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Discrete singularities method in problems of seismic and impulse impacts on reservoirs

D. V. Krutchenko[1)], E. A. Strelnikova[1),2)], Yu. S. Shuvalova[3)]
[1]*"Podgorny Institute For Mechanical Engineering Problems" National Academy of Sciences of Ukraine,*
[2]*V. N. Karazin Kharkiv National University,*
[3]*Ukrainian State University of Railway Transport*

A numerical method to simulate impulse and seismic effects on storages filled with a liquid has been proposed. The liquid is supposed to be ideal, incompressible, and its current is irrotational. The fluid pressure satisfies the Cauchy-Lagrange integral. To determine it, a system of integral equations has been obtained. Its numerical solution is obtained by the boundary element method. The eigenvalues and the forms of liquid vibrations have been obtained. The proposed method has made it possible to estimate the level of the free surface under the action of a suddenly applied force.

Key words: *tanks with liquid, method of integral equations, free and forced oscillations*

Предложен численный метод, для моделирования импульсного и сейсмического воздействия на хранилища с жидкостью. Предполагается что жидкость идеальная, несжимаемая, а её течение безвихревое. Давление жидкости удовлетворяет интегралу Коши-Лагранжа. Для его определения получена система интегральных уравнений. Её численное решение получено методом граничных элементов. Получены собственные значения и формы колебаний жидкости. Предложенный метод позволил оценить уровень свободной поверхности при внезапно приложенной нагрузке.

Ключевые слова*: резервуары с жидкостью, метод интегральных уравнений, свободные и вынужденные колебания*

Запропоновано чисельний метод для моделювання імпульсу і сейсмічної дії на сховища з рідиною. Припускається, що рідина ідеальна, нестислива, а її рух є безвихровим. Тиск рідини задовольняє інтегралу Коші-Лагранжа. Для його визначення отримана система інтегральних рівнянь. Її чисельний розв’язок отримано методом граничних елементів. Отримано власні значення і форми коливань рідини. Запропонований метод дозволив визначити рівень вільної поверхні при раптово прикладеному навантаженні.

Ключові слова*: резервуары с жидкостью, метод интегральных уравнений, свободные и вынужденные колебания*

1. Problem statement and its topicality

Containers and tanks for storing oil, flammable and poisonous liquids are widely used in various fields of engineering such as power engineering and transportation, as well as, in aircraft industry, chemical, oil and gas industry. These tanks usually operate under increased technological loadings and they are filled with oil, flammable or toxic agents. As a result of a sudden action of earthquakes, shockwaves or other force majeure circumstances the liquid stored in tanks may be affected by intensive sloshing.

Sloshing is a phenomenon observed in a number of industrial facilities: containers for storage of liquefied gas, oil or fuel tanks, tanks of cargo tankers. It is known that partially filled tanks are affected by especially intensive sloshing. It can lead to high pressure on tank walls, to destruction of structures or loosing stability, and to leakage of dangerous contents, that in turn, can result in serious ecological consequences.

The analysis of research devoted to the problems of liquid sloshing in tanks is given in R. A. Ibrahim's works [1, 2]. The works [3-5] also deal with liquid sloshing in cylindrical tanks under seismic loadings.

In this paper the problem concerning liquid vibrations in a shell of revolution is considered. We designate a moistened shell surface by S_1, and a free surface by S_0. Suppose the Cartesian coordinate system 0xyz is connected with the shell, the liquid free surface S_0 coincides with the x0y plane at the state of rest (fig. 1)

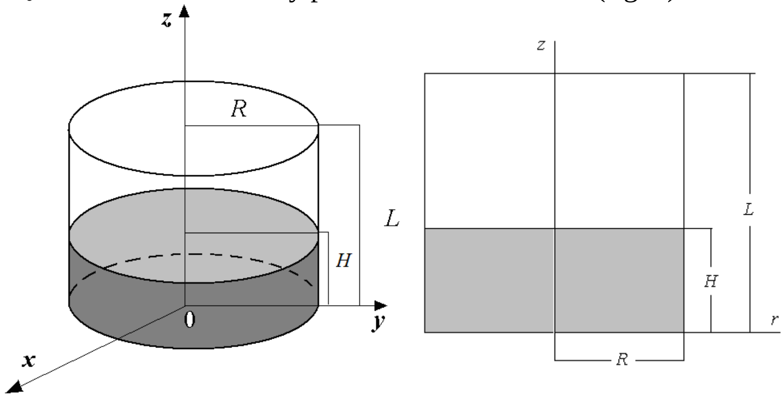

Fig.1. Fluid-filled cylindrical shell and its sketch

Suppose that the liquid is an ideal and incompressible one and its movement starts from the state of rest and is irrotational. Then there exists a liquid velocity potential Φ

$$V_x = \frac{\partial \Phi}{\partial x}; V_y = \frac{\partial \Phi}{\partial y}; V_z = \frac{\partial \Phi}{\partial z},$$

which satisfies the Laplace's equation.

We determine the pressure *p* on shell walls from the linearized Cauchy-Lagrange's integral by the following formula

$$p = -\rho_l \left(\frac{\partial \Phi}{\partial t} + gz \right) + p_0 + a_s(t)x,$$

Here Φ is the velocity potential, g is the acceleration of gravity, z is a point vertical coordinate in the liquid, ρ_l is the liquid density, p_0 is an atmospheric pressure, $a_s(t)$ is a function which characterizes the external influence (a horizontal seism or an impulse).

On the free surface of liquid the following conditions are to be satisfied:

$$\left.\frac{\partial \Phi}{\partial n}\right|_{S_0} = \frac{\partial \zeta}{\partial t}; \quad p - p_0|_{S_0} = 0,$$

where the function ζ describes the form and location of the free surface.

Thus, for the potential we have the following boundary problem

$$\nabla^2 \Phi = 0; \left.\frac{\partial \Phi}{\partial \mathbf{n}}\right|_{S_1} = 0; \left.\frac{\partial \Phi}{\partial n}\right|_{S_0} = \frac{\partial \zeta}{\partial t}; \quad p - p_0|_{S_0} = 0; \left.\frac{\partial \Phi}{\partial t} + g\zeta + a_s(t)x\right|_{s_0} = 0.$$

Having determined the velocity potential Φ and the function ζ, we establish the height of raising the free surface and determine the liquid pressure on shell walls.

2. The mode superposition method

Consider the potential **Φ** in the next form

$$\Phi = \sum_{k=1}^{M} \dot{d}_k \varphi_k \,. \tag{1}$$

For the functions φ consider the following boundary problems:

$$\nabla^2 \varphi_k = 0\,,\ \left.\frac{\partial \varphi_k}{\partial \mathbf{n}}\right|_{S_1} = 0\,, \tag{2}$$

$$\left.\frac{\partial \varphi_k}{\partial \mathbf{n}}\right|_{S_0} = \frac{\partial \varsigma}{\partial t};\quad \frac{\partial \varphi_k}{\partial t} + g\zeta = 0\,. \tag{3}$$

Let us differentiate the second relation in (3) and substitute it for the received equality $\frac{\partial \varsigma}{\partial t}$ from the first relation. Further we present the functions φ_k in the following form $\varphi_k(t,x,y,z) = e^{i\chi_k t}\varphi_k(x,y,z)$. We come to the eigenvalue

$$\frac{\partial \varphi_k}{\partial n} = \frac{\chi_k^2}{g}\varphi_k\,. \tag{4}$$

As the equation for the free surface we obtain the expression

$$\zeta = \sum_{k=1}^{M} d_k \frac{\partial \varphi_k}{\partial n}\,. \tag{5}$$

In cylindrical coordinates system we have the following expression

$$\varphi_k(r,z,\theta) = \varphi_k(r,z)\cos\alpha\theta \tag{6}$$

Here α is a harmonica number. Thus, frequencies and modes of free vibrations are considered separately for different α.

We present φ as potentials of simple and double layers [5]

$$2\pi\varphi(P_0) = \iint_S \frac{\partial \varphi}{\partial n}\frac{1}{|P-P_0|}dS - \iint_S \varphi \frac{\partial}{\partial n}\frac{1}{|P-P_0|}dS\,. \tag{7}$$

Here $S = S_1 \cup S_0$; points P and P_0 belong to the surface S. By $|P - P_0|$ we denote the Cartesian distance between points P and P_0.

With the boundary conditions (2), (3) we come to the system of the integral equations in the form [6, 7]:

$$\begin{cases} 2\pi\varphi_1 + \iint_{S_1} \varphi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_1 - \frac{\kappa^2}{g}\iint_{S_0}\varphi_0 \frac{1}{r}dS_0 + \iint_{S_0}\varphi_0 \frac{\partial}{\partial z}\left(\frac{1}{r}\right)dS_0 = 0, \\ -\iint_{S_1}\varphi_1 \frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_1 - 2\pi\varphi_0 + \frac{\kappa^2}{g}\iint_{S_0}\varphi_0\frac{1}{r}dS_0 = 0. \end{cases} \tag{8}$$

Here for convenience we denote values of potential on the free surface by φ_0 and by φ_1 on the shell walls.

We look for the solution of the system (8) in form (9).

Previously, having integrated the equation (8) by the variable θ, we have obtained the following system of one-dimensional singular equations.

$$2\pi\varphi(z_0)+\int_\Gamma \varphi(z)Q(z,z_0)r(z)d\Gamma-\int_0^R q(\rho)\Psi(P,P_0)\rho d\rho=\int_\Gamma w(z)\Psi(P,P_0)r(z)d\Gamma_1;\ P_0\in S_1, \quad (9)$$

$$\int_\Gamma \varphi(z)Q(z,z_0)r(z)d\Gamma-\int_0^R q(\rho)\Psi(P,P_0)\rho d\rho=\int_\Gamma w(z)\Psi(P,P_0)r(z)d\Gamma_1;\ P_0\in S_0.$$

Here

$$Q(z,z_0)=\frac{4}{\sqrt{a+b}}\left\{\frac{1}{2r}\left[\frac{r^2-r_0^2+(z_0-z)^2}{a-b}\mathrm{E}_\alpha(k)-\mathrm{F}_\alpha(k)\right]n_r+\frac{z_0-z}{a-b}\mathrm{E}_\alpha(k)n_z\right\};$$

$$\Psi(P,P_0)=\frac{4}{\sqrt{a+b}}\mathrm{F}_\alpha(k);\ \mathrm{E}_\alpha(k)=(-1)^\alpha\left(1-4\alpha^2\right)\int_0^{\pi/2}\cos 2\alpha\psi\sqrt{1-k^2\sin^2\psi}\,d\psi;$$

$$\mathrm{F}_\alpha(k)=(-1)^\alpha\int_0^{\pi/2}\frac{\cos 2\alpha\psi d\psi}{\sqrt{1-k^2\sin^2\psi}};\quad a=\rho^2+\rho_0^2+\left(z^*-z_0\right)^2;\quad b=2\rho\rho_0;\ k^2=\frac{2b}{a+b}.$$

To define potentials φ_k we use representation (9) and introduce the following integral operators:

$$A\psi_1=2\pi\psi_1+\iint_{S_1}\psi_1\frac{\partial}{\partial n}\frac{1}{r(P,P_0)}dS_1;\ B\psi_0=\iint_{S_0}\psi_0\frac{1}{r}dS_0;\ C\psi_0=\iint_{S_0}\psi_0\frac{\partial}{\partial z}\left(\frac{1}{r}\right)dS_0;$$

$$D\psi_1=-\iint_{S_1}\psi_1\frac{\partial}{\partial n}\frac{1}{|P-P_0|}dS_1;\ F\psi_0=\iint_{S_0}\psi_0\frac{1}{r}dS_0. \quad (10)$$

Then the boundary value problem (2)-(5) takes the form

$$A\psi_1=\frac{\kappa^2}{g}B\psi_0-C\psi_0;\ P_0\in S_1;\qquad D\psi_1=2\pi E\psi_0-\frac{\kappa^2}{g}F\psi_0;\qquad P_0\in S_0.$$

After excluding function ψ_1 from these relations we obtain the following eigenvalue problem

$$(DA^{-1}C+E)\psi_0-\lambda(DA^{-1}B+F)\psi_0=0;\quad \lambda=\frac{\chi^2}{g}$$

Its solution gives natural modes and frequencies of liquid sloshing in a rigid tank.

The evaluation of integral operators in (10) is carried out by the method proposed in [8-10].

3. Reducing the dynamic problem to the differential equation system

Having defined the basic functions φ_k, let us substitute them in expressions for velocity potential (1) and for the free surface elevation (5). Then substitute the received relations for the boundary condition on the free surface.

$$\left.\frac{\partial \Phi}{\partial t} + g\zeta + a_s(t)x\right|_{S_0} = 0 .$$

As in a cylindrical system of coordinates there is $x = r\cos\theta$, we are only interested in the first harmonica, i.e. in the formula (6) we only consider α=1. We come to the following equation on the surface S_0

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + g\sum_{k=1}^{M} d_k \frac{\partial \varphi_k}{\partial n} + a_s(t)r = 0 .`$$

Due to validity of the relation (4) on the surface S_0 the equality given above takes the form

$$\sum_{k=1}^{M} \ddot{d}_k \varphi_k + \sum_{k=1}^{M} \chi_k^2 d_k \varphi_k + a_s(t)r = 0 . \qquad (11)$$

Accomplishing the dot product of equality (11) by $\varphi_l \; \left(l = \overline{1, M}\right)$ and having used orthogonality of its own modes, we receive the system of ordinary differential equations of the second order

$$\ddot{d}_k + \chi_k^2 d_k + a_s(t)F_k = 0; \quad F_k = \frac{(r, \varphi_k)}{(\varphi_k, \varphi_k)}; \quad k = \overline{1, M} . \qquad (12)$$

Suppose that before applying the horizontal impulse the tank has been at the state of rest. Then we are to solve (12) under zero initial conditions. The operational method is applied here for solving the system (12).

The following values for the coefficients $d_k(t), \quad k = \overline{1, M}$ are obtained:

$$d_k(t) = \begin{cases} \dfrac{1}{\chi_k^2} - \dfrac{1}{\chi_k^2}\cos(\chi_k t) \quad 0 \le t \le T \\ \dfrac{1}{\chi_k^2} - \dfrac{1}{\chi_k^2}\cos(\chi_k t) - \dfrac{1}{\chi_k^2} + \dfrac{1}{\chi_k^2}\cos\chi_k (t - T) \;\; t > T \end{cases}$$

Substituting these coefficients in the relation (5), one can obtain the time-history of the free surface elevation.

4. Analysis of the numerical results

Let us consider the cylindrical shell with a flat bottom partially filled with the liquid. The tank parameters are following: radius is *R* = 1 m, thickness is *h*= 0.01m, length is *L* = 2 m, filling level is *H* =0.8m.

For carrying out the calculations we accepted different numbers of the basic functions.

Fig. 2 shows the time-history of the free surface elevation in the point B with *r*=1.5 (see fig. 1)**.** Here the only one (*M*=1) basic function is used in (5).

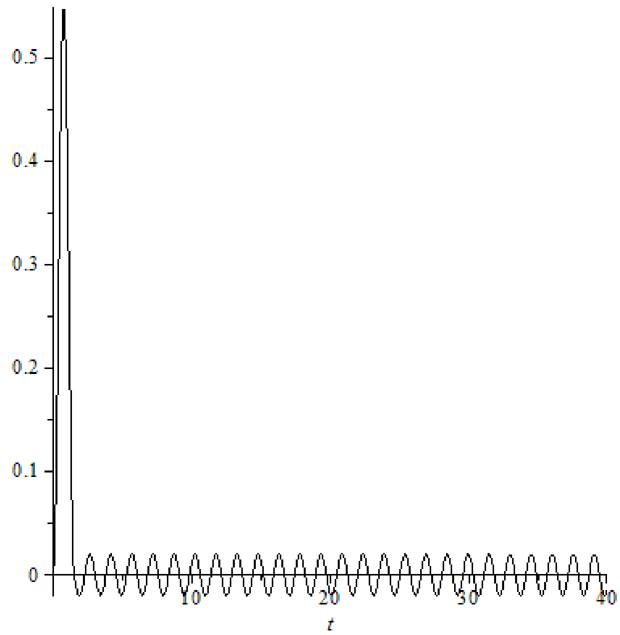

Fig.2. Time –history of the free surface under impulse loading, M=1.

On fig. 3 the free surface elevation in the point B with r=1.5 point, depending on time is shown. Here we use three basic functions ($M = 3$ in (5)).

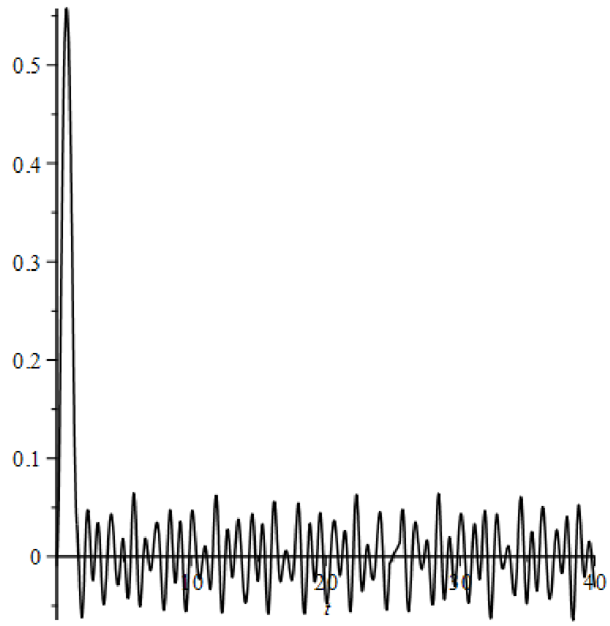

Fig.3. Time –history of the free surface under impulse loading, M=3

Further increasing in number of basic functions has not lead to the essential change of the results.

Conclusion

The developed method allows us to estimate the level of the free surface elevation under suddenly enclosed loadings. This approach can be easy generalized for elastic tanks with elastic baffles. The tank geometry can be changed easily, therefore the results could be obtained for conical, spherical and compound shells. It can allow us to make recommendations about installation of protective elements (covers, partitions).

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