0{,}3468}{\tau_2} + \frac{0{,}3564}{\tau_2^3}\right)\frac{\pi_2}{\tau_2} + \frac{1}{2}\left(\frac{0{,}0273}{\tau_2} - \frac{0{,}117}{\tau_2^3}\right)\left(\frac{\pi_2}{\tau_2}\right)^2\right];$$

$$\delta_1 = \tau_1\left[\left(\frac{0{,}3468}{\tau_1} + \frac{0{,}3564}{\tau_1^3}\right)\frac{\pi_1}{\tau_1} + \frac{1}{2}\left(\frac{0{,}0273}{\tau_1} - \frac{0{,}117}{\tau_1^3}\right)\left(\frac{\pi_1}{\tau_1}\right)^2\right];$$

$$\tau_2 = \frac{T_2}{T_c};\ \tau_1 = \frac{T_1}{T_c};$$

$$\pi_2 = \frac{p_2}{p_c};\ \pi_1 = \frac{p_1}{p_c};$$

where R – gas constant NG, J/(kg·K);

δ_1, δ_2 – corrections of a specific enthalpy of NG which lead values of an enthalpy from a condition of a perfect gas to a condition of a real gas;

τ_1, τ_2 – the specified NG temperature on an entrance of CS and an exit from CS;

π_1, π_2 – reduced pressure of NG on an entrance of CS and an exit from CS;

T_c – critical temperature of NG, K;

p_c – critical pressure of NG, K.

Knowing h_{2a} and h_1 values, it is possible to find specific work of A_a (fig. 1):

$$A_a = h_{2a} - h_1,$$

and then to define specific work of A_p and further to determine polytrophic EF CS by a formula: $\eta = A_p / A_a$ [5, 6].

For specific works as A_p and A_a (fig. 1) we will write down formulas taking into account coefficient of a polytope n and coefficient of an adiabatic k [5]:

$$A_p = \frac{n}{n-1} R\left(T_2 - T_1\right); \tag{3}$$

$$A_a = \frac{k}{k-1} R\left(T_{2a} - T_1\right). \tag{4}$$

We will transform a formula (3) taking into account dependence of the NG parameters at polytrophic process [5, 6]:

$$A_p = \frac{n}{n-1} R\,T_1\left(\frac{T_2}{T_1} - 1\right);$$

$$A_p = \frac{n}{n-1} R\,T_1\left(\left[\frac{p_2}{p_1}\right]^{\frac{n-1}{n}} - 1\right). \tag{5}$$

Also we will transform a formula (4) taking into account dependence of the NG parameters at adiabatic process [5, 6]:

$$A_a = \frac{k}{k-1} R T_1 \left(\frac{T_{2a}}{T_1} - 1 \right);$$

$$A_a = \frac{k}{k-1} R T_1 \left(\left[\frac{p_{2a}}{p_1} \right]^{\frac{k-1}{k}} - 1 \right). \qquad (6)$$

Formulas (5) and (6) are used more often than formulas (3) and (4) since look more presentably though the result of calculations of A_p and A_a turns out identical.

In a formula of $\eta = A_p / A_a$ [5, 6] we will substitute the equations (3), (4) and we will receive:

$$\eta = \frac{\frac{n}{n-1} R(T_2 - T_1)}{\frac{k}{k-1} R(T_{2a} - T_1)}; \quad \eta = \frac{n}{n-1} \frac{k-1}{k} \frac{T_2 - T_1}{T_{2a} - T_1},$$

and at $T_2 \approx T_{2a}$ assumption, we will receive the simplified formula

$$\eta = \frac{n}{n-1} \frac{k-1}{k}. \qquad (7)$$

The formula (7) is used in the standard [3] where for the CS parameters the following equations are given:

$$\frac{k}{k-1} = 4{,}6 + 0{,}0041(t_{av} - 0) + \\ + 3{,}9(\Delta_{air} - 0{,}5) + 5{,}0\left(\frac{n-1}{n} - 0{,}3\right); \qquad (8)$$

$$t_{av} = \frac{t_1 + t_2}{2};$$

$$\frac{n-1}{n} = \frac{\lg \frac{T_2}{T_1}}{\lg \frac{p_2}{p_1}} = \frac{\ln \frac{T_2}{T_1}}{\ln \frac{p_2}{p_1}}; \quad \frac{n}{n-1} = \frac{\ln \frac{p_2}{p_1}}{\ln \frac{T_2}{T_1}}, \qquad (9)$$

where t_1, t_2 – NG temperature on an entrance and an exit of CS, ºC; t_{av} – average temperature of NG, ºC; Δ_{air} – relative density of NG by air.

The equation (8) is convenient that having defined k / (k-1) value and n / (n-1) value, it is possible to find η on a formula (7) at once.

The following equation for the NG parameters is given in literature [4]:

Table 1

Methods of calculation of values η

Methods	Description of calculation of parameters NG	Calculation η
Method **1** according to MM of the standard [1], rules of calculation [2]	We determine h_{2a}, h_1 values by formulas (1), (2) and we calculate $A_a = h_{2a} - h_1$. We determine $\frac{n}{n-1}$ by a formula (9) and we calculate A_p by a formula (5).	$\eta = \frac{A_p}{A_a}$
Method **2** according to MM of the standard [3]	We determine $\frac{k}{k-1}$ by a formula (8). We determine $\frac{n}{n-1}$ by a formula (9).	On a formula (7)
Method **3** according to MM [4]	We determine $\tilde{n}_p$ by a formula (11) and we calculate $\frac{k}{k-1} = \frac{c_p}{R}$ We determine $\frac{n}{n-1}$ by a formula (9).	On a formula (7)
Method **4** according to a formula (7)	We set value k for NG taking into account values of the following parameters: T_1, T_2, p_1, p_2. We determine $\frac{n}{n-1}$ by a formula (9).	On a formula (7)

$$Mc_{p} = 5{,}5 + (5{,}6 + 0{,}017\, t_2)\Delta_{air}\,, \qquad (10)$$

where M – molar mass of NG, kg/kmol; c_{p} – heat capacity of NG with a constant pressure if NG has properties of a perfect gas, kcal/(kg·ºC).

Dimension of the equation (10) kcal/(kmol·ºC). In literature [4] for Ì ñð two dimensions (kcal/(kmol·ºC) and kcal/(kmol·K)) that can be a typo or accounting not of temperature, and an interval of temperatures when intervals of degrees Kelvin and Celsius are equal:

$$\Delta T = T_2 - T_1 = (t_2 + 273{,}5) - (t_1 + 273{,}5) =$$
$$= t_2 - t_1 = \Delta t\,,$$

therefore for an interval we can use dimension as kcal/(kmol·ºC), and kcal/(kmol·K).

We will accept assumption that ñð ≈ ñðk where ñak – heat capacity of NG with a constant pressure and adiabatic process of compression. We will separate both members of equation (10) into M and we will substitute in the equation (10) instead of value of an interval of ΔÒ:

$$c_{pk} = \frac{1}{M}\left[5{,}5 + (5{,}6 + 0{,}017\Delta Ò)\Delta_{air}\right],$$

where dimension of c_{pk} – kcal/(kg·K).

If in the equation to use ΔT, then great values of c_{pk} therefore we will use not ΔT, but ΔT/2 turn out and we will rewrite (10) in a look:

$$c_{ak} = \frac{1}{M}\left[5{,}5 + \left(5{,}6 + 0{,}017\frac{\Delta T}{2}\right)\Delta_{air}\right],$$

and for change of dimension of c_{pk} of calories in Joules, we will increase the right member of equation on 4187 J (1 kcal = 4187 J):

$$c_{pk} = \frac{4187}{M}\left[5{,}5 + \left(5{,}6 + 0{,}017\frac{\Delta T}{2}\right)\Delta_{air}\right], \qquad (11)$$

where dimension of c_{pk} – J/(kg·K).

Replacement ΔT on ΔT/2 brought closer value c_{p} to c_{pk} value that we will check further at practical calculations of η.

For c_{pk} we will write down expression [6]:

$$c_{ak} = \frac{k}{k-1}R$$

and we will receive a formula

$$\frac{k}{k-1} = \frac{c_{pk}}{R}\,.$$

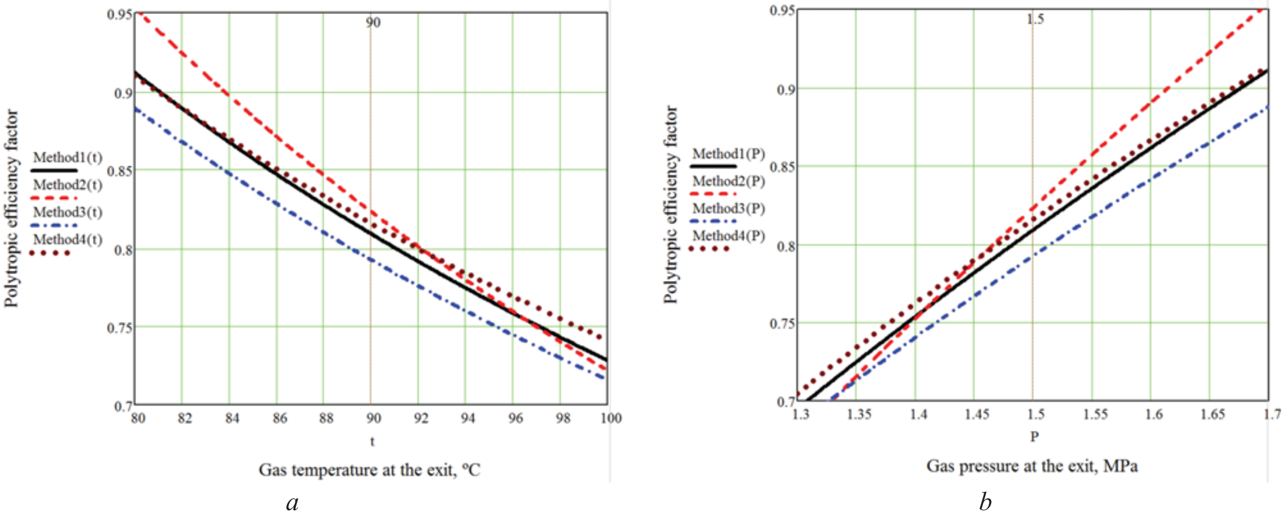
In the grant "Centrifugal gas compressors" developed by Solar for the size A_a the following formula is given:

$$A_a = \frac{R_{air}T_1}{\left(\frac{k-1}{k}\right)\Delta_{air}}\left(\left[\frac{p_2}{p_1}\right]^{\frac{k-1}{k}} - 1\right); \qquad (12)$$

$$\frac{R_{air}}{\Delta_{air}} = R\,,$$

where R_{air} – gas constant of air, J/(kg·K).

The formula for k not to be given in a grant of "Solar". It is possible to assume that the constant value k is used or the program of calculation of $A_{à}$ uses different values k in dependence on parameters of gas and modes of behavior of CS.



a *b*

Fig. 2 Calculation of η by four methods: a - input data: $p_1 = 0{,}6$ MPa; $p_2 = 1{,}6$ MPa; $t_1 = 4$ ºC; $t_2 = 90$ ºC; $\rho_{\text{s}} = 0{,}73$ kg/m³; b - the calculated parameters: $\eta_1 = 0{,}8091$; $\eta_2 = 0{,}8232$; $\eta_3 = 0{,}7925$; $\eta_4 = 0{,}8159$; n = 1,3804; $k_2 = 1{,}2934$; $k_3 = 1{,}2794$; $k_4 = 1{,}2900$.

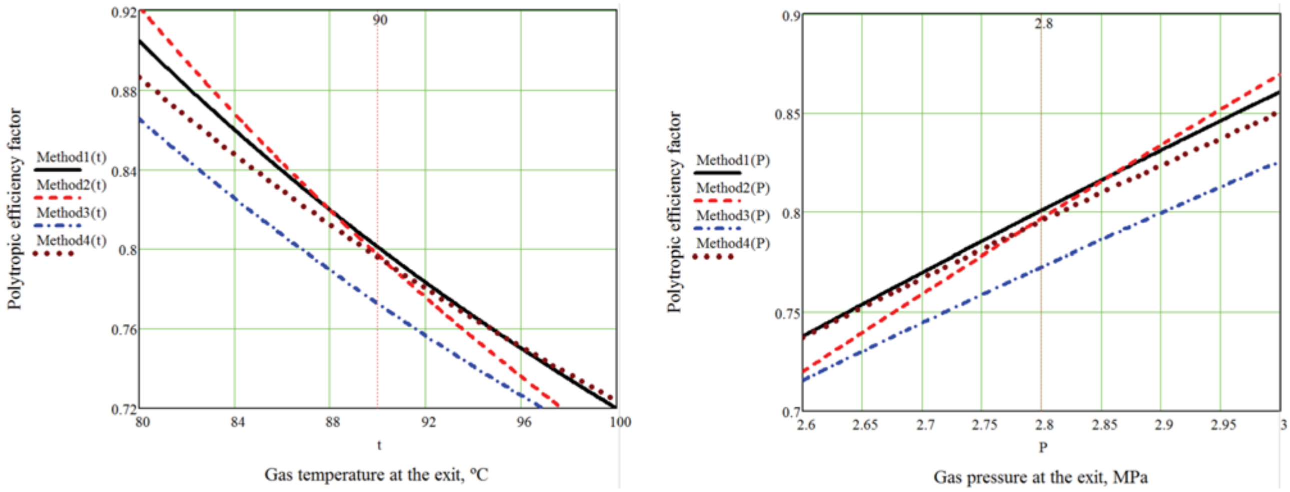


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Fig. 3 Calculation of η by four methods:a - input data: $p_1 = 1$ MPa; $p_2 = 2{,}8$ MPa; $t_1 = 3$ ºC; $t_2 = 90$ ºC; $\rho_{г} = 0{,}73$ kg/m³; b - the calculated parameters: $\eta_1 = 0{,}8009$; $\eta_2 = 0{,}7969$; $\eta_3 = 0{,}7725$; $\eta_4 = 0{,}7957$; n = 1,3938; $k_2 = 1{,}2906$; $k_3 = 1{,}2792$; $k_4 = 1{,}2900$.

We will rewrite a formula (12), having received the equation similar to the equation (6):

$$A_a = \frac{k}{k-1} T_1 R \left(\left[\frac{p_2}{p_1} \right]^{\frac{k-1}{k}} - 1 \right),$$

where assumption is accepted that outlet pressures from the compressor at polytrophic and adiabatic process of compression of gas are equal, that is $p_2 \approx p_{2a}$ (fig. 1).

For A_p we will write down a formula (at $p_2 \approx p_{2a}$):

$$A_p = \frac{n}{n-1} T_1 R \left(\left[\frac{p_2}{p_1} \right]^{\frac{n-1}{n}} - 1 \right)$$

and for definition of $\eta = A_p / A_a$, we will receive:

$$\eta = \frac{\dfrac{n}{n-1}}{\dfrac{k}{k-1}} \cdot \frac{\left[\dfrac{p_2}{p_1} \right]^{\frac{n-1}{n}} - 1}{\left[\dfrac{p_2}{p_1} \right]^{\frac{k-1}{k}} - 1}. \qquad (13).$$

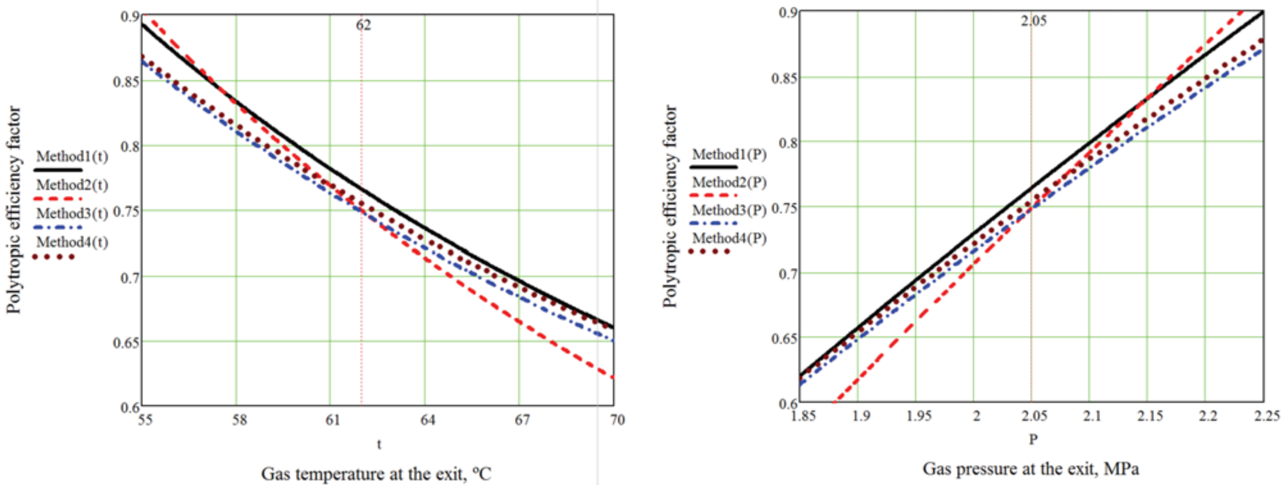

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Fig. 4 Calculation of η by four methods: a - input data: $p_1 = 2{,}2$ MPa; $p_2 = 3{,}75$ MPa; $t_1 = 33$ ºC; $t_2 = 83$ ºC; $\rho_{г} = 0{,}73$ kg/m³; b - the calculated parameters: $\eta_1 = 0{,}7658$; $\eta_2 = 0{,}7494$; $\eta_3 = 0{,}7484$; $\eta_4 = 0{,}7546$; n = 1,4243; $k_2 = 1{,}2874$; $k_3 = 1{,}2869$; $k_4 = 1{,}2900$.

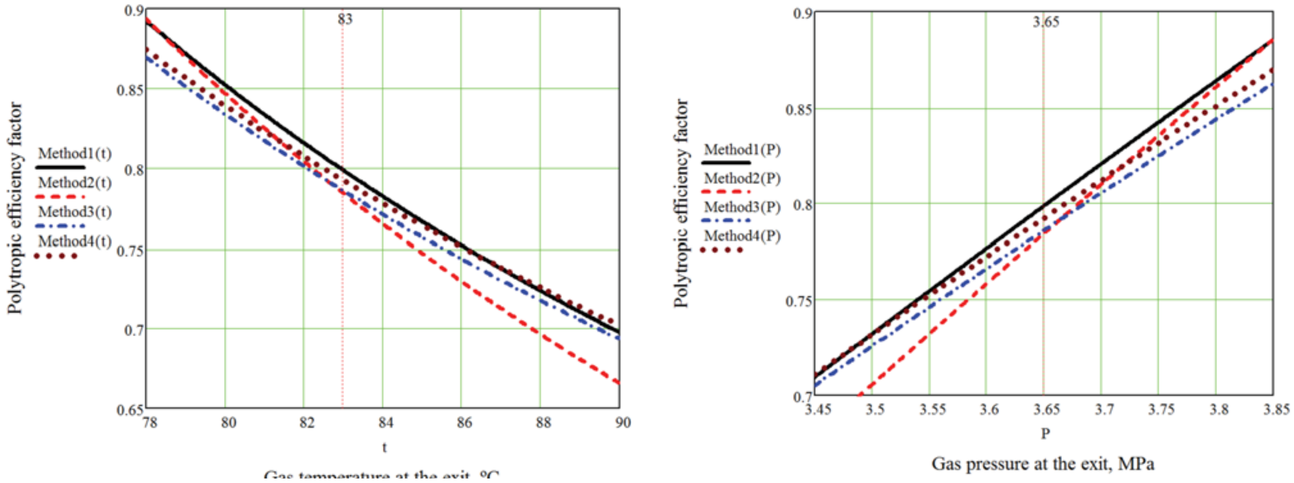

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Fig. 5 Calculation of η by four methods: a - input data: $p_1 = 2{,}2$ MPa; $p_2 = 3{,}75$ MPa; $t_1 = 33$ ºC; $t_2 = 83$ ºC; $\rho_{\$}$ = 0,73 kg/m³; b - the calculated parameters: $\eta_1 = 0{,}7998$; $\eta_2 = 0{,}7844$; $\eta_3 = 0{,}7955$; $\eta_4 = 0{,}7925$; n = 1,3960; $k_2 = 1{,}2862$; $k_3 = 1{,}2914$; $k_4 = 1{,}2900$.

We will enter the following designations:

$$\frac{\dfrac{n}{n-1}}{\dfrac{k}{k-1}} = S_1 < 1; \quad \frac{\left[\dfrac{p_2}{p_1}\right]^{\frac{n-1}{n}} - 1}{\left[\dfrac{p_2}{p_1}\right]^{\frac{k-1}{k}} - 1} = S_2 > 1.$$

also we will write down a formula (13) in a look

$$\eta = S_1 S_2 .$$

As $S_1 < 1$, and $S_2 > 1$, it is possible to receive result when $S_1 S_2 \geq 1$. At practical calculations for a formula (13), values η = 1,01…1,03 are received that confirms the made assumptions of great value of the work $S_1 S_2$.

Methods of calculation polytrophic EF CS

From the analyzed MM four methods of calculation of η shown in table 1 are allocated. The most difficult calculations are carried out in a method 1, the simplest – in a method 4.

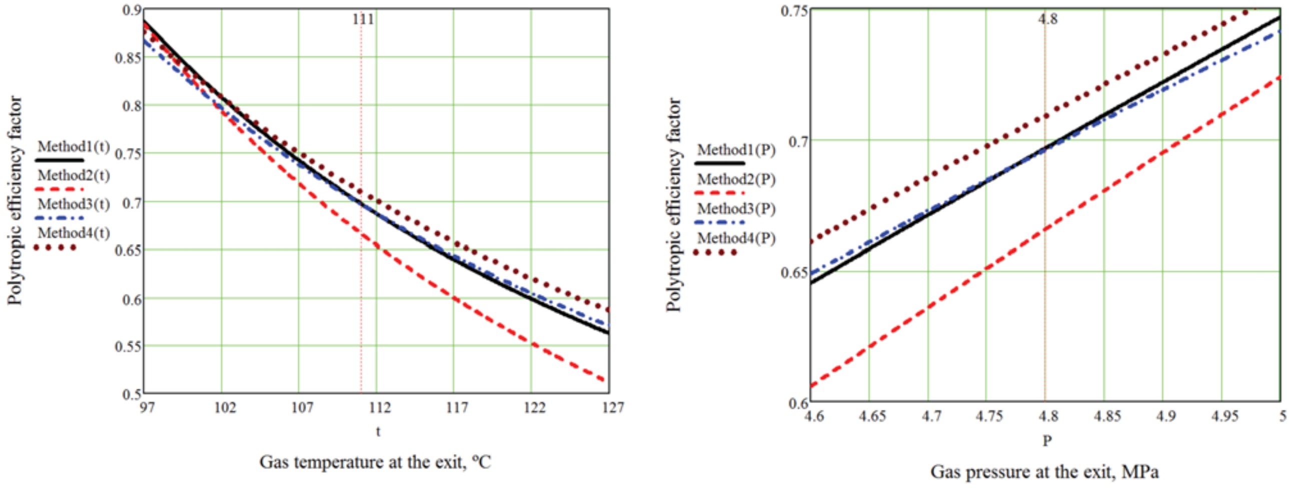

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Fig. 6 – Calculation of η by four methods:a - input data: $p_1 = 2{,}65$ MPa; $p_2 = 4{,}9$ MPa; $t_1 = 43$ ºC; $t_2 = 111$ ºC; $\rho_{\$}$ = 0,73 kg/m³; b - the calculated parameters: $\eta_1 = 0{,}6967$; $\eta_2 = 0{,}6658$; $\eta_3 = 0{,}6962$; $\eta_4 = 0{,}7093$; n = 1,4640; $k_2 = 1{,}2675$; $k_3 = 1{,}2831$; $k_4 = 1{,}2900$.

The η values calculated by methods 1-4 (table 1) are given in fig. 2-6.

In fig. 2-6 absolute pressures $ð_1$, $ð_2$ are given and the following parameters are used: $\rho_ȼ$ – NG density under standard conditions (20 ºC, 101325 Pa); η_1 – the η value calculated by a method 1; η_2 – the η value calculated by a method 2; η_3 – the η value calculated by a method 3; η_4 – the η value calculated by a method 4; k_2 – value k when calculating for a method 2; k_3 – value k when calculating for a method 3; $k_4 = 1{,}29$ – the constant value k accepted in a method 4.

The k_1 value (value k when calculating for a method 1) in fig. 2-6 isn't present as η_1 is calculated not by a formula (7) and as the relation $À_ï / À_à$ (table 1).

In calculations and schedules of fig. 2-6 the following constant parameters were accepted:

- for a method 3: M = 17,4 kg/kmol;
- for a method 4: k = 1,29.

In fig. 2-6 not only η values at the fixed t_2 value are shown, but also schedules of $\eta(t_2)$ and $\eta(p_2)$ where t_2 and p_2 change in the range of near real value (a dotted line on graphics) given in basic data. Schedules allow to analyze more precisely features of change of η for each method that shows features of MM of calculation of the NG parameters.

Replacement ΔT on ΔT/2 in a formula (11) showed good results in calculations of η (fig. 2-6), but we will note that with entrance pressure 0,5…1,0 MPa the method 3 gives the minimum η values.

Good results for calculation of η were shown by the simplest method 4 (table 1, fig. 2-6) which it is possible to apply under operating conditions CS. At $p_1 = 2{,}65$ MPa the method 4 gives the overestimated values (fig. 2-6) and it is better to use it at $p_1 = 0{,}5\text{-}2{,}3$ MPa.

By a method 2 at $p_1 = 2{,}65$ MPa the smallest polytrophic EF (fig. 2-6) therefore at $p_1 > 2{,}2$ MPa are better to use a method 3 which is simpler in calculations, than a method 1 was received.

Conclusions

1. According to the analyzed MM four methods of calculation of polytrophic EF CS (table 1) are described.
2. Practical calculations of polytrophic EF CS for four methods are carried out. Schedules at change of temperature and outlet pressure of CS are constructed that allows to compare the nature of change of η for each method.
3. It is shown that it is possible to use the simplest fourth method for calculation of polytrophic EF CS with inlet pressures of CS about 0,5-2,3 MPa.
4. With inlet pressures of CS more than 2,3 MPa it is better to use the third method of calculation (table 1).

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