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The Type-Conversion of Oscillations at the Excitation of Nonlinear Layered Media

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The present paper focuses on the development of a mathematical model, an effective algorithm and a self-consistent numerical analysis of the multifunctional properties of resonant scattering and generation of oscillations by nonlinear, cubically polarizable layered structures. It presents results of the numerical analysis characterizing the type-conversion of the generation/scattering oscillations of the nonlinear layered structures for one/two-sided acting fields at the generation/scattering frequency were taken into account and could be observed. These effects were observed at a symmetry violation of the nonlinear problem.

Key words: *cubically polarizable medium, resonance scattering, generation of oscillations, self-consistent analysis, a type-conversion of the oscillations.*

Ця праця зосереджена на розвитку математичної моделі, ефективного алгоритму та взаємоузгодженому чисельному аналізу багатофункціональних властивостей резонансного розсіяння та генерації коливань нелінійними, кубічне поляризуємими шарованими структурами. У праці наведені результати чисельного аналізу що характеризують перетворення типу коливань генерації/розсіяння для одно/двох сторонніх полів збудження нелінійних шарованих структур на частотах генерації/розсіяння. Ці ефекти спостерігаються при порушенні симетрії нелінійної задачі.

Ключові слова: *кубічне поляризуєма середа, резонансне розсіяння, генерація коливань, взаємоузгоджений аналіз, перетворення типу коливань.*

Данная работа сосредоточена на развитии математической модели, эффективного алгоритма и самосогласованного числового анализа мульти функциональных свойств резонансного рассеяния и генерации колебаний нелинейными, кубически поляризуемыми слоистыми структурами. В работе приведены результаты численного анализа, характеризующие преобразование типа генерируемых/рассеянных колебаний для одно/двух стороннего поля возбуждения нелинейных слоистых структур на частотах генерации/рассеяния. Эти эффекты наблюдались при нарушении симметрии нелинейной задачи.

Ключевые слова: *кубически поляризуемая среда, резонансное рассеяние, генерация колебаний, само согласованный анализ, преобразование типа колебаний.*

1. Introduction

Nonlinear dielectrics with controllable permittivity have a great application prospect in electronics and device technology [1-5]. The present paper focuses on the development of a mathematical model, an effective algorithm and a self-consistent numerical analysis of the multifunctional properties of resonant scattering and generation of oscillations by nonlinear, cubically polarizable layered structures [6-9]. The multifunctionality of the cubically polarizable layered media will be caused by the nonlinear mechanism between interacting oscillations - the incident oscillations exciting the nonlinear layer from the upper and lower half-spaces as well as the scattered and generated oscillations at the frequencies of excitation/scattering and generation. The study of the resonance properties of scattering and generation of oscillations by a nonlinear layered structure with a controllable permittivity in dependence on the variation of the intensities of the components of the exciting wave package is of particular interest.

2. Statement of the problem

The problem of resonant scattering and generation of harmonic oscillations by a nonlinear, nonmagnetic, isotropic, linearly E-polarized $\mathbf{E}=\left(E_x,0,0\right)^{\mathsf{T}}$, $\mathbf{H}=\left(0,H_y,H_z\right)^{\mathsf{T}}$, cubically polarizable $\mathbf{P}^{(\mathrm{NL})}=\left(P_x^{(\mathrm{NL})},0,0\right)^{\mathsf{T}}$, layered dielectric structure (see Fig. 1) is investigated in a self-consistent formulation [6-9]. The time dependency has the form $\exp(-in\omega t)$, $n=1,2,\ldots$

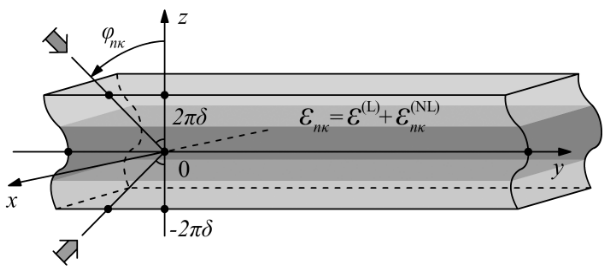

Fig. 1. The nonlinear layered dielectric structure.

The variables x,y,z,t denote dimensionless spatial-temporal coordinates such that the thickness of the layer is equal to $4\pi\delta$, with $\delta>0$; $\omega=\kappa c$ is the dimensionless circular frequency; $n\kappa=n\omega/c=2\pi/\lambda_{n\kappa}$ are dimensionless frequencies. This parameter characterizes the ratio of the true thickness h of the layer to the lengths of the incident waves $\lambda_{n\kappa}$, i.e. $h/\lambda_{n\kappa}=2n\kappa\delta$. Where $c=(\varepsilon_0\mu_0)^{-1/2}$ is a dimensionless parameter, the absolute value of which is equal to the velocity of light within the medium containing the layer, $\operatorname{Im} c=0$, ε_0 and μ_0 are the material parameters of the medium. The absolute values of the true variables x',y',z',t',ω' are given by the formulas $\left(x',y',z',t'\right)^{\mathsf{T}}=\dfrac{h}{4\pi\delta}\left(x,y,z,t\right)^{\mathsf{T}}$ and $\omega'=\dfrac{4\pi\delta}{h}\omega$.

The incidence of a packet of plane waves onto the layer at the angles $\left\{\varphi_{n\kappa},\pi-\varphi_{n\kappa}:\left|\varphi_{n\kappa}\right|<\pi/2\right\}_{n=1}^{3}$ and with respect to the amplitudes $\left\{a_{n\kappa}^{\mathrm{inc}},b_{n\kappa}^{\mathrm{inc}}\right\}_{n=1}^{3}$ at the frequencies $\left\{n\kappa\right\}_{n=1}^{3}$ is considered, where the excitation field consists of a strong field at the frequency κ (generating a field at the triple frequency) and of weak fields at the frequencies 2κ, 3κ (influencing on the process of generation of the third harmonic):

$$\left\{ \overline{E}_1^{\text{inc}}(n\kappa; y,z) \right\}_{n=1}^{3} \cup \left\{ \underline{E}_1^{\text{inc}}(n\kappa; y,z) \right\}_{n=1}^{3},$$

here

$$\left\{ \begin{Bmatrix} \overline{E}_1^{\text{inc}}(n\kappa; y,z) \\ \underline{E}_1^{\text{inc}}(n\kappa; y,z) \end{Bmatrix} = \begin{Bmatrix} a_{n\kappa}^{\text{inc}} \\ b_{n\kappa}^{\text{inc}} \end{Bmatrix} \exp\left[i\left(\Phi_{n\kappa} y \mp \Gamma_{n\kappa} \cdot (z \mp 2\pi\delta) \right) \right], \quad z \gtrless \pm 2\pi\delta \right\}_{n=1}^{3}.$$

In such a situation, taking into account Kleinman's rule (i.e. the equality of all the susceptibility tensor components $\chi_{xxxx}^{(3)}$ at the multiple frequencies [10]), the problem under consideration can be described by a system of nonlinear boundary value problems [6-9]

$$\begin{aligned} &\left[\nabla^2 + (n\kappa)^2 \varepsilon_{n\kappa}\left(z, \alpha(z), \{E_x(m\kappa; y,z)\}_{m=1}^3 \right) \right] E_x(n\kappa; y,z) \\ &= -(n\kappa)^2 \alpha(z) \left[\delta_n^1 E_x^{\,2}(2\kappa; y,z) E_x^{\,*}(3\kappa; y,z) \right. \\ &\left. + \delta_n^3 \left\{ \frac{1}{3} E_x^{\,3}(\kappa; y,z) + E_x^{\,2}(2\kappa; y,z) E_x^{\,*}(\kappa; y,z) \right\} \right], \quad n = 1, 2, 3, \end{aligned} \tag{1}$$

together with the following generalized boundary conditions:

(C1) $E_x(n\kappa; y,z) = U(n\kappa; z)\exp(i\Phi_{n\kappa} y)$, the quasi-homogeneity condition w.r.t. y,

(C2) $\Phi_{n\kappa} = n\Phi_\kappa$ or $\varphi_{n\kappa} = \varphi_\kappa$, the condition of phase synchronism of waves [6],

(C3) $\mathbf{E}_{\text{tg}}(n\kappa; y,z)$ and $\mathbf{H}_{\text{tg}}(n\kappa; y,z)$ are continuous at the boundaries of the layered structure,

(C4) $E_x^{\text{scat/gen}}(n\kappa; y,z) = \begin{Bmatrix} a_{n\kappa}^{\text{scat/gen}} \\ b_{n\kappa}^{\text{scat/gen}} \end{Bmatrix} \exp\left[i\left(\Phi_{n\kappa} y \pm \Gamma_{n\kappa}(z \mp 2\pi\delta) \right) \right], \quad z \gtrless \pm 2\pi\delta$ for $\operatorname{Im}\Gamma_{n\kappa} \equiv 0$ and $\operatorname{Re}\Gamma_{n\kappa} > 0$, the radiation condition w.r.t. the scattered/generated fields.

Here: $\nabla^2 = \partial^2/\partial y^2 + \partial^2/\partial z^2$, δ_n^k – Kronecker's symbol, $\mathbf{E}_{\text{tg}}(n\kappa; y,z)$ and $\mathbf{H}_{\text{tg}}(n\kappa; y,z)$ – the tangential components of the vectors of the full electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$, $\Gamma_{n\kappa} = \sqrt{(n\kappa)^2 - \Phi_{n\kappa}^{\,2}}$ and $\Phi_{n\kappa} = n\kappa\sin(\varphi_{n\kappa})$ – the transverse and longitudinal propagation constants of the nonlinear structure

$$\begin{aligned} &\varepsilon_{n\kappa} = \left\{ 1, |z| > 2\pi\delta; \text{ and } \varepsilon^{(\text{L})} + \varepsilon_{n\kappa}^{(\text{NL})}, |z| \le 2\pi\delta \right\}, \\ &\varepsilon^{(\text{L})} = 1 + 4\pi\chi_{11}^{(1)}(z), \\ &\varepsilon_{n\kappa}^{(\text{NL})} = \alpha(z) \left[\sum_{m=1}^{3} |E_1(m\kappa; y,z)|^2 \right. \\ &\left. + \left\{ \delta_n^1 \frac{\left[E_1^{\,*}(\kappa; y,z) \right]^2}{E_1(\kappa; y,z)} + \delta_n^2 \frac{E_1^{\,*}(2\kappa; y,z)}{E_1(2\kappa; y,z)} E_1(\kappa; y,z) \right\} E_1(3\kappa; y,z) \right], \end{aligned} \tag{2}$$

$\alpha(z)=6\pi\chi^{(3)}_{xxxx}(z)$ – the function of cubic susceptibility of the nonlinear medium, $\chi^{(1)}_{xx}$ and $\chi^{(3)}_{xxxx}$ – components of the susceptibility tensors of the nonlinear medium.

The sought complex Fourier amplitudes of the total scattered and generated fields in the problem (1), (C1)-(C4) at the multiple frequencies $\{n\kappa\}_{n=1}^{3}$ can be represented in the form

$$E_x(n\kappa;y,z)=U(n\kappa;z)\exp(i\Phi_{n\kappa}y)$$
$$=\begin{cases} a^{\text{inc}}_{n\kappa}\exp(i(\Phi_{n\kappa}y-\Gamma_{n\kappa}(z-2\pi\delta))) \\ \quad +a^{\text{scat/gen}}_{n\kappa}\exp(i(\Phi_{n\kappa}y+\Gamma_{n\kappa}(z-2\pi\delta))), & z>2\pi\delta, \\ U(n\kappa;z)\exp(i\Phi_{n\kappa}y), & |z|\le 2\pi\delta, \\ b^{\text{inc}}_{n\kappa}\exp(i(\Phi_{n\kappa}y+\Gamma_{n\kappa}(z+2\pi\delta))) \\ \quad +b^{\text{scat/gen}}_{n\kappa}\exp(i(\Phi_{n\kappa}y-\Gamma_{n\kappa}(z+2\pi\delta))), & z<-2\pi\delta. \end{cases} \qquad (3)$$

Taking into consideration (3), the nonlinear system (1), (C1)-(C4) is equivalent to a system of nonlinear boundary-value problems of Sturm-Liouville type, see [6-9],

$$\left[d^2/dz^2+\Gamma^2_{n\kappa}-(n\kappa)^2\{1-\varepsilon_{n\kappa}(z,\alpha(z),U(\kappa;z),U(2\kappa;z),U(3\kappa;z))\}\right]U(n\kappa;z)$$
$$=-(n\kappa)^2\alpha(z)\Big(\delta^1_n U^2(2\kappa;z)U^*(3\kappa;z)$$
$$+\delta^3_n\left\{\frac{1}{3}U^3(\kappa;z)+U^2(2\kappa;z)U^*(\kappa;z)\right\}\Big),\quad |z|\le 2\pi\delta, \qquad (4)$$

$$[i\Gamma_{n\kappa}-d/dz]U(n\kappa;2\pi\delta)=2i\Gamma_{n\kappa}\overline{U}^{\text{inc}}(n\kappa;2\pi\delta),$$
$$[i\Gamma_{n\kappa}+d/dz]U(n\kappa;-2\pi\delta)=2i\Gamma_{n\kappa}\underline{U}^{\text{inc}}(n\kappa;-2\pi\delta),\quad n=1,2,3,$$

and also to a system of one-dimensional nonlinear integral equations w.r.t. the unknown functions $U(n\kappa;\cdot)\in L_2(-2\pi\delta,2\pi\delta)$, see [6-9],

$$U(n\kappa;z)+\frac{i(n\kappa)^2}{2\Gamma_{n\kappa}}\int_{-2\pi\delta}^{2\pi\delta}\exp(i\Gamma_{n\kappa}|z-\xi|)\left[1-\varepsilon_{n\kappa}\left(\xi,\alpha(\xi),\{U(m\kappa;\xi)\}_{m=1}^{3}\right)\right]U(n\kappa;\xi)d\xi$$
$$=\frac{i(n\kappa)^2}{2\Gamma_{n\kappa}}\int_{-2\pi\delta}^{2\pi\delta}\exp(i\Gamma_{n\kappa}|z-\xi|)\alpha(\xi)\Big[\delta^1_n U^2(2\kappa;\xi)U^*(3\kappa;\xi)$$
$$+\delta^3_n\left\{\frac{1}{3}U^3(\kappa;\xi)+U^2(2\kappa;\xi)U^*(\kappa;\xi)\right\}\Big]d\xi \qquad (5)$$
$$+\overline{U}^{\text{inc}}(n\kappa;z)+\underline{U}^{\text{inc}}(n\kappa;z),\quad n=1,2,3.$$

Here: $\overline{U}^{\text{inc}}(n\kappa;z)=a^{\text{inc}}_{n\kappa}\exp[-i\Gamma_{n\kappa}(z-2\pi\delta)]$, $\underline{U}^{\text{inc}}(n\kappa;z)=b^{\text{inc}}_{n\kappa}\exp[i\Gamma_{n\kappa}(z+2\pi\delta)]$.

The solution of the problem (1), (C1)-(C4), represented in (3), can be obtained from (4) or (5) using the formulas

$$U(n\kappa;2\pi\delta)=a^{\text{inc}}_{n\kappa}+a^{\text{scat/gen}}_{n\kappa},\quad U(n\kappa;-2\pi\delta)=b^{\text{inc}}_{n\kappa}+b^{\text{scat/gen}}_{n\kappa},\quad n=1,2,3.$$

3. Self-consistent analysis of the system of nonlinear integral equations

According to [6-9], the application of suitable quadrature rules to the system nonlinear integral equations (5) leads to a system of complex-valued nonlinear algebraic equations of the second kind

$$\begin{aligned}&\left[\mathbf{I}-\mathbf{B}_{n\kappa}\left(\mathbf{U}_{\kappa},\mathbf{U}_{2\kappa},\mathbf{U}_{3\kappa}\right)\right]\mathbf{U}_{n\kappa}\\ &=\delta_n^1\mathbf{C}_{\kappa}\left(\mathbf{U}_{2\kappa},\mathbf{U}_{3\kappa}\right)+\delta_n^3\mathbf{C}_{3\kappa}\left(\mathbf{U}_{\kappa},\mathbf{U}_{2\kappa}\right)+\overline{\mathbf{U}}_{n\kappa}^{\text{inc}}+\underline{\mathbf{U}}_{n\kappa}^{\text{inc}},\quad n=1,2,3\end{aligned} \tag{6}$$

where $\mathbf{U}_{n\kappa}=\{U_l(n\kappa)\}_{l=1}^N\approx\{U(n\kappa;z_l)\}_{l=1}^N$ – the vectors of the unknown approximate values of the solution, $\left\{\{z_l\}_{l=1}^N: z_1=-2\pi\delta<...<z_l<...<z_N=2\pi\delta\right\}$ – a discrete set of interpolation nodes, $\mathbf{I}=\{\delta_l^m\}_{l,m=1}^N$ – the identity matrix, $\mathbf{B}_{n\kappa}\left(\mathbf{U}_{\kappa},\mathbf{U}_{2\kappa},\mathbf{U}_{3\kappa}\right)$ – nonlinear matrices, $\mathbf{C}_{\kappa}\left(\mathbf{U}_{2\kappa},\mathbf{U}_{3\kappa}\right)$, $\mathbf{C}_{3\kappa}\left(\mathbf{U}_{\kappa},\mathbf{U}_{2\kappa}\right)$ – the vectors of the right-hand sides determined by the choice of the quadrature rule, and $\overline{\mathbf{U}}_{n\kappa}^{\text{inc}}=\left\{a_{n\kappa}^{\text{inc}}\exp\left[-i\Gamma_{n\kappa}\left(z_l-2\pi\delta\right)\right]\right\}_{l=1}^N$, $\underline{\mathbf{U}}_{n\kappa}^{\text{inc}}=\left\{b_{n\kappa}^{\text{inc}}\exp\left[+i\Gamma_{n\kappa}\left(z_l+2\pi\delta\right)\right]\right\}_{l=1}^N$ – the vectors induced by the incident wave packets at the multiple frequencies $n\kappa$, $n=1,2,3$.

A solution of (6) can be found iteratively by the help of a block Jacobi method, where at each step a system of linearized algebraic equations is solved [6-9]:

$$\left\{\begin{array}{l}\left\{\begin{array}{l}\left[\mathbf{I}-\mathbf{B}_{\kappa}\left(\mathbf{U}_{\kappa}^{(s-1)},\mathbf{U}_{2\kappa}^{(S_2(q))},\mathbf{U}_{3\kappa}^{(S_3(q))}\right)\right]\mathbf{U}_{\kappa}^{(s)}\\ \quad=\mathbf{C}_{\kappa}\left(\mathbf{U}_{2\kappa}^{(S_2(q))},\mathbf{U}_{3\kappa}^{(S_3(q))}\right)+\overline{\mathbf{U}}_{\kappa}^{\text{inc}}+\underline{\mathbf{U}}_{\kappa}^{\text{inc}}\end{array}\right\}_{s=1}^{S_1(q):\,\eta_1(S_1(q))<\xi}\\ \left\{\begin{array}{l}\left[\mathbf{I}-\mathbf{B}_{2\kappa}\left(\mathbf{U}_{\kappa}^{(S_1(q))},\mathbf{U}_{2\kappa}^{(s-1)},\mathbf{U}_{3\kappa}^{(S_3(q))}\right)\right]\mathbf{U}_{2\kappa}^{(s)}\\ \quad=\overline{\mathbf{U}}_{2\kappa}^{\text{inc}}+\underline{\mathbf{U}}_{2\kappa}^{\text{inc}}\end{array}\right\}_{s=1}^{S_2(q):\,\eta_2(S_2(q))<\xi}\\ \left\{\begin{array}{l}\left[\mathbf{I}-\mathbf{B}_{3\kappa}\left(\mathbf{U}_{\kappa}^{(S_1(q))},\mathbf{U}_{2\kappa}^{(S_2(q))},\mathbf{U}_{3\kappa}^{(s-1)}\right)\right]\mathbf{U}_{3\kappa}^{(s)}\\ \quad=\mathbf{C}_{3\kappa}\left(\mathbf{U}_{\kappa}^{(S_1(q))},\mathbf{U}_{2\kappa}^{(S_2(q))}\right)+\overline{\mathbf{U}}_{3\kappa}^{\text{inc}}+\underline{\mathbf{U}}_{3\kappa}^{\text{inc}}\end{array}\right\}_{s=1}^{S_3(q):\,\eta_3(S_3(q))<\xi}\end{array}\right\}_{q=1}^{Q} \tag{7}$$

with

$$\eta_n(s)=\left\|\mathbf{U}_{n\kappa}^{(s)}-\mathbf{U}_{n\kappa}^{(s-1)}\right\|\Big/\left\|\mathbf{U}_{n\kappa}^{(s)}\right\|,\ n=1,2,3.$$

Given a relative error tolerance $\xi>0$, the terminating index $Q\in\mathrm{N}$ and $Q\geq 2$ is defined by the requirement

$$\max\{\eta_1(Q),\eta_2(Q),\eta_3(Q)\}<\xi\ .$$

Finally we mention that the classification of scattered and generated fields of the dielectric layer by the $H_{m,l,p}$-type adopted in our paper is identical to that given in [6-9, 11]. In the case of E-polarization, $H_{m,l,p}$ (or $TE_{m,l,p}$) denotes the type of polarization of the wave field under investigation. The subscripts indicate the number of local maxima of $|E_x|$ (or $|U|$, as $|U| = |E_x|$, see (3)) in the dielectric layer, i.e. along the coordinate axes x, y и z (see Fig. 1). Since the considered waves are homogeneous along the x-axis and quasi-homogeneous along the y-axis, we actually study fields of the type $H_{0,0,p}$ (or $TE_{0,0,p}$), where the subscript p is equal to the number of local maxima of the function $|U|$ of the argument z in $[-2\pi\delta, 2\pi\delta]$.

4. The type-conversion of the generated oscillations of a decanalizing nonlinear layer media

The Fig. 2 show the properties of a decanalizing ($\alpha(z) < 0$) nonlinear dielectric structure with the parameters $\varepsilon^{(L)}(z) = 16$, $\alpha(z) = -0.01$, $\delta = 0.5$. The excitation of the nonlinear layer takes place from above by only one strong top electromagnetic field at $\varphi_\kappa = 0^0$ and the basic frequency $\kappa = \kappa^{\text{inc}} = 0.375$, i.e. $\left\{a_\kappa^{\text{inc}} \neq 0, b_\kappa^{\text{inc}} = a_{2\kappa}^{\text{inc}} = b_{2\kappa}^{\text{inc}} = a_{3\kappa}^{\text{inc}} = b_{3\kappa}^{\text{inc}} = 0\right\}$.

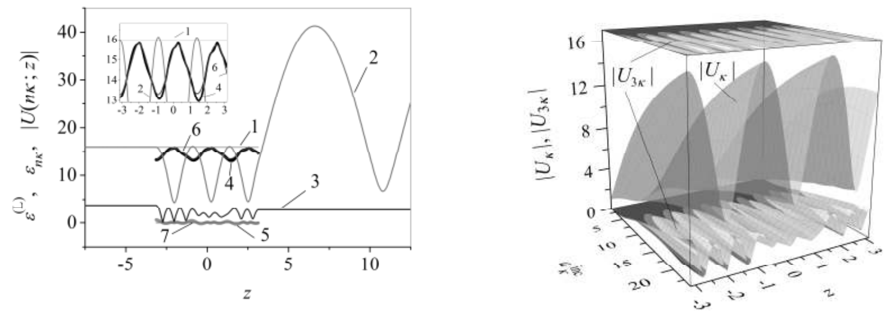

Fig. 2. Curves: 1 – $\varepsilon^{(L)}$, 2 – $|U(\kappa; z)|$, 3 – $|U(3\kappa; z)|$, 4 – $\text{Re}(\varepsilon_\kappa)$, 5 – $\text{Im}(\varepsilon_\kappa)$, 6 – $\text{Re}(\varepsilon_{3\kappa})$, 7 – $\text{Im}(\varepsilon_{3\kappa}) \equiv 0$ at $a_\kappa^{\text{inc}} = 24$ (left); and surfaces of scattered $\left|U_\kappa\left[a_\kappa^{\text{inc}}, z\right]\right|$ and generated $\left|U_{3\kappa}\left[a_\kappa^{\text{inc}}, z\right]\right|$ fields in the nonlinear layer (right).

In Fig. 2 we see a symmetry violation in the generated field in the radiation zone (graphs no. 3, left). In particular, inside the decanalizing layer the symmetry violation is accompanied by the presence of an inflection point $z \approx 1{,}25$, where $|U(3\kappa; z)| = 1{,}81$ for $a_\kappa^{\text{inc}} = 24$, see graph no. 3 (left) and the surface $\left|U_{3\kappa}\left[a_\kappa^{\text{inc}}, z\right]\right|$ (right). In the considered ranges of amplitudes a_κ^{inc}, the plane waves exciting the nonlinear layer under the angle φ_κ produce a scattered field U_κ of the type $H_{0,0,4}$. The generated field $U_{3\kappa}$ changes its type with increasing amplitude a_κ^{inc}. The generation of a third harmonic field $U_{3\kappa}$ is observed in the range $a_\kappa^{\text{inc}} \in [4, 24]$, (right). Here it is of the type $H_{0,0,10}$ for $a_\kappa^{\text{inc}} \in [4, 23)$ and of the type $H_{0,0,9}$ for $a_\kappa^{\text{inc}} \in [23, 24]$. *The type-conversion of the generated oscillations from* $H_{0,0,10}$ *to* $H_{0,0,9}$ *with increasing* a_κ^{inc} is due to the loss of one maximum point of the function $|U(3\kappa; z)|$ for $z \in [-2\pi\delta, 2\pi\delta]$ at the inflection point $z = 1.15$ for $a_\kappa^{\text{inc}} = 23$, see the point with coordinates $(a_\kappa^{\text{inc}} = 23, z = 1.15, |U_{3\kappa}| = 1.61)$ on the surface $|U_{3\kappa}[a_\kappa^{\text{inc}}, z]|$ (right).

The increase in the intensity of the excitation field leads to critical inflection points of the function (the absolute value of the amplitude of the scattered/generated field) identifying the type of oscillation. If in these points the local maximum of the function along the characteristic spatial coordinate of the investigated structure (the transverse coordinate along the height of the nonlinear layer) is lost, then the effect of type-conversion of the radiation field occurs. The amplitudes of the incident field, for which the described effect is observed, can be called the threshold of the considered types of oscillations.

5. The type-conversion of the scattering oscillations at the two-sided excitation of nonlinear layered structures

The Fig. 3 show the properties of the nonlinear structures with the parameters:

$$\{\varepsilon^{(L)}(z), \alpha(z)\} = \begin{cases} \{\varepsilon^{(L)} = 16, \alpha = \alpha_1\}, & z \in [-2\pi\delta, z_1 = -2\pi\delta/3); \\ \{\varepsilon^{(L)} = 64, \alpha = \alpha_2\}, & z \in (z_1 = -2\pi\delta/3, z_2 = 2\pi\delta/3); \\ \{\varepsilon^{(L)} = 16, \alpha = \alpha_3\}, & z \in [z_1 = 2\pi\delta/3, 2\pi\delta); \end{cases}$$

at $\delta = 0.5$, $\alpha_1 = \alpha_3 = 0.01$, $\alpha_2 = -0.01$. The excitation takes place from above and below by electromagnetic fields at the basic frequency $\kappa = \kappa^{\text{inc}} = 0.25$ and $\{a_\kappa^{\text{inc}} = 38, b_\kappa^{\text{inc}} \neq 0, a_{2\kappa}^{\text{inc}} = b_{2\kappa}^{\text{inc}} = a_{3\kappa}^{\text{inc}} = b_{3\kappa}^{\text{inc}} = 0\}$.

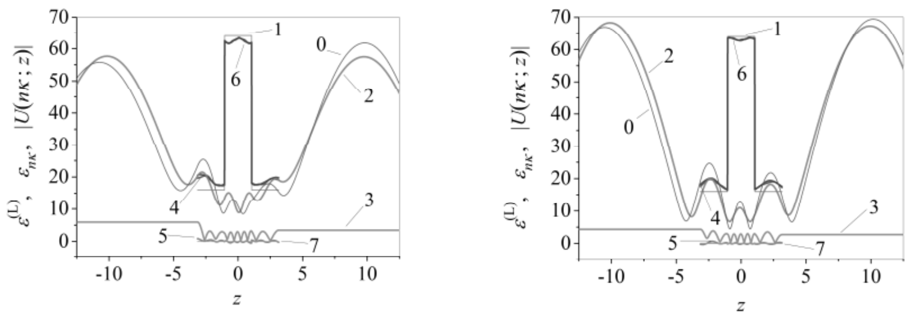

Fig. 3. Curves: 0 – $|U(\kappa; z)|$ for $\alpha(z) = 0$,1 – $\varepsilon^{(L)}$,2 – $|U(\kappa; z)|$,3 – $|U(3\kappa; z)|$,4 – $\text{Re}(\varepsilon_\kappa)$,5 – $\text{Im}(\varepsilon_\kappa)$,6 – $\text{Re}(\varepsilon_{3\kappa})$,7 – $\text{Im}(\varepsilon_{3\kappa}) \equiv 0$ at $\{\varphi_\kappa, 180^0 - \varphi_\kappa\}$ with $\varphi_\kappa = 0^0$ and (left): $\{a_\kappa^{\text{inc}} = 38, b_\kappa^{\text{inc}} = 20\}$; (right): $\{a_\kappa^{\text{inc}} = 38, b_\kappa^{\text{inc}} = 30\}$.

The absolute values of the amplitudes of the total scattering $|U(\kappa;z)|$ and generation $|U(3\kappa;z)|$ fields, for different variants of two-sided $\{a_\kappa^{\text{inc}}, b_\kappa^{\text{inc}}\}$ normal $\{\varphi_\kappa, 180^0 - \varphi_\kappa\}\big|_{\varphi_\kappa = 0^0}$ excitation of the nonlinear structure are illustrated by the graphs no. 2 and no. 3 in Fig. 3. They may be identified as oscillations of the types $\{H_{0,0,4}$ and $H_{0,0,9}\}$ (left), $\{H_{0,0,3}$ and $H_{0,0,9}\}$ (right). We mention a resonance effect of type-conversion of the total scattered field $H_{0,0,4} \Leftrightarrow H_{0,0,3}$. The resonant type-conversion of oscillations, which is observed for the two-sided excitation of both linear and nonlinear structures, occurs if the symmetry of the excitation is violated.

The resonant type-conversion of oscillations, which is observed for the two-sided excitation of nonlinear structures, occurs if the symmetry of the nonlinear structures or excitation is violated. The fundamental difference in the occurrence of this effect between the nonlinear and the linear situations consists in the presence of the nonlinear part $\varepsilon_{n\kappa}^{(\text{NL})}$ of the dielectric permittivity $\varepsilon_{n\kappa}$, $n = 1, 3$, see (2).

We mention that the behaviour of the quantity $\varepsilon_{n\kappa}^{(\text{NL})} = \varepsilon_{n\kappa} - \varepsilon^{(\text{L})}$ can be estimated easily by means of the graphs nos. 4, 5, 6, 7 and 1 in Figs. 2 (left) and 3 (left/*right*). The graph no. 1 depicts the dielectric permittivity $\varepsilon^{(\text{L})}$ of a linear non-absorbing $\text{Im}\left(\varepsilon^{(\text{L})}\right) \equiv 0$ structure. The graphs nos. 4, 5, 6, 7 show the real and imaginary parts of the nonlinear dielectric permittivity $\varepsilon_{n\kappa}$, $n = 1, 3$, for the excitation variants under consideration, see Figs. 2 (left) and 3 (left/*right*). In particular, $\text{Im}\left(\varepsilon_\kappa^{(\text{NL})}\right)$ takes positive and negative values along the height of the nonlinear layer, for all the considered excitation variants of the nonlinear structure. The variation of this quantity characterizes the energy consumption of the nonlinear medium which is spent for the third harmonic generation.

The numerical results for the scattering and generation of a wave package by a nonlinear cubically polarizable layer are obtained by means of the solution of the system of nonlinear integral equations (5). Applying Simpson´s quadrature rule, the system (5) is reduced to a system of nonlinear algebraic equations (6). The numerical solution of (6) is carried out using a self-consistent iterative algorithm (7) based on a block Jacobi method. In the investigated range of problem parameters the dimension of the algebraic systems was 301 and 501 in the case of single-layered (Section 4) and three-layered (Section 5) structures, respectively. The relative error of the calculations did not exceed 10^{-7}.

6. Conclusion

The type-conversion effect was observed at a symmetry violation of the nonlinear problem caused by different amplitudes of the excitation fields. For the first time, two-sided acting fields at the scattering frequency were taken into account and a type-conversion of the oscillations could be observed. The latter effect was observed at a symmetry violation of the nonlinear problem caused by different amplitudes of the excitation fields. This effect may serve as a basis for numerical and analytical methods for the synthesis and analysis of nonlinear structures in the vicinity of critical points of the amplitude-phase dispersion, similar to the approach developed in the papers [12]. That is, mathematical models for the control of anomalous scattering and generation properties of nonlinear structures via the variation of amplitudes in a one/two-sided excitation of a nonlinear structure at scattering and generation frequencies near the resonance frequencies of the linearized spectral problems can be created.

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