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Heterogeneities in antenna cavity and the scattering properties of some special antennas. Numerical analysis, part I

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Целью работы являлось проведение численного анализа влияния включения диэлектрических вставок и дополнительных цилиндрических преград в полость цилиндрических антенн с конечным числом продольных щелей на технические характеристики описанных устройств. В работе для реализации цели были использованы математические модели рассеяния Е-поляризованных волн на базе граничных сингулярных интегральных уравнений. Дискретизация построенных интегральных уравнений проведена по методу дискретных особенностей. С использованием построенных дискретных моделей получены графики зависимостей поперечников полного рассеяния от размера дополнительной преграды и заполнения полости антенны диэлектриками различной проницаемости.

Ключевые слова: *сингулярное интегральное уравнение, щелевая антенна, численное решение, задача дифракции.*

Метою цієї роботи було проведення чисельного аналізу впливу включень діелектричних вставок та додаткових циліндричних перешкод у порожнині циліндричних антен з скінченою кількістю подовжних щілин на технічні характеристики описаних пристроїв. В роботі для реалізації цілі були використані математичні моделі розсіяння Е-поляризованих хвиль на базі граничних сингулярних інтегральних рівнянь. Дискретизація побудованих інтегральних рівнянь проведена за методом дискретних особливостей. Використовуючи побудовані дискретні моделі отримані графіки залежностей поперечника повного розсіяння від розмір додаткової перешкоди та заповнення порожнини антен діелектриками різної проникності.

Ключові слова: *сингулярне інтегральне рівняння, чисельний розв'язок, задача дифракції.*

The aim of the work was carrying out the numerical analysis of influence of dielectric insertions and additional cylindrical obstacles in the cavity of cylindrical antennas with a finite number of longitudinal slots on technical characteristics of the described devices. For this aim implementation, in this work, mathematical models developed on the base of boundary singular integral equations were applied to E-polarized plane wave scattering. Discretization of the derived integral equations was carried out by the method of discrete singularities. With the help of the obtained discrete models, the authors have plotted the graphs for dependency of total diffusion cross-section on additional obstacle size. Computations were done for different antenna cavity fillings.

Key words: *singular integral equations, numerical solution, slot antennas, diffraction problem.*

Introduction

Derivation of mathematical models in order to calculate different types of antenna devices is an important part of contemporary research in the field of high frequency radioelectronics. Various appliances that scatter and transmit electromagnetic waves can be modelled with open conducting screens of different configurations. Such devices include numerous aperture and flush antennas as well as leaky wave ones. Such slot antennas do not perturb aerodynamics of the objects they are installed on.

Aero- and hydrodynamic peculiarities of submarines, planes, rockets, and other mobile objects require these very structural features of antennas [1-7].

The hollow structure of antennas in earlier-built models does not fully describe real objects. Consequently, derivation of models for slotted antennas filled with dielectric and ones containing additional reflecting structures in their cavity draw the models nearer to real devices [2-5].

The methods, worked out recently by Shestopalov V.P., Nosich A.I., Levin L., Ziolkowski R.W., Voytovich N.N. to calculate technical characteristics of cylindrical structures with circular cross-section and longitudinal slots have a number of shortages [1,2,6]. These methods are applicable to the structures that have one slot only, it is impossible to extend their usage to a greater number of slots.

The discrete vortex method was used in the works of Belocerkovskiy S.M., Lifanov I.K., Dovgiy S.A. for calculations in applied aerodynamics. Yu.V. Gandel, V.O. Mishenko, V.D. Dushkin, and their students used the method of discrete singularities to solve problems of diffraction on multislotted arrays and for electromagnetic field computation in waveguides. The same tools have been successfully used to solve a set of problems [8-10]. In the author's works fulfilled under scientific supervision of Yu.V. Gandel, [8-9], the method of discrete singularities was used for problems related to wave diffraction on hollow cylinders with multiple slots. In these works, the results of near and far field computation for the structures with different number of slots are discussed. As it was said before, of special interest are the filled structures; and introduction of an additional reflector will allow to extend the line of structures under estimation. This was the reason to carry out the work in attempt to obtain evaluation formulas for analysis of dependence of total diffusion cross-sections on different coefficient of dielectric permittivity and on the size of the introduced additional reflector.

The statement of a boundary value problem for Maxwell's equations

Let us outline the geometry of the problem. We will use the following notation: the slot traces on cylindrical surface, that are the arcs S_{R_2} of the external circle with the radius R_2 centered at the origin of the coordinates, will be denoted as (a_q, b_q) (fig. 1-2). Then

$$L = \bigcup_{i=1}^{p} (\alpha_i, \beta_i), \; CL = [-\pi, \pi] \setminus L .$$

The stationary case $\mathbf{E}(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}) e^{-i\omega t}$, $\mathbf{H}(\mathbf{x}, t) = \mathbf{H}(\mathbf{x}) e^{-i\omega t}$ was considered (monochromatic waves with angular frequency ω). The wave vector of a plane wave is directed perpendicularly to the generatrices of cylinders. In the outer cylinder, several longitudinal slots are cut. The space between the cylinders is filled with dielectric.

The fields in the inner and in the outer part of the considered electrodynamic structure satisfy the Maxwell's equations [6]:

$$rot\mathbf{E} = i\omega\mu\mathbf{H}, \; rot\mathbf{H} = -i\omega\varepsilon\mathbf{E} \qquad (1)$$

It is assumed, that electrical-field sources are absent, thus the following condition is satisfied:

$$div\mathbf{E} = 0. \quad (2)$$

Over the perfectly conducting surfaces the following boundary condition is true:

$$\left[\mathbf{E}(r,\varphi),\mathbf{n}\right]_{\substack{r=R_1 \\ \varphi\in[0,2\pi)}} = 0, \ \left[\mathbf{E}(\mathrm{r},\varphi),\mathbf{n}\right]_{\substack{r=R_2 \\ \varphi\in L}} = 0. \quad (3)$$

here $\mathbf{n}$ is the outward vector normal to the cylindrical surfaces.

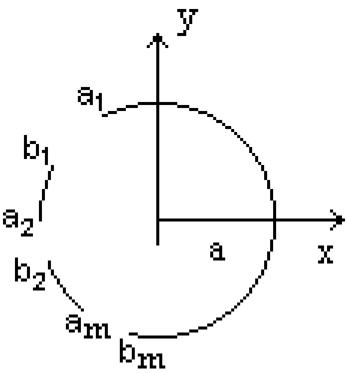

Fig.1. Cross-section of a cylindrical antenna with a finite number of longitudinal slots, circle in the section

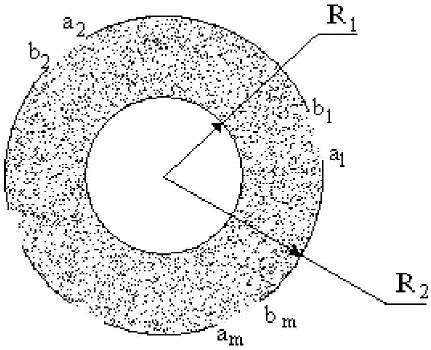

Fig.2. Section of a cylindrical metal structure filled with dielectric

Considering the case when electromagnetic field does not depend on the coordinate z, we obtain the following two-dimensional problem:

The system of Maxwell's equations (1), falls into two independent subsystems [6,8], one of which can be reduced to the Neumann boundary value problem for Helmholtz equation (H-polarization); and the other one to the Dirichlet boundary value problem for Helmholtz equation (E- polarization). For both components of the total field, it is enough to find a single function, which is the longitudinal component of the magnetic field $H_z(r,\varphi)$ or electric field $E_z(r,\varphi)$, correspondingly. One of the components found, another can be restored uniquely.

A mathematical model for the Dirichlet problem

Let us consider a E- polarized wave. In this case the component $E_z(r,\varphi)$ meets the following conditions:

– Helmholtz equation

$$\Delta E_z(r,\varphi) + k^2 E_z(r,\varphi) = 0, \quad (4)$$

here $k^2 = \varepsilon\mu\omega^2$;

– boundary condition

$$E_z(R_2,\varphi) = -E_{0z}(\mathrm{R}_2,\varphi), \varphi \in L, \quad (5)$$

here the given incident field $E_{0z}(r,\varphi) = e^{ikr\cos\varphi}$, $k = \dfrac{\omega}{c}$, c is the speed of light in vacuum;

– Sommerfeld radiation condition and Meixner's edge condition

$$\frac{\partial E_z(r,\varphi)}{\partial r} - ikE_z(r,\varphi) = o\left(\frac{1}{\sqrt{r}}\right),\ r\to\infty;\ \int_\Omega\left[k^2|E_z|^2 + |\nabla E_z|^2\right]ds < \infty \qquad (6)$$

for any bounded area $\Omega\subset R^2$.

As for the Dirichlet problem $u(r,\varphi) = E_z(r,\varphi)$, let us denote limitations for the sought function $u(r,\varphi)$ over the inner ($R_1 < r < R_2$) and the outer ($r > R_2$) areas of the ring by $u^+(r,\varphi)$ and $u^-(r,\varphi)$, correspondingly; and for Neumann problem $u(r,\varphi) = H_z(r,\varphi)$. If the functions $u^+(r,\varphi)$, $R_1 < r < R_2$ and $u^-(r,\varphi)$, $r > R_2$ satisfy the Helmholtz equation, and the so-called matching conditions are fulfilled:

$$u^+(r,\varphi)\Big|_{r=R_2} = u^-(r,\varphi)\Big|_{r=R_2},\ \varphi\in C\overline{L}, \qquad (7)$$

and

$$\left.\frac{\partial u^+(r,\varphi)}{\partial r}\right|_{r=R_2} = \left.\frac{\partial u^-(r,\varphi)}{\partial r}\right|_{r=R_2},\ \varphi\in C\overline{L}, \qquad (8)$$

then there exists such function $u(r,\varphi)$, which satisfies the Helmholtz equation in the area $\{r > R_1,\ \varphi\in[0,2\pi)\}$ excluding union of arcs L of the circle S_{R_2}; and $u(r,\varphi) = u^+(r,\varphi)$ *for* $R_1 < r < R_2$ while $u(r,\varphi) = u^-(r,\varphi)$ *for* $r > R_2$.

Let us write down the paired Fourier series for the electric field component $E_z(r,\varphi)$. Acting in the same way as in [8], let us write down the Fourier representations for the fields:

$$E_z^+(\rho,\varphi) = \sum_{n=-\infty}^{\infty} C_n^+ V_n^E\left(k^+\rho\right)e^{in\varphi},\ \rho\in[R_1,R_2], \qquad (9)$$

here $V_n^E\left(k^+\rho\right) = \dfrac{J_n\left(k^+R_1\right)Y_n\left(k^+\rho\right) - Y_n\left(k^+R_1\right)J_n\left(k^+\rho\right)}{J_n\left(k^+R_1\right)Y_n\left(k^+R_2\right) - Y_n\left(k^+R_1\right)J_n\left(k^+R_2\right)}$,

$$E_z^-(\rho,\varphi) = \sum_{n=-\infty}^{\infty} C_n^- H_n^{(1)}\left(k^-\rho\right)e^{in\varphi},\ \rho > R_2, \qquad (10)$$

here $J_n(z)$, $Y_n(z)$, $H_n^{(1)}(z)$ are the Bessel, Neumann and Hankel functions (1st kind) of order n.

Using the matching condition (7) and the boundary conditions (5) we derive the formula for the Fourier coefficients:

$$C_n^+ = C_n^- H_n^{(1)}\left(k^-R_2\right) \equiv C_n,\ n\in\mathbf{Z}. \qquad (11)$$

The modified paired Fourier series with the introduced coefficients C_n is as follows:

$$\begin{cases} \sum\limits_{n=-\infty}^{\infty} B_n e^{in\varphi} = 0, & \varphi \in L \\ \sum\limits_{n=-\infty}^{\infty} B_n \Gamma_n^E e^{in\varphi} = -E_{0z}(R_2, \varphi), & \varphi \in CL \end{cases} . \qquad (12)$$

here $B_n = C_n \left(k^+ V_n^{E'}\left(k^+ R_2\right) - k^- \dfrac{H_n^{(1)'}\left(k^- R_2\right)}{H_n^{(1)}\left(k^- R_2\right)} \right),\ n \in \mathbf{Z}$,

$\Gamma_n^E = \left(k^+ V_n^{E'}\left(k^+ R_2\right) - k^- \dfrac{H_n^{(1)'}\left(k^- R_2\right)}{H_n^{(1)}\left(k^- R_2\right)} \right)^{-1}$, and Γ_n^E has the estimation $\Gamma_n^E - \dfrac{A_1}{|n|} = O\left(\dfrac{1}{n^2}\right)$, where $A_1 = \dfrac{k^- R_2}{2}$

Let us rewrite the second equation from (12) in the form of equality:

$$A_1 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{B_n}{|n|} e^{in\varphi} + B_0 \Gamma_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} B_n \left(\Gamma_n - A_1 \frac{1}{|n|} \right) e^{in\varphi} = -E_{0z}(R_2, \varphi),\ \varphi \in CL, \qquad (13)$$

Acting in the same way as in [8-9,11], let us introduce an unknown function $v(\varphi) = \sum\limits_{n=-\infty}^{\infty} B_n e^{in\varphi}$, with which all the unknown coefficients are expressed as $B_n = \dfrac{1}{2\pi} \int\limits_{CL} v(\varphi) e^{-in\varphi} d\varphi$. Meixner's edge condition (6) will be fulfilled if restriction of the function $v(\theta)$ over the arc (α_q, β_q) is written as

$$v(\theta)\Big|_{(\alpha_q, \beta_q)} = \frac{w_q(\theta)}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}},\ \alpha_q < \theta < \beta_q, \qquad (14)$$

where $w_q(\theta),\ \theta \in [\alpha_q, \beta_q]$ is a Holder continuous function.

In the work [9], with the help of parametric representation of integral operator with logarithmic kernel, the equation (13) was transformed into the following integral equation:

$$-\frac{A_1}{\pi} \sum_{q=1}^{m} \int_{\alpha_q}^{\beta_q} w_q(\theta) \ln|\theta - \varphi| \frac{d\theta}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}} - \frac{\Gamma_0}{2\pi} \sum_{q=1}^{m} \int_{\alpha_q}^{\beta_q} w_q(\theta) \frac{d\theta}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}} -$$

$$-\frac{A_1}{\pi} \sum_{q=1}^{m} \int_{\alpha_q}^{\beta_q} w_q(\theta) \ln\left| \frac{\sin\frac{\theta - \varphi}{2}}{\frac{\theta - \varphi}{2}} \right| \frac{d\theta}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}} + \frac{1}{\pi} \sum_{q=1}^{m} \int_{\alpha_q}^{\beta_q} \frac{K(\theta, \varphi) w_q(\theta) d\theta}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}} = .$$

$$= -u_0(r, \varphi)\Big|_{r=R_2},\qquad \varphi \in CL. \qquad (15)$$

Differentiating the equation (15) with respect to parameter t_0, we obtain a singular integral equation. CL is a union of disjoint intervals. Let us introduce the functions: $g_q : (-1,1) \to (\alpha_q, \beta_q) : t \mapsto \theta = \frac{\beta_q - \alpha_q}{2} t + \frac{\beta_q + \alpha_q}{2}$ taking into account that $\frac{w_q(\theta)}{\sqrt{(\beta_q - \theta)(\theta - \alpha_q)}} \equiv \frac{2}{\beta_q - \alpha_q} \frac{\gamma_q(t)}{\sqrt{1-t^2}}$, $K(g_p(t), g_q(t_0)) = \tilde{K}_{pq}(t, t_0)$.

Such substitution transforms the integral equation over the system of intervals into the system of integral equations over the standard interval $(-1,1)$. This system of singular integral equations is written as:

$$\frac{A_1}{\pi} \text{v.p.} \int_{-1}^{1} \frac{\gamma_q(t)}{t - t_0} \frac{dt}{\sqrt{1-t^2}} + \frac{A_1}{\pi} \frac{\beta_q - \alpha_q}{2} \sum_{\substack{p=1 \\ p \neq q}}^{m} \int_{-1}^{1} \gamma_p(t) \frac{1}{g_p(t) - g_q(t_0)} \frac{dt}{\sqrt{1-t^2}} +$$

$$+ \frac{A_1}{2\pi} \frac{\beta_q - \alpha_q}{2} \sum_{p=1}^{m} \int_{-1}^{1} \gamma_p(t) \left[ctg \frac{g_p(t) - g_q(t_0)}{2} - \frac{2}{g_p(t) - g_q(t)} \right] \frac{dt}{\sqrt{1-t^2}} +$$

$$\frac{1}{\pi} \sum_{p=1}^{m} \int_{-1}^{1} \frac{Z_{pq}(t, t_0) \gamma_p(t) dt}{\sqrt{1-t^2}} = f_q'(t_0), \tag{16}$$

where $Z_{pq}(t, t_0) = \left(\tilde{K}_{pq}(t, t_0) \right)'_{t_0} = \frac{b_q - a_q}{2} \sum_{n=1}^{\infty} n \left(\Gamma_n - B \frac{1}{n} \right) \sin\left(n \left(g_p(t) - g_q(t_0) \right) \right)$,

$q = 1, \ldots, m$,

and the additional condition is:

$$\frac{A_1}{\pi} \left(\pi \ln 2 - \ln \left| \frac{\beta_q - \alpha_q}{2} \right| \right) \int_{-1}^{1} \gamma_q(t) \frac{dt}{\sqrt{1-t^2}} -$$

$$\frac{A_1}{\pi} \sum_{\substack{p=1 \\ p \neq q}}^{m} \int_{-1}^{1} \int_{-1}^{1} \ln \left| g_p(t) - g_q(t_0) \right| \frac{dt_0}{\sqrt{1-t_0^2}} \gamma_p(t) \frac{dt}{\sqrt{1-t^2}} -$$

$$- \frac{\Gamma_0}{2} \sum_{p=1}^{m} \int_{-1}^{1} \gamma_p(t) \frac{dt}{\sqrt{1-t^2}} - \frac{A_1}{\pi} \sum_{p=1}^{m} \int_{-1}^{1} \int_{-1}^{1} \ln \left| \frac{\sin \frac{g_p(t) - g_q(t_0)}{2}}{\frac{g_p(t) - g_q(t_0)}{2}} \right| \frac{dt_0}{\sqrt{1-t_0^2}} \gamma_p(t) \frac{dt}{\sqrt{1-t^2}} +$$

$$+ \frac{1}{\pi} \sum_{p=1}^{m} \int_{-1}^{1} \int_{-1}^{1} \tilde{K}_{pq}(t, t_0) \frac{dt_0}{\sqrt{1-t_0^2}} \gamma_p(t) \frac{dt}{\sqrt{1-t^2}} = \int_{-1}^{1} f_q(t_0) \frac{dt_0}{\sqrt{1-t_0^2}}, \ q = 1, \ldots, m. \tag{17}$$

At that the integral $\text{v.p.} \int_{-1}^{1} \frac{\gamma(t)}{t - t_0} \frac{dt}{\sqrt{1-t^2}}$ should be understood in the sense of the Cauchy principal value.

Discrete mathematical model

In course of discretization of the integral equation (16) and the additional condition (17), the unknown function $\gamma_p(t)$ is substituted with an interpolating polynomial, and the integrals within the equation are replaced with interpolatory quadrature formulae [10], but first the equation kernels should be substituted with their interpolating polynomials having the same nodes, as it was done, for example, in [8].

$$\begin{cases} \dfrac{B}{N_q}\sum\limits_{i=1}^{N_q}\dfrac{\gamma_q\left(t_i^{N_q}\right)}{t_i^{N_q}-t_{0j}^{N_q}}+\sum\limits_{p=1}^{m}\dfrac{1}{N_p}\sum\limits_{i=1}^{N_p}G_{pq}\left(t_i^{N_p},t_{0j}^{N_q}\right)\gamma_p\left(t_i^{N_p}\right)=f_q'\left(t_{0j}^{N_q}\right) \\ \dfrac{B}{N_q}\left(\pi\ln 2-\ln\left|\dfrac{\beta_q-\alpha_q}{2}\right|\right)\sum\limits_{i=1}^{N_q}\gamma_q\left(t_i^{N_q}\right)+ \\ \qquad +\sum\limits_{p=1}^{m}\dfrac{1}{N_p}\sum\limits_{i=1}^{N_p}Q_{pq}\left(t_i^{N_p}\right)\gamma_p\left(t_i^{N_p}\right)=\int\limits_{-1}^{1}f_q(t_0)\dfrac{dt_0}{\sqrt{1-t_0}} \end{cases} \tag{18}$$

here $j=1,...,N_q-1$, $q=1,...,m$, and the functions $G_{pq}\left(t_i^{N_p},t_{0j}^{N_q}\right)$, $Q_{pq}\left(t_i^{N_p}\right)$ are expressed in terms of ones described above (see for example [8]), and $t_k^N=\cos\dfrac{2k-1}{2N}\pi$ are Chebyshev polynomial zeros of the I type and $t_{0j}^N=\cos\dfrac{j\pi}{N}$ are Chebyshev polynomial zeros of the II type.

Having solved the linear equation system (18), we find the values of the interpolating polynomial in the points t_k^N with prescribed accuracy. Unknown coefficients C_n are calculated with the help of this formula:

$$C_n=B_n\Gamma_n^E=\frac{\Gamma_n^E}{2}\sum_{q=1}^{m}\sum_{k=1}^{N_q}\gamma_q\left(t_k^{N_q}\right)e^{-ing_q\left(t_k^{N_q}\right)}$$

Let us derive the expression for the E- polarized wave power scattered in space:

$$\sigma=\mathrm{Re}\int\limits_0^{2\pi}E_z(r,\varphi)H_\varphi^*(r,\varphi)rd\varphi, \tag{19}$$

here r is the radius of any cylindrical surface that encloses the screen.

Using the representation of the field component $E_z^-(r,\varphi)$ [8], and the relationships among the components in Maxwell's equations, we obtain the component $H_\varphi(r,\varphi)$ of the field in the following form

$$H_\varphi(r,\varphi)=\frac{1}{i\omega\mu}\sum_{n=-\infty}^{\infty}kC_n^-H_n^{(1)\prime}(kr)e^{in\varphi} \tag{20}$$

Substituting expressions for the components $H_\varphi(r,\varphi)$ (20) and $E_z(r,\varphi)$ into the power equation (19) and utilizing the $H_\varphi(r,\varphi)$ property of being a complex conjugate, we obtain:

$$\sigma = \frac{4}{k}\sum_{n=-\infty}^{\infty}\left|\frac{A_n\Gamma_n}{H_n^{(1)}(ka)}\right|^2,\ A_n = \frac{1}{2}\sum_{q=1}^{m}\frac{1}{n_q}\sum_{j=1}^{n_q}\gamma_q\left(t_j^{n_q}\right)e^{-ing_q\left(t_j^{n_q}\right)},\quad n\in\mathbf{Z} \qquad (21)$$

Using the found values of interpolating polynomial as the unknown function $\gamma_p(t)$ values, we find the power, scattered into space.

Now we can construct graphs of functional characteristics: power, directivity diagram for the component of electric field, and also the field in the near zone.

The results of numerical calculations

As the result of calculations (21), graphs were obtained for total diffusion cross-section dependency on dielectric characteristics of inner filling of the antenna. Besides, the radius of the inner cylinder was changing. As an example, see figures 3-5 that show the graphs for plane wave incidence onto the structure with three slots in the outer cylinder.

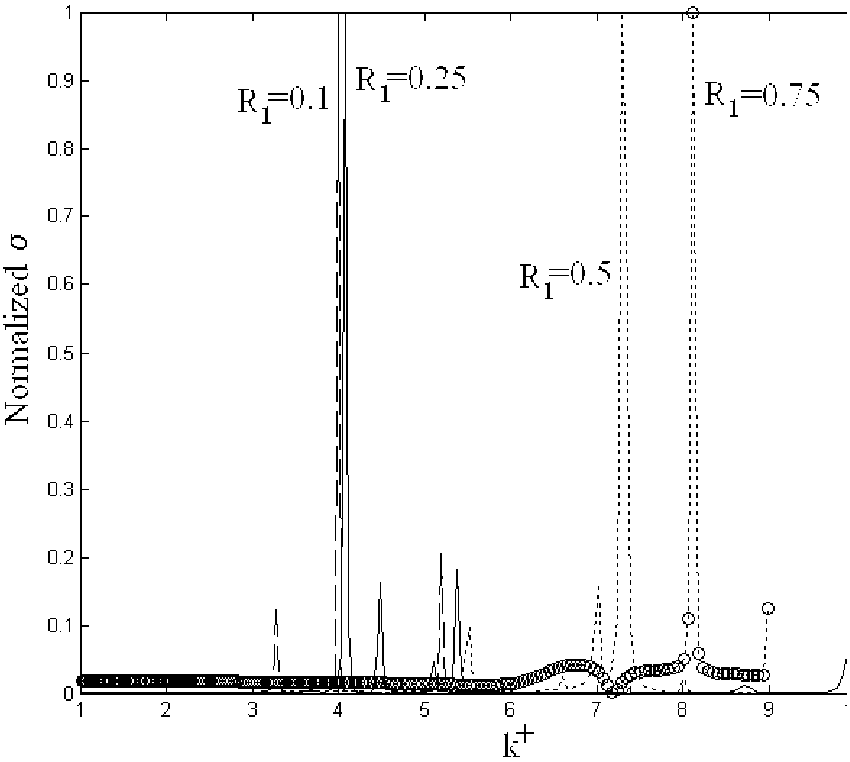

Fig.3. Normalized power for incident wave number κ=1

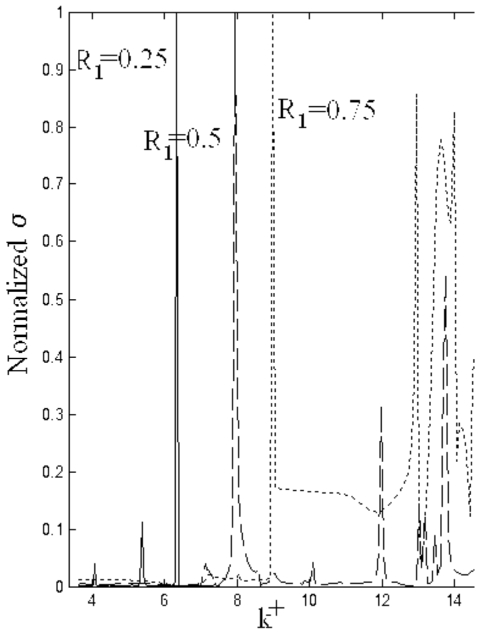

Fig.4. Normalized power-for incident wave number κ=4

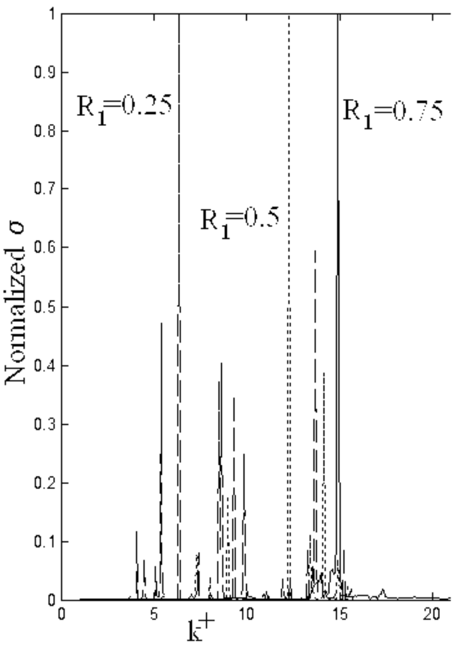

Fig.5. Normalized power-for incident wave number κ=6

Summary

In the work, the formula was derived for numerical evaluation of total diffusion cross-sections of plane waves scattered on cylindrical structures with a finite number of longitudinal slots, cylindrical reflecting structure coaxial with the outer cylinder, and filled with homogeneous dielectric inserts.

On the basis of the obtained formula, analysis of diffracted field characteristics in dependence on the coefficient of dielectric filling was performed.

The influence of the additional reflector in the antenna cavity as well as the dependence of characteristics on the inner cylinder size were shown.

In the future, it is interesting to perform similar analysis for H-polarized wave and the structures with impedance boundary condition.

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