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PAIRS OF COMPACT CONVEX SETS: CATEGORICAL PROPERTIES

Lidiya BAZYLEVYCH[1], Oleksandr SAVCHENKO[2], Mykhailo ZARICHNYI[3]

[1] *National University "Lviv Polytechnica", 12 Stepana Bandery Str., 79000 Lviv, e-mail: izar@litech.lviv.ua*
[2] *Kherson State Agrarian University, 23 Rozy Liuksemburg Str., 73006 Kherson, e-mail: savchenko1960@rambler.ru*
[3] *Ivan Franko National University of Lviv, Universytetska Str., 1, 79000 Lviv, e-mail: izar@litech.lviv.ua*

The main result of this note is to demonstrate that the construction of the space of pairs of compact convex subsets in normed spaces determines a monad in the category of normed spaces and bounded linear operators.

Key words: compact convex set, Banach space, category, monad, lattice.

1. Introduction

The pairs of convex subsets in linear spaces find numerous applications in different areas of mathematics. In particular, they are used in the quasidifferential calculus [4], mathematical economics (in investigations of the Aumann integral [2]).

Different authors (see, e.g., [5, 6, 7]) considered the linear space of the (equivalence classes of) pairs of convex sets. This construction was considered from the categorical point of view in the realm of fuzzy metric spaces by the second named author [10]. In [10], a fuzzy norm on the mentioned linear space was defined and it was proved that the functor of the pairs of polyhedral convex sets (i.e., the convex hulls of finite subsets) determines a monad in a suitable category.

The aim of the present paper is to demonstrate that the construction of the (normed) linear space of the (equivalence classes of) pairs of compact convex sets generates a monad in the category of normed linear spaces. We also discuss the category of algebras determined by this monad.

2. Preliminaries

2.1. **Pairs of compact convex sets.** For every linear topological space X, let $\mathrm{cc}(X)$ denote the set of nonempty compact convex subsets in X. As usual, $+$ stands also for the Minkowski addition: $A+B=\{a+b \mid a\in A,\ b\in B\}$, for every $A,B\in \mathrm{cc}(X)$.

Consider the following equivalence relation $\sim$ on the set $\mathcal{L}(X)=\mathrm{cc}(X)\times \mathrm{cc}(X)$:

$$(A,B)\sim(C,D),\ \text{if } A+D=B+C.$$

The equivalence class containing (A,B) is denoted by $[A,B]$. The quotient set $\mathcal{K}(X)=\mathcal{L}(X)/\sim$ is a linear space with respect to the addition

$$[A,B]+[C,D]=[A+C,B+D]$$

and multiplication by scalar defined by the formula

$$\lambda[A,B]=\begin{cases}[\lambda A,\lambda B], & \text{if } \lambda>0,\\ [-\lambda B,-\lambda A], & \text{if } \lambda<0\end{cases}$$

(see [7]).

Suppose now that $(X,\|\cdot\|)$ is a normed space. Denote by d_H the Hausdorff metric on $\mathrm{cc}(X)$ with respect to the metric d induced by the norm $\|\cdot\|$. It is known (see, e.g., [7]) that the following function $\|\cdot\|'$ is a norm on $\mathcal{K}(X)$: $\|[A,B]\|'=d_H(A,B)$.

Let $\tilde{\mathcal{K}}(X)$ denote the completion of $\mathcal{K}(X)$ with respect to the norm $\|\cdot\|'$. Given a bounded linear operator $f\colon X\to Y$, let a map $\tilde f\colon \mathcal{K}(X)\to\mathcal{K}(Y)$ be defined as follows: $\tilde f([A,B])=[f(A),f(B)]$. It is easy to check that $\tilde f$ is a well-defined linear operator.

For the seek of notational simplicity, in the sequel we will denote all the norms simply by $\|\cdot\|$.

Лема 1. *$\tilde f$ is bounded and $\|\tilde f\|=\|f\|$.*

Доведення. Let $\|[A,B]\|\le 1$. Then $d_H(A,B)\le 1$ and therefore $d_H(f(A),f(B))\le\|f\|$, which is equivalent to $\|[f(A),f(B)]\|\le\|f\|$. Thus $\|\tilde f\|\le\|f\|$. The reverse inequality is obvious. □

This allows us to extend $\tilde f$ and to obtain a bounded linear operator $\tilde{\mathcal{K}}(X)\to\tilde{\mathcal{K}}(Y)$. We denote it by $\tilde{\mathcal{K}}(f)$.

Denote by **Ban** the category of Banach spaces and bounded linear operators. We therefore obtain a functor $\tilde{\mathcal{K}}\colon \mathbf{Ban}\to\mathbf{Ban}$.

2.2. **Monads and algebras.** Recall some necessary definitions concerning the monads; see, e.g., [1] for details.

Означення 1. *A monad $\mathbb{T}=(T,\eta,\mu)$ in a category $\mathcal{C}$ consists of an endofunctor $T\colon\mathcal{C}\to\mathcal{C}$ and natural transformations $\eta\colon 1_{\mathcal{C}}\to T$ (unit), $\mu\colon T^2=T\circ T\to T$ (multiplication) that satisfy the following relations: $\mu\circ T\eta=\mu\circ\eta T=\mathbf{1}_T$ and $\mu\circ\mu T=\mu\circ T\mu$. In other* *words, the following two diagrams are commutative:*

$$\begin{array}{ccc} T^3(X) & \xrightarrow{T(\mu_X)} & T^2(X) \\ \downarrow{\scriptstyle \mu_{T(X)}} & & \downarrow{\scriptstyle \mu_X} \\ T^2(X) & \xrightarrow[\mu_X]{} & T(X) \end{array}, \qquad \begin{array}{ccccc} T(X) & \xrightarrow{T(\eta_X)} & T^2(X) & \xleftarrow{\eta_{T(X)}} & T(X) \\ & \searrow{\scriptstyle 1} & \downarrow{\scriptstyle \mu_X} & \swarrow{\scriptstyle 1} & \\ & & T(X) & & \end{array}$$

The left side diagram is referred to as the associativity and the second one as the two-side unit.

Означення 2. *Let $\mathbb{T} = (T, \eta, \mu)$ be a monad on a category $\mathcal{C}$. A pair (X, ξ), where X is an object of $\mathcal{C}$ and $\xi\colon T(X) \to X$ a morphism of is called a $\mathbb{T}$-algebra if the diagrams*

$$\begin{array}{ccc} X & \xrightarrow{\eta_X} & TX \\ & \searrow{\scriptstyle 1} & \downarrow{\scriptstyle \xi} \\ & & X \end{array}, \qquad \begin{array}{ccc} T^2X & \xrightarrow{\mu_X} & TX \\ \downarrow{\scriptstyle T\xi} & & \downarrow{\scriptstyle \xi} \\ TX & \xrightarrow[\xi]{} & X \end{array}$$

are commutative.

Означення 3. *A morphism $\varphi\colon X \to Y$ is said to be a morphism of $\mathbb{T}$-algebras $(X, f) \to (Y, g)$, if the diagram*

$$\begin{array}{ccc} TX & \xrightarrow{T\varphi} & TY \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle g} \\ X & \xrightarrow[\varphi]{} & Y \end{array} \tag{1}$$

commutes.

$\mathbb{T}$*-algebras and their morphisms form a category which we denote by* $\mathcal{C}^{\mathbb{T}}$.

3. Result

Given a Banach space X, denote by $\eta_X : X \to \mathcal{K}(X)$ the map acting by the formula: $\eta_X(x) = [\{x\}, \{0\}]$.

Твердження 1. $\eta = (\eta_X)$ *is a natural transformation*

Доведення. We first note that, for any $x, y \in X$, we have

$$d(\eta_X(x), \eta_X(y)) = \|[\{x\}, \{0\}] - [\{y\}, \{0\}]\| = \|[\{x\}, \{y\}]\| = d_H(\{x\}, \{y\}) = d(x, y).$$

Clearly, η_X is a linear map. Given a bounded linear operator $f\colon X \to Y$, we obtain, for any $x \in X$,

$$\mathcal{K}(f)\eta_X(x) = \mathcal{K}(f)([\{x\}, \{0\}]) = [\{f(x)\}, \{0\}] = \eta_Y f(x),$$

i.e., η is a natural transformation. □

By completing, one also obtains a natural transformation from the identity functor to the functor $\tilde{\mathcal{K}}$. We keep the notation η for this transformation.

By $\mathrm{conv}(A)$ we denote the convex hull of a set A in a linear space. If $A = \{x_1, \dots, x_n\}$, then we denote it convex hull by $\langle x_1, \dots, x_n \rangle$. For any linear space X, let $\mathcal{K}_p(X)$ denote the set of equivalence classes of pairs of convex polyhedra in X, i.e., convex hulls of nonempty finite sets in X.

Лема 2. *Let Y be a dense set in* $\mathrm{cc}(X)$*, where X is a Banach space. Then the set $\{[A,B]\in\mathcal{K}(X)\mid A,B\in Y\}$ is dense in $\mathcal{K}(X)$. In particular, if Z is a dense subset in X, then the set*

$$\{[A,B]\in\mathcal{K}(X)\mid A,B \text{ are convex polyhedra with vertices in } Z\}$$

is dense in $\mathcal{K}(X)$.

Доведення. The proof follows that of the corresponding result from from [9]. □

For any subsets $A,B\subset X$, let $A\vee B=\mathrm{conv}(A\cup B)$.
For any $[A,B],[C,D]\in\mathcal{K}(X)$ let

$$[A,B]\oplus[C,D]=[(A+D)\vee(B+C),B+D].$$

The space $\mathcal{K}(X)$ forms a vector lattice with respect to the operation $\oplus$.
Recall the construction from [10]. Suppose that $[\mathcal{A},\mathcal{B}]\in\mathcal{K}_p^2(X)$, then there exist

$$[A_i,C_i],[B_j,D_j]\in\mathcal{K}_p(X),\ i=1,\dots,k,\ j=1,\dots,l,$$

such that

$$\mathcal{A}=\langle[A_1,C_1],\dots,[A_k,C_k]\rangle,\ \mathcal{B}=\langle[B_1,D_1,],\dots,[B_l,D_l]\rangle.$$

Then we let

$$\mu_X([\mathcal{A},\mathcal{B}])=([A_1,C_1]\oplus\dots\oplus[A_k,C_k])+([D_1,B_1,]\oplus\dots\oplus[D_l,B_l]).$$

The following lemma is proved in [10].

Лема 3. *Let*

$$M=\langle[A_1,B_1],\dots,[A_k,B_k]\rangle\subset L_p(X).$$

Then

$$\begin{aligned}\sup M=&[(A_1+B_2+\dots+B_k)\vee(B_1+A_2+B_3\dots+B_k)\\&\vee(B_1+B_2+\dots+A_k),B_1+\dots+B_k].\end{aligned}$$

This lemma implies the following formula (see [10]):

$$\mu_X([\mathcal{A},\mathcal{B}])=\max(\mathcal{A})-\max(\mathcal{B}).$$

Лема 4. *The map μ_X is a linear operator of norm 1.*

Доведення. This is proved in [10]. □

Now note that $\mathcal{K}_p(X)$ is a dense subset of the space $\mathcal{K}(X)$ and therefore in $\tilde{\mathcal{K}}(X)$. Using Lemma 2 we conclude that the set $\mathcal{K}_p^2(X)=\mathcal{K}_p(\mathcal{K}_p(X))$ is dense in $\tilde{\mathcal{K}}^2(X)$. Therefore one can extend the natural transformation μ to a unique natural transformation from $\tilde{\mathcal{K}}^2(X)$ to $\tilde{\mathcal{K}}(X)$. We keep the same notation μ also for this extended transformation.

Теорема 1. *The triple $\mathbb{K}=(\tilde{\mathcal{K}},\eta,\mu)$ is a monad on the category* **Ban**.

Доведення. The commutativity of the diagrams from the definition of monads is proved in [10] for the case of the classes of equivalence of pairs of polyhedral compact convex sets. Since these classes form, by Lemma 2, a dense subset in the spaces $\tilde{\mathcal{K}}^3(X)$, one can conclude that the diagram representing the associativity property of the multiplication is also commutative.

□

Let X be a Banach lattice. We refer to [8] for the basic facts concerning Banach lattices. We denote the supremum of $x, y \in X$ by $x \oplus y$. Recall that X is an AM-space if $\|x \oplus y\| = \max\{\|x\|, \|y\|\}$ for all $x, y \geq 0$.

Теорема 2. *The category of $\mathbb{K}$-algebras is isomorphic to the category of Banach lattices and linear lattice homomorphisms.*

Доведення. Let (X, ξ) be a $\mathbb{K}$-algebra. Define an operation $\oplus\colon X \times X \to X$ by the formula

$$x \oplus y = \xi(\eta_X(x) \oplus \eta_X(y)) = \xi([\langle x, y\rangle, \{0\}]).$$

Note that

$$\begin{aligned}\|x \oplus y\| \leq & d_H(\langle x, y\rangle, \{0\}) \leq \max\{d(tx + (1-t)y, 0) \mid t \in [0,1]\} \\ \leq & \max\{t\|x\| + (1-t)\|y\| \mid t \in [0,1]\} \leq \max\{\|x\|, \|y\|\}.\end{aligned}$$

Clearly, $x \oplus x = x$, for any $x \in X$.

Let $x, y, z \in X$. We are going to prove that $(x \oplus y) \oplus z = x \oplus (y \oplus z)$. Consider the element

$$\alpha = [\langle [\langle x, y\rangle, \{0\}], [\{z\}, \{0\}]\rangle, \mathcal{K}(\eta_X)(\eta_X(0))] \in \mathcal{K}^2(X).$$

Then

$$\begin{aligned}\mu_X(\alpha) = & \max\{[\langle x, y\rangle, \{0\}], [\{z\}, \{0\}]\} - \max \mathcal{K}(\eta_X)(\eta_X(0))] = [\langle x, y, z\rangle, \{0\}] - [\{0\}, \{0\}] \\ = & [\langle x, y, z\rangle, \{0\}].\end{aligned}$$

On the other hand,

$$\mathcal{K}(\xi)(\alpha) = [\langle \xi([\langle x, y\rangle, \{0\}]), \xi([\{z\}, \{0\}])\rangle, \eta_X(0))] = [\langle x \oplus y, z\rangle, \{0\}].$$

Therefore, from the definition of algebra it follows that

$$(x \oplus y) \oplus z = \xi([\langle x \oplus y, z\rangle, \{0\}]) = \xi([\langle x, y, z\rangle, \{0\}]).$$

One can similarly prove that

$$x \oplus (y \oplus z) = \xi([\langle x, y, z\rangle, \{0\}]).$$

One can easily prove that $(X, \oplus)$ is a Banach lattice.

Now let $(X, \oplus)$ be a Banach lattice. Define $\xi\colon \mathcal{K}(X) \to X$ by the formula: $\xi([A, B]) = \max A - \max B$. First note that ξ is well-defined. Indeed, if $[A, B] = [C, D]$, then $A + D = B + C$ and therefore

$$\max A + \max D = \max(A + D) = \max(B + C) = \max B + \max C.$$

We have $\xi\eta_X(x) = \xi([\{x\}, \{0\}]) = x - 0 = x$, for every $x \in X$.

Now let $\alpha = [\mathcal{A}, \mathcal{B}] \in \mathcal{K}_p^2(X)$,

$$\mathcal{A} = \langle [A_1, B_1], \ldots, [A_m, B_m]\rangle,\ \mathcal{B} = \langle [C_1, D_1], \ldots, [C_n, D_n]\rangle.$$

Then

$$\mu_X(\alpha) = \max \mathcal{A} - \max \mathcal{B} = [A_1, B_1] \oplus \cdots \oplus [A_m, B_m] - [C_1, D_1] \oplus \cdots \oplus [C_n, D_n]$$

and, since ξ is a lattice homomorphism,

$$\xi\mu_X(\alpha) = \max_i(\max A_i - \max B_i) - \max_j(\max C_j - \max D_j).$$

On the other hand,

$$\mathcal{K}(\xi)(\alpha) = [\langle \max A_1 - \max B_1, \dots, \max A_m - \max B_m\rangle, \\ \langle \max C_1 - \max D_1, \dots, \max C_n - \max D_n\rangle]$$

and therefore

$$\xi\mathcal{K}(\xi)(\alpha) = \max_i(\max A_i - \max B_i) - \max_j(\max C_j - \max D_j) = \xi\mu_X(\alpha).$$

Note that any nonexpanding lattice preserving linear operator generates a morphism of the corresponding $\mathbb{K}$-algebras.

We are going to show that the described correspondences between the Banach lattices and $\mathbb{K}$-algebras are inverse to each other. We temporarily denote by $\oplus_\xi$ the lattice operation on X that corresponds to the $\mathbb{K}$-algebra (X, ξ) and by $\xi_\oplus$ the structure map of the $\mathbb{K}$-algebra that corresponds to the lattice operation $\oplus$.

Given $(X, \oplus)$, for any $x, y \in X$ we obtain

$$x \oplus_\xi y = \xi([\langle x, y\rangle], \{0\}]) = \max\langle x, y\rangle - \max\{0\} = x \oplus y.$$

On the other hand, given (X, ξ), for any $A = \langle a_1, \dots, a_m\rangle \subset X$, $A = \langle b_1, \dots, b_n\rangle \subset X$ we obtain

$$\xi_\oplus([A, B]) = \max\langle a_1, \dots, a_m\rangle - \max\langle b_1, \dots, b_n\rangle = a_1 \oplus \cdots \oplus a_m - b_1 \oplus \cdots \oplus b_n \\ = \xi([\langle a_1, \dots, a_m\rangle, \{0\}]) - \xi([\langle b_1, \dots, b_b\rangle, \{0\}]) = \xi([A, B]).$$

This finishes the proof of the theorem. □

4. Pairs of compact convex subsets of constant width

A compact convex subset A in the Euclidean space $\mathbb{R}^n$ is said to be a set of constant width $d > 0$ if $A - A = \overline{B_d(0)}$ (the closed ball of radius d centered at the origin). Topology of the hyperspace of compact convex sets of constant width in $\mathbb{R}^n$, $n \geq 2$, was studied in [3].

Let S^{n-1} denote the unit sphere in $\mathbb{R}^n$. Given a compact convex body $A \subset \mathbb{R}^n$, define the support function $h_A\colon S^{n-1} \to \mathbb{R}$ by the formula: $h_A(v) = \max\{\langle x, v\rangle \mid x \in A\}$, for $v \in S^{n-1}$ (here $\langle\cdot,\cdot\rangle$ stands for the inner product in $\mathbb{R}^n$). Then one can reformulate the definition of the bodies of constant width, namely, A is of constant width $\lambda > 0$ if $|h_A(v) - h_A(-v)| = \lambda$, for every $v \in S^{n-1}$.

Denote by $\mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$ the family

$$\{[A, B] \in \mathcal{K}(\mathbb{R}^n) \mid [A, B] = [C, D], \text{ where } C, D \text{ are of constant width}\}.$$

Твердження 2. *The set $\mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$ is a closed linear subspace in $\mathcal{K}(\mathbb{R}^n)$.*

Доведення. Suppose that $[A_i, B_i] \in \mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$, $i = 1, 2$. Then there exist compact convex sets C_i, B_i of constant width in $\mathbb{R}^n$ such that $[A_i, B_i] = [C_i, D_i]$, $i = 1, 2$. We obtain

$$[A_1, B_1] + [A_2, B_2] = [C_1, D_1] + [C_2, D_2] = [C_1 + C_2, D_1 + D_2] \in \mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n),$$

because the Minkowski sum of bodies of constant width is again of constant width. It is also easy to demonstrate that $\alpha[A, B] \in \mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$, for any $[A, B] \in \mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$ and $\alpha \in \mathbb{R}$.

Closedness of $\mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$ in $\mathcal{K}(\mathbb{R}^n)$ easily follows from the fact that the set $\mathrm{cw}(\mathbb{R}^n)$ of compact convex bodies of constant width is closed in the hyperspace $\mathrm{cc}(\mathbb{R}^n)$.

□

Let $\tilde{\mathcal{K}}_{\mathrm{cw}}(\mathbb{R}^n)$ denote the completion of the space $\mathcal{K}_{\mathrm{cw}}(\mathbb{R}^n)$. We obtain a functor $\tilde{\mathcal{K}}_{\mathrm{cw}}$ from the category of finite-dimensional Euclidean spaces and orthoprojectors to the category **Ban**. This follows from the fact that the orthogonal projection of any body of constant width is also a body of constant width.

5. Remarks and open questions

It looks plausible that the results of this note can be generalized, on one hand, to the case of locally convex spaces and, on the other hand, to the case of convex bounded subsets in normed (locally convex) spaces.

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ПАРИ КОМПАКТНИХ ОПУКЛИХ МНОЖИН: КАТЕГОРНІ ВЛАСТИВОСТІ

Лідія БАЗИЛЕВИЧ[1], Олександр САВЧЕНКО[2], Михайло ЗАРІЧНИЙ[3]

[1] *Національний університет "Львівська політехніка", вул. Степана Бандери, 12, Львів, 79000, e-mail: izar@litech.lviv.ua*
[2] *Херсонський державний аграрний університет, вул. Рози Люксембург, 23, Херсон, 73006, e-mail: savchenko1960@rambler.ru*
[3] *Львівський національний університет імені Івана Франка, вул. Університетська, 1, Львів, 79000, e-mail: mzar@litech.lviv.ua*

Основний результат полягає в тому, що конструкція пар компактних опуклих підмножин у нормованих просторах визначає монаду в категорії нормованих просторів і обмежених лінійних операторів.

Ключові слова: компактна опукла множина, банаховий простір, категорія, монада, ґратка.

ПАРЫ КОМПАКТНЫХ ВЫПУКЛЫХ МНОЖЕСТВ: КАТЕГОРНЫЕ СВОЙСТВА

Лидия БАЗИЛЕВИЧ[1], Александр САВЧЕНКО[2], Михаил ЗАРИЧНЫЙ[3]

[1] *Национальный университет "Львовская политехника", ул. Степана Бандеры, 12, Львов, 79000, e-mail: izar@litech.lviv.ua*
[2] *Херсонский государственный аграрный университет, ул. Розы Люксембург, 23, Херсон, 73006*
[3] *Львовский национальный университет имени Ивана Франко, ул. Университетская, 1, Львов, 79000*

Основной результат состоит в том, что конструкция пространства пар компактных выпуклых множеств в нормированном пространстве определяет монаду в категории нормированных пространств и ограниченных линейных операторов.

Ключевые слова: компактное выпуклое множество, банахово пространство, категория, монада, решетка.