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Wind farm deployment by wind turbines of different power curves under a 2-way routine of energy-and-costs constrained optimization

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The parameters of the efficient wind farm deployment are determined as the estimation of the probability density function for area wind statistics and an appropriate set of wind turbines based either on the desired annual energy output or the given financial resources. The deployment routine is stated in four items. The initial data/conditions and number of wind turbines which might be used for building a wind farm are followed by the estimation. The efficiency is achieved via energy-and-costs optimization. Therefore we can maximize the power output while minimizing the costs or controlling their total sum.

Key words: *wind turbine, wind farm, wind statistics, Weibull law, power curve, efficiency, energy-and-costs optimization.*

Вітрова енергетика та її впровадження інтенсивно розвивається з початком теперішньої декади. Сумарна потужність вітрової енергії, яка виробляється на сьогодні у світі, у 30 разів більша за такий же показник 2000 року. Проте швидке зростання і розповсюдження вітрової енергетики обмежується рядом факторів, пов'язаних зі значними обсягами необхідного фінансування для монтажу й експлуатації вітрових генераторів, транспортуванням виробленої енергії, а також потенційним негативним впливом на навколишнє середовище. Доцільність початкового інвестування та відповідних фінансових витрат залежить від того, наскільки ефективно використовується генерована потужність. Різниця між планованою річною потужністю і тією, що виробляється вітровою електростанцією, має бути якнайменшою. Для цього слід вивчати статистику розподілу вітрів місцевості, на якій будуватиметься електростанція. З іншого боку, може бути поставлена задача максимізації потужності за наданого фінансування. Обидва підходи потребують чіткого переліку параметрів ефективних дій. Відтак визначаються параметри ефективного розгортання вітрової електростанції як оцінка функції щільності імовірності для зональної статистики вітру та прийнятна множина вітрогенераторів на основі даних річної необхідної енергії або початкових фінансових ресурсів. Програма розгортання викладена у чотирьох пунктах. Оцінка слідує за наданням початкових даних/умов та числом вітрогенераторів, що могли би бути використані для будівництва електростанції. Ефективність досягається за оптимізацією енергії і витрат. При цьому вихідна потужність максимізується разом з мінімізацією витрат або контролем їх суми.

Ключові слова: *вітрогенератор, вітрова електростанція, статистика розподілу вітрів, закон Вейбулла, крива потужності, ефективність, оптимізація енергії і витрат.*

Определяются параметры эффективного развёртывания ветровой электростанции как оценка функции плотности вероятности для зональной статистики ветра и соответствующее множество ветрогенераторов на основе данных годовой необходимой энергии или начальных финансовых ресурсов. Программа развёртывания изложена в четырёх пунктах. Оценка следует за заданием начальных данных/условий и числом ветрогенераторов, что могли бы быть использованы для строительства электростанции. Эффективность достигается при оптимизации энергии и расходов. При этом выходная мощность максимизируется вместе с минимизацией расходов или контролем их суммы.

Ключевые слова: *ветрогенератор, ветровая электростанция, статистика распределения ветров, закон Вейбулла, кривая мощности, эффективность, оптимизация энергии и расходов.*

Determining parameters of efficient deployment of wind farms (WFs)

Wind energy deployment has been increased intensively since the 2010s. As of December 2017, the global wind power cumulative capacity is 30 times larger than that of 2000 [1, 2]. The growth would have been faster if there were not a few formidable obstacles. WF consisting of many wind turbines (WTs) is technically hard to build [1, 3]. Joining the power grid requires extra equipment or WF control systems [1] to meet the technical standards set by the operator of a transmission line. Some negative influence of WFs on environment (nature) still exists, e. g. noising, impact on wildlife, affecting weather in WFs' immediate vicinity, fire danger due to lightning strikes [1]. Thus, determining parameters of efficient WF deployment (WFD) remains an open issue.

Existing methods of meeting wind statistics for increasing efficiency of WFD

The troubles mentioned above can be partially solved with additional financial resources. They may be produced owing to building highly efficient WFs. Increasing efficiency of WFD is possible by meeting wind statistics. Otherwise, WTs are mostly useless [1, 3]. In [3], reference to wind speed (WS) is received from a power-speed scheduler for the pitch controller intended to regulate the power. Wind statistics in [3] are met after the event. In [2], aerodynamic interactions (known as the wake effect) among WTs are considered for taking into account the reduction of WS's for the downstream WTs due to the wake effect caused by the upwind ones. WS's in [2] are sectioned by wind directions, each of

which has its own probability density function (PDF) modeled as the Weibull law (that theoretically reflects actual wind statistics [2])

$$p(s) = (b/a) \cdot (s/a)^{b-1} \cdot e^{-(s/a)^b} \quad (1)$$

by WS s in meters per second (m/s), the shape and scale parameters b and a . WTs in [2] have various power curves (PCs), just like in [4] suggesting a diversification in PCs to increase the expected power output (EPO) of a WF. However, neither [2] nor [4] suggests a way to get the estimates of the parameters b and a . Moreover, selection of WTs with appropriate PCs would ensure conforming to the wind statistics.

Goal of determining parameters of the efficient WFD and tasks to be fulfilled

The goal is to determine parameters of the efficient WFD. Eventually, a routine of WFD will be itemed. To achieve the goal, the following three tasks must be fulfilled:

1. To list initial data/conditions. They may be optional, but there is to be the planned EPO.
2. To state rules for meeting wind statistics and finding an appropriate set of WTs.
3. To item the routine of WFD and consider an example of its using.

Initial data/conditions and their interpretation

An EPO r_0 planned/required in megawatts (MW) for a definite area is counted on an annual desired energy (ADE) E_0 given in MWh. Then the planned/required EPO is

$$r_0 = E_0 / (24 \cdot 365.25) = E_0 / 8766.$$

On the other hand, an amount of initial financial resources (IFR) can be given, which may be spent on WFD. Denote that amount by C_0 . Costs of buying and installing WTs are included in that amount, which can be denoted also by $c(r_0)$ to show its issue from r_0 . Obviously, values E_0 and C_0 cannot be given simultaneously, unless magnitudes of E_0 and C_0 are overestimated beforehand. Normally, either E_0 or C_0 is given first. Henceforward, one of two ways is then considered:

1. If E_0 is given first, then a problem is to minimize the costs of WFD. In this case, an optimal combination of WTs is searched for. If all WTs are identical, this is a mono-WF, otherwise — this is a multi-WF consisting of diverse WTs having various PCs [4].

2. If C_0 is given first, then a problem is to maximize an EPO of a WF. It does not matter whether the WF will happen to be a mono-WF or a multi-WF. The only reason is the EPO maximization, with no regards whether the maximized EPO is sufficient for covering energy demands or not. If insufficient, the value C_0 may be increased.

Nevertheless, neither of these ways can guarantee that magnitudes E_0 and C_0 will be reached exactly. Therefore, along with E_0 , a minimal ADE E_{\min} should be given as well, whose EPO is $r_{\min} = E_{\min} / 8766$. This EPO might be called an EPO lower threshold. The corresponding costs are subsequently estimated via r_0 and $c(r_0)$ using a rough supposition about the proportionality between EPOs and their costs, where $c(r_0)$ is taken closer similarly to a maximum of costs for EPOs approximating to r_0 .

Rules for meeting wind statistics based on estimating parameters of PDF (1)

If data about relative frequency of WS have been obtained over a year (or a few years) of observations across the area, then parameters b and a for PDF (1) are found via approximating to the histogram. Let $\{s_i\}_{i=1}^Q$ be a set of WS's in m/s, which are observed and fixed/registered, if any, where difference $\delta_s = s_{i+1} - s_i < 1$ is constant $\forall i = \overline{1, Q-1}$. WS's $s_1 = 0$ or near 0 and $s_Q = 30$ or greater, if the area is offshore. If t_i is a number of times when WS s_i was registered, then the relative frequency is

$$f(s_i) = n_i / \sum_{q=1}^Q n_q.$$

Then linearly and successively connected points $\{s_i, f(s_i)/\delta_s\}_{i=1}^Q$ constitute a polyline $\tilde{p}(s)$, which is an approximation to PDF (1):

$$\tilde{p}(s_i) = f(s_i)/\delta_s \quad \forall i = \overline{1, Q-1} \quad \text{by} \quad \sum_{q=1}^Q \tilde{p}(s_q) \delta_s = \sum_{q=1}^Q f(s_q) = 1. \quad (2)$$

The parameters b and a are found for (2) by solving a nonlinear curve-fitting problem

$$\arg \min_{a>0, b>0} \sum_{i=1}^Q [\tilde{p}(s_i) - p(s_i)]^2 = \arg \min_{a>0, b>0} \sum_{i=1}^Q [\tilde{p}(s_i) - (b/a) \cdot (s_i/a)^{b-1} \cdot e^{-(s_i/a)^b}]^2. \quad (3)$$

If WS data are poorer, then solution of problem (3) is inconsistent. In this case, the average WS (AWS) \tilde{s} is calculated in shorter periods (say, a few days or weeks). Then we set $b = 1.667$ or $b = 2$ as a default (e. g., see [4]). Parameter a is found as

$$\arg \min_{a>0} \left| \tilde{s} - \int_0^{\infty} (b/a) \cdot (s/a)^{b-1} \cdot e^{-(s/a)^b} ds \right|. \quad (4)$$

Surely, solutions of problems (3) and (4) are rather estimates of the parameters b and a , so let us denote them by \tilde{b} (the same for the default value) and \tilde{a} , respectively.

Finding an appropriate set of WTs for increasing efficiency of WFD

Having $b = \tilde{b}$ and $a = \tilde{a}$ in (1), the next step is the calculation of EPO. Considering a series of K WTs having different PCs, denote the PC (in MW) of the k -th WT by $w(k, s)$ for $k = \overline{1, K}$. An EPO of a mono-WF built of N_1 WTs (this number is not constrained from above) labeled by $\#k_0$ is [4]

$$r(k_0, N_1) = N_1 \cdot m(k_0) \quad \text{by} \quad m(k) = \int_0^{\infty} p(s) w(k, s) ds. \quad (5)$$

The selection of $k_0 \in \{ \overline{1, K} \}$ is a separate task, but not the difficult one. For instance, k_0 can correspond to a high or the highest nominal power, which is

$$\bar{w}(k) = \max_{s \in [0; \infty)} w(k, s)$$

for the k -th WT. Mono-WFs appear as not so efficient as they “focus” on a specific range of WS’s tied to the PC of a single WT. A multi-WF may rectify this as it has various PCs and that widens the active range of WS’s. Suppose the k_l -th WT is installed in $n(k_l)$ places,

$$n(k_l) \in \square \quad \text{and} \quad \{k_l\}_{l=1}^L = I \subset \{ \overline{1, K} \} \quad \text{by} \quad L \in \{ \overline{1, K} \}.$$

Thus, a multi-WF is built of

$$N(I) = \sum_{l=1}^L n(k_l)$$

WTs by having L different PCs, where $D(L, I) = \{n(k_l)\}_{l=1}^L$ and $N(I)$ along with N_1 is not constrained. Herein, EPO of the multi-WF is [4]

$$r(L, I, D(L, I)) = \sum_{l=1}^L n(k_l) m(k_l). \quad (6)$$

For further consideration, denote EPO by (5) and (6) just shortly by \tilde{r} .

When E_0 is given first, we should determine a subset $R = \{ \tilde{r}_u^* \}_{u=1, \overline{U}}$ of those EPOs by (5) and (6) that $\tilde{r} \in [r_{\min}; r_0]$. If $R = \emptyset$ then E_0 must be re-given as greater or the EPO lower threshold must be set lower (or both of them are corrected). Formally, $U \in \square$, but if $U = 1$ then those EPO values are probably to be corrected as well. If \tilde{r}_u^* -th EPO in R is a mono-WF of $N_1^{(u)}$ WTs labeled by $\#k_0^{(u)}$, then its costs are

$$c(\tilde{r}_u^*) = N_1^{(u)} \left(v_{\text{buy}}(k_0^{(u)}) + v_{\text{ins}}(k_0^{(u)}) \right) \quad (7)$$

by the respective costs $v_{\text{buy}}(k_0^{(u)})$ and $v_{\text{ins}}(k_0^{(u)})$ of buying and installing the $k_0^{(u)}$ -th WT. If \tilde{r}_u^* -th EPO in R is a multi-WF built of

$$N(I_*) = \sum_{l=1}^L n_*(k_l^{(u)})$$

WTs, then its costs are

$$c(\tilde{r}_u^*) = \sum_{l=1}^L n_*(k_l^{(u)}) \left(v_{\text{buy}}(k_l^{(u)}) + v_{\text{ins}}(k_l^{(u)}) \right). \quad (8)$$

Eventually, a two-criterion problem (2CP) is to be solved:

$$\begin{aligned} \min_{u=1, U} \frac{r_0 - \tilde{r}_u^*}{r_0 - r_{\min}} = \min_{u=1, U} \tilde{d}_u, \quad \min_{u=1, U} \frac{c(\tilde{r}_u^*) - c(r_{\min})}{c(r_0) - c(r_{\min})} = \min_{u=1, U} \tilde{c}_u \\ \text{by } c(r_0) = \frac{r_0}{\max_{u=1, U} \tilde{r}_u^*} \max_{u=1, U} c(\tilde{r}_u^*), \quad c(r_{\min}) = \frac{r_{\min}}{\min_{u=1, U} \tilde{r}_u^*} \min_{u=1, U} c(\tilde{r}_u^*). \end{aligned} \quad (9)$$

2CP (9) may not have a solution because as \tilde{r}_u^* approaches r_0 , the costs $c(\tilde{r}_u^*)$ are very likely to increase, and so minima in (9) are not reached at the same $\tilde{r}_{u_*}^*$ by $u_* \in \{1, U\}$. A solution to 2CP (9) is rather searched on a plane of U points $(\tilde{d}_u, \tilde{c}_u)$. A Pareto-efficient point $(\tilde{d}_{u_*}, \tilde{c}_{u_*})$ is chosen so that (the shortest distance to the zeros)

$$\min_{u=1, U} \sqrt{\tilde{d}_u^2 + \tilde{c}_u^2} = \sqrt{\tilde{d}_{u_*}^2 + \tilde{c}_{u_*}^2}. \quad (10)$$

When C_0 is given first, we do not have a financial lower threshold. We just find

$$r_{\max} = \max \left\{ \max_{k_0 \in \{1, K\}} \max_{N_1} r(k_0, N_1), \max_{D(L, I)} \max_{I \in \{1, K\}} \max_L r(L, I, D(L, I)) \right\} \quad (11)$$

which is reachable either at some $k_0^* \in \{1, K\}$ and N_1^* or at sets L_* , I_* , and $D_*(L_*, I_*) = \{n_*(k_l)\}_{l=1}^{L_*}$ by controlling that

$$C_0 \dots N_1^* \left(v_{\text{buy}}(k_0^*) + v_{\text{ins}}(k_0^*) \right), \quad C_0 \dots \sum_{l=1}^{L_*} n_*(k_l) \left(v_{\text{buy}}(k_l) + v_{\text{ins}}(k_l) \right). \quad (12)$$

For further, denote those costs by $c(\tilde{r})$. A combination of those arguments under the first or second group of maxima in (11) might be called shortly a WF. If maximum (11) is reached at a few WFs, then a WF with minimal costs is chosen.

A routine of WFD and examples of the efficient WFD and IFR utilization

The costs of buying and installing WTs being known, such a routine consists of four items:

1. To give a value of either E_0 or C_0 . A minimal ADE is requested along with E_0 .
2. To determine a number K of different WTs to be used for building a WF.
3. To estimate the parameters b and a for obtaining PCs of those K WTs.
4. To solve 2CP (9) for the given r_0 , r_{\min} and assessed $c(r_0)$, or find (11) by (12).

For considering an example of using the routine, we put $a = 5$ (corresponding to AWS 4.43 m/s) and $b = 2$ optionally, without tethering to a region. Let a minimal ADE be 90 % of ADE. We take five known and widespread WTs: Enercon E82 E2 (2.3 MW), Gamesa G128-4.5 MW, Nordex N90/2500 (2.5 MW), REpower MM82 (2 MW), Vestas V112-3.0 MW. Their characteristics and PCs are downloadable from [5], though prices for buying and installing these WTs are still hidden. They can be

only roughly estimated via general information about WFD. So, having enumerated the WTs from #1 to #5, respectively, the costs for (7) and (8) are (in million euros):

$$v_{\text{buy}}(1) = 3.1, \quad v_{\text{buy}}(2) = 7.72, \quad v_{\text{buy}}(3) = 3.25, \quad v_{\text{buy}}(4) = 2.68, \quad v_{\text{buy}}(5) = 5.1,$$

where

$$v_{\text{ins}}(k) = (0.17v_{\text{buy}}(k))^2 \text{ for } k = \overline{1, 5}$$

(installation is a few times cheaper, but installing bigger WTs becomes more expensive). Three examples of solutions of 2CP (9) in the sense of efficiency by (10) are shown in Fig. 1 – 3 for $L \in \{\overline{1, 4}\}$. Three examples of the solution for problem (11) by controlling (12) are shown in Fig. 4 – 6.

The examples in Fig. 1 – 6 show that the solutions of both 2CP (9) by (10) and problem (11) by (12) are very effective because they allow either having the closest approach to ADE or utilizing the given IFR almost completely. By this, the efficient WFD does not necessarily imply a multi-WF – apparently, efficient mono-WFs exist (Fig. 6). Building a multi-WF of WTs having a great number of different PCs is unlikely as well.

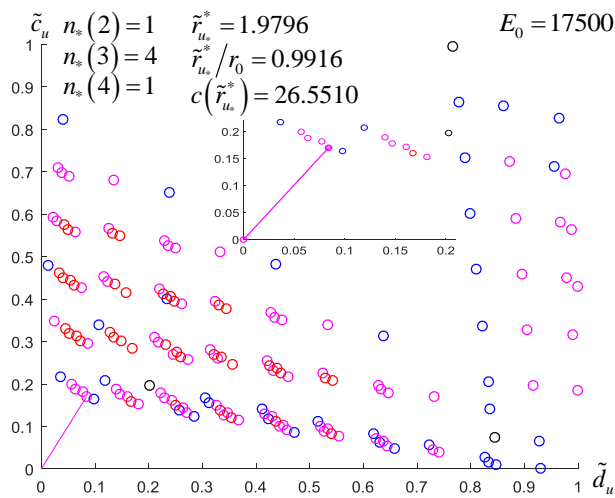


Fig. 1. WF of 6 WTs having 3 different PCs

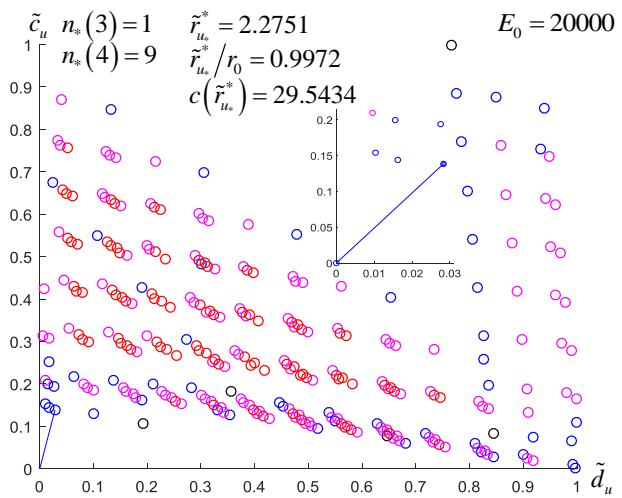


Fig. 2. WF of 10 WTs having 2 different PCs

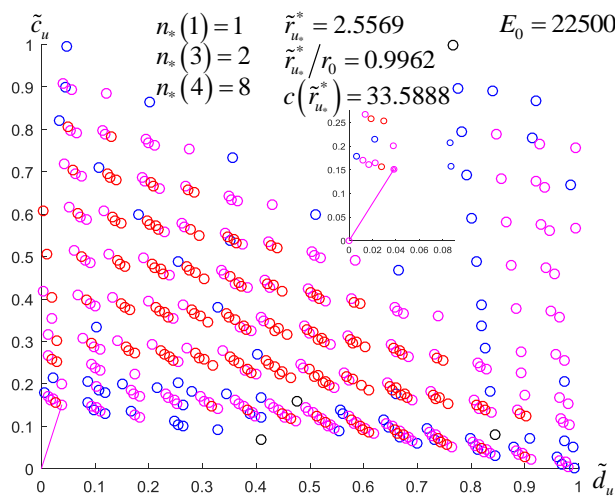


Fig. 3. WF of 11 WTs having 3 different PCs

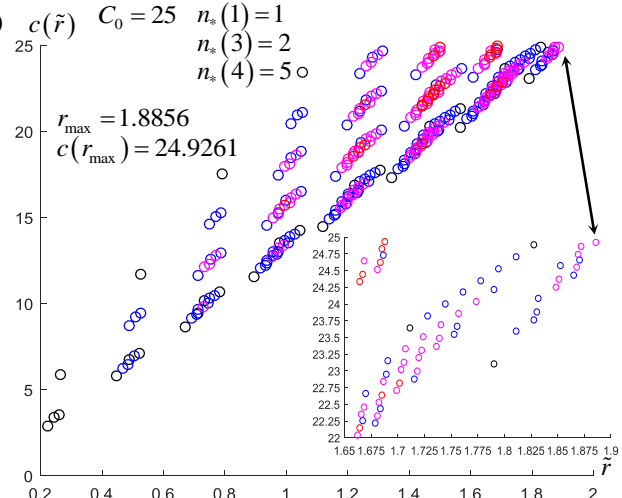


Fig. 4. WF of 8 WTs for up to 25 mln euros

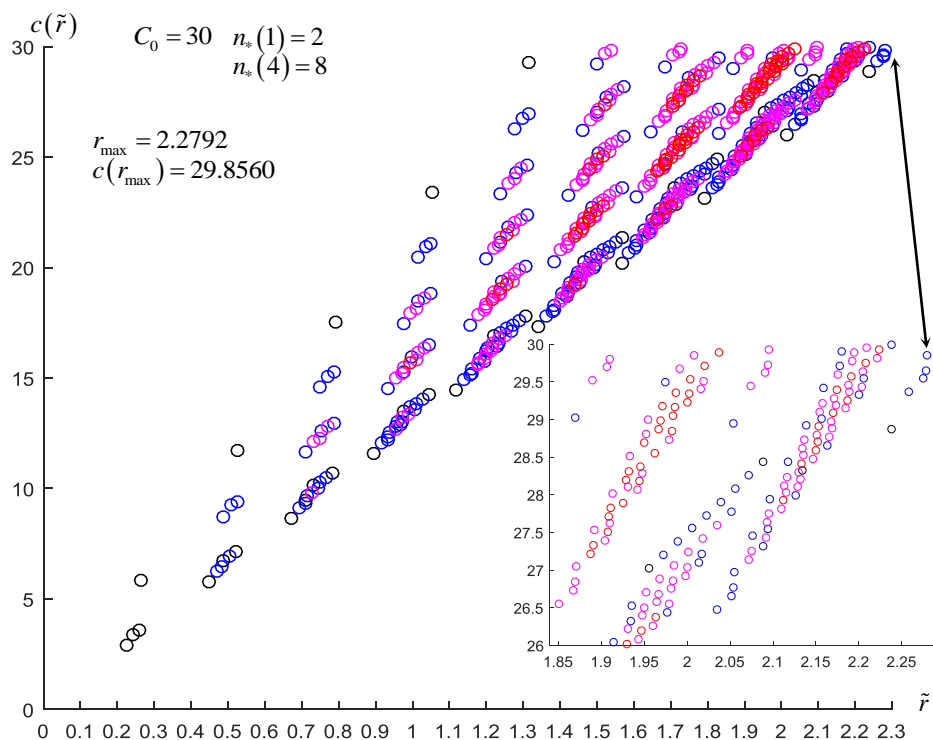


Fig. 5. WF of 10 WT's for up to 30 mln euros

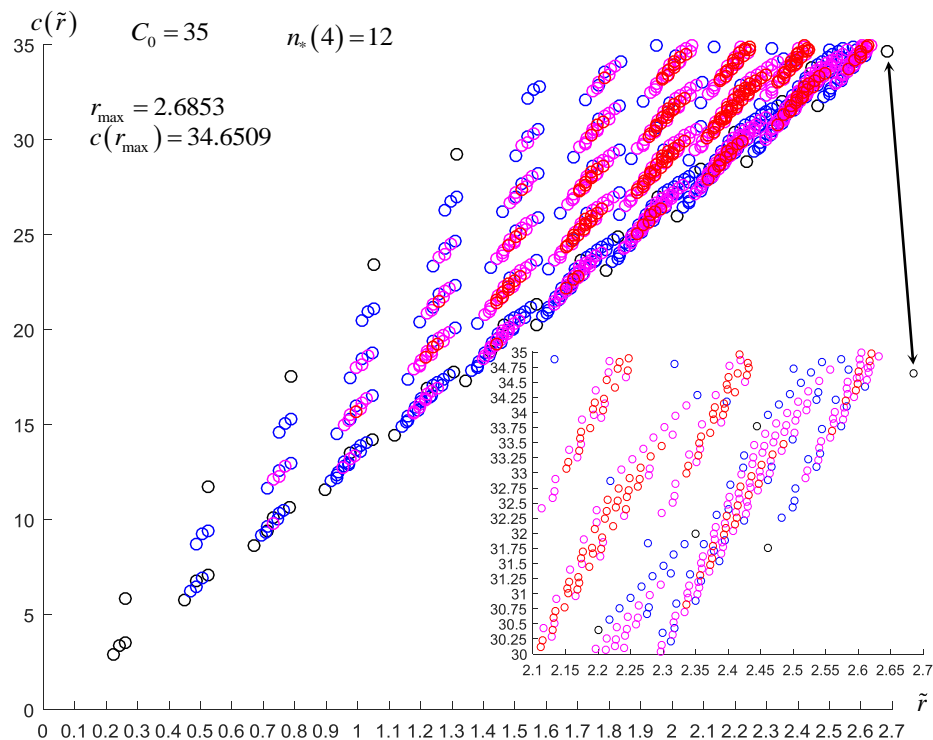


Fig. 6. WF of 12 WT's for up to 35 mln euros

Conclusion and suggestion for the further research in increasing efficiency of WFD

The parameters of the efficient WFD are a PDF (1) estimation for an area and an appropriate set of WT's based on the given either ADE or IFR. The number K of different WT's to be used for building a WF should be taken as big as possible. This is realizable, if those K types of WT's are commercially available. The efficiency can be achieved via energy-and-costs optimization: either 2CP (9) by (10) or

problem (11) by (12) is solved. Service costs for the WTs are not considered as they can be included into the power grid user's charge later. A further research might be focused on trying to turn on/off some WTs in a multi-WF depending on wind seasonality.

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