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ON INITIAL-BOUNDARY VALUE PROBLEM FOR NONLINEAR INTEGRO-DIFFERENTIAL STOKES SYSTEM

Oleh BUHRII, Mariana KHOMA

Ivan Franko National University of Lviv,
Universytetska Str., 1, Lviv, 79000
e-mail: oleh.buhrii@lnu.edu.ua, marianna.khoma88@gmail.com

Some nonlinear integro-differential Stokes system is considered. The initial-boundary value problem for this system is investigated and the existence and uniqueness of the weak solution for the problem is proved.

Key words: evolution Stokes system, integro-differential equation, initial-boundary value problem, weak solution.

1. Introduction

Let $n \in \mathbb{N}$ and $T > 0$ be fixed numbers, $n \geq 2$, $\Omega \subset \mathbb{R}^n$ be a bounded domain with the smooth boundary $\partial\Omega$, $Q_{0,T} := \Omega \times (0,T)$, $\Sigma_{0,T} := \partial\Omega \times (0,T)$, $\Omega_\tau := \{(x,t) \mid x \in \Omega, t = \tau\}$, $\tau \in [0,T]$. We seek a weak solution $\{u, \pi\}$ of the problem

$$u_t - \sum_{i,j=1}^{n} \Big(A_{ij}(x,t)u_{x_i}\Big)_{x_j} + G(x,t)|u|^{q-2}u + \int_\Omega \mathfrak{Z}(x,t,y)u(y,t)\,dy + \nabla\pi(x,t) = F(x,t), \qquad (x,t) \in Q_{0,T}, \tag{1}$$

$$\operatorname{div} u = 0, \qquad (x,t) \in Q_{0,T}, \tag{2}$$

$$\int_\Omega \pi(x,t)\,dx = 0, \quad t \in (0,T), \tag{3}$$

$$u|_{\Sigma_{0,T}} = 0, \tag{4}$$

$$u|_{t=0} = u_0(x), \quad x \in \Omega. \tag{5}$$

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Here $u = (u_1, \ldots, u_n) : Q_{0,T} \to \mathbb{R}^n$ is the velocity field, $|u| = (|u_1|^2 + \ldots + |u_n|^2)^{1/2}$, $\operatorname{div} u = \dfrac{\partial u_1}{\partial x_1} + \ldots + \dfrac{\partial u_n}{\partial x_n}$, $\pi : Q_{0,T} \to \mathbb{R}$ is the pressure, $\nabla\pi = \left(\dfrac{\partial \pi}{\partial x_1}, \ldots, \dfrac{\partial \pi}{\partial x_n}\right)$, and $q > 1$ is some number which is called an exponent of the nonlinearity of system (1).

The linearized version of the Navier-Stokes system is called the Stokes system. It is well known that these equations describe the time evolution of the solutions to the mathematical models of the viscous incompressible fluids. For more details about the physical meaning of the Navier-Stokes and Stokes systems see [1], [2], etc. The initial-boundary value problem for the Stokes system is considered in [3], [4], [5], [6], [7], [8], [9] (see also the references given there).

To take into account of some elasticity aspect of the non-Newtonian viscous fluids, the well-known classical Navier-Stokes equations are perturbed by an integral term which means the past history of the fluid (see [10]). The problems for the Navier-Stokes system with the integral memory term of the type

$$u_t + \sum_{k=1}^{n} v_k v_{x_k} - \alpha\,\Delta u - \int_0^t K_1(t,\tau)\Delta u\, d\tau - \int_\Omega K_2(t,y)\Delta u\, dy + \nabla\pi = F,$$

where Δu is a Laplacian, is considered in [11] if either $K_2 \equiv 0$, or $\alpha = 0$ and $K_1 \equiv 0$.

We perturb the classical Stokes equations by the monotonous nonlinear term and the linear integral term. We seek a weak solution to the initial-boundary value problem (1)-(5). As we know this problem is not studied yet. The paper is organized as follows. In Section 2, we formulate the considered problem and main results. The auxiliary statements are given in Section 3. Finally, in Section 4 we prove the main results.

2. Statement of problem and formulation of main results

Let $(\cdot,\cdot)_{\mathbb{R}^n}$ be a scalar product in the space $\mathbb{R}^n$,

$$(u,v)_\Omega := \int_\Omega (u(x), v(x))_{\mathbb{R}^n}\, dx, \quad u = (u_1, \ldots, u_n),\ v = (v_1, \ldots, v_n) : \Omega \to \mathbb{R}^n. \qquad (6)$$

Take $s \in \mathbb{N}$. Let us consider the Sobolev space $[H^s(\Omega)]^n$ with the scalar product

$$((u,v))_s := \sum_{i=1}^{n} (u_i, v_i)_{H^s(\Omega)}, \quad u, v \in [H^s(\Omega)]^n. \qquad (7)$$

Let $C_{\mathrm{div}} := \{u \in [C_0^\infty(\Omega)]^n \ \mid\ \operatorname{div} u = 0\}$,

$$H \text{ is the closure of } C_{\mathrm{div}} \text{ in } [L^2(\Omega)]^n, \qquad (8)$$

$$Z_s \text{ is the closure of } C_{\mathrm{div}} \text{ in } [H^s(\Omega)]^n, \qquad (9)$$

where $||h; H|| := ||h; [L^2(\Omega)]^n|| = \sum_{l=1}^{n} ||h_l; L^2(\Omega)||, \quad h = (h_1, \ldots, h_n) \in H$, and $||z; Z_s|| := \sqrt{((z,z))_s}, \quad z = (z_1, \ldots, z_n) \in Z_s$.

By definition, put

$$V := Z_1 \cap [L^q(\Omega)]^n, \quad U(Q_{0,T}) := L^2(0,T; Z_1) \cap [L^q(Q_{0,T})]^n.$$

Assume that the following conditions are fulfilled.

(A): A_{ij} is an n-order square matrix with the elements from $L^\infty(Q_{0,T})$; $A_{ij}=A_{ji}$ $(i,j=\overline{1,n})$; for a.e. $(x,t)\in Q_{0,T}$ and for every $\xi^1,\dots,\xi^n\in\mathbb{R}^n$, we get

$$a_0\sum_{i=1}^n|\xi^i|^2\le\sum_{i,j=1}^n\Big(A_{ij}(x,t)\xi^i,\xi^j\Big)_{\mathbb{R}^n}\le a^0\sum_{i=1}^n|\xi^i|^2\quad(0<a_0\le a^0<+\infty);$$

(G): G is an n-order square matrix, $G=diag(g_1,\dots,g_n)$, $g_l\in L^\infty(Q_{0,T})$, and $0<g_0\le g_l(x,t)\le g^0<+\infty$ for a.e. $(x,t)\in Q_{0,T}$, where $l=\overline{1,n}$;

(E): $\mathfrak{Z}$ is an n-order square matrix with the elements from $L^\infty(Q_{0,T}\times\Omega)$;

(F): $F\in L^2(0,T;H)$;

(U): $u_0\in H$.

We define the operators $A(t):V\to V^*$, $\mathcal{A}:U(Q_{0,T})\to[U(Q_{0,T})]^*$, $E(t):[L^2(\Omega)]^n\to[L^2(\Omega)]^n$, and $\mathtt{E}:[L^2(Q_{0,T})]^n\to[L^2(Q_{0,T})]^n$ by the rules:

$$\langle A(t)z,w\rangle_V:=\int\limits_\Omega\Bigg[\sum_{i,j=1}^n\Big(A_{ij}(x,t)z_{x_i}(x),w_{x_j}(x)\Big)_{\mathbb{R}^n}+$$
$$+\Big(G(x,t)|z(x)|^{q-2}z(x),w(x)\Big)_{\mathbb{R}^n}\Bigg]\,dx,\quad z,w\in V,\quad t\in(0,T),\tag{10}$$

$$\langle\mathcal{A}u,v\rangle_{U(Q_{0,T})}:=\int\limits_0^T\langle A(t)u(t),v(t)\rangle_V\,dt,\quad u,v\in U(Q_{0,T}),\tag{11}$$

$$(E(t)z)(x):=\int\limits_\Omega\mathfrak{Z}(x,t,y)z(y)\,dy,\quad x\in\Omega,\quad z\in[L^2(\Omega)]^n,\quad t\in(0,T),\tag{12}$$

$$(\mathtt{E}u)(x,t):=\big(E(t)u(t)\big)(x)=\int\limits_\Omega\mathfrak{Z}(x,t,y)u(y,t)\,dy,\ \ (x,t)\in Q_{0,T},\ u\in[L^2(Q_{0,T})]^n.\tag{13}$$

Let

$$q>1,\qquad s\in\mathbb{N},\qquad s\ge\max\left\{2,\ \frac{n}{2},\ n\left(\frac12-\frac1q\right)\right\},\qquad h=\min\left\{2,\frac{q}{q-1}\right\}.\tag{14}$$

Note that (14) implies that $Z_s\ \overline{\circlearrowleft}\ (Z_1\cap[L^q(\Omega)]^n)\ \overline{\circlearrowleft}\ V$.

Definition 1. A pair of the functions $\{u,\pi\}$ is called a *weak solution* of problem (1)–(5), if $u\in U(Q_{0,T})\cap C([0,T];Z_s^*)$, $u_t\in[U(Q_{0,T})]^*$, $\pi\in L^h(Q_{0,T})$, u satisfies (5) in Z_s^*, for $v\in V$ and $t\in(0,T)$ we have

$$\langle u_t(t),v\rangle_V+\langle A(t)u(t),v\rangle_V+(E(t)u(t),v)_\Omega=(F(t),v)_\Omega,\tag{15}$$

π satisfies (1) in $D^*(Q_{0,T})$, and π satisfies (3) in $D^*(0,T)$.

Theorem 1 (existence)**.** *Let conditions* **(A)**–**(U)** *hold. Then problem* (1)–(5) *has a weak solution* $\{u,\pi\}$*. Moreover,* $u\in L^\infty(0,T;H)$ *and* $\nabla\pi\in L^h(0,T;[W^{-1,s}(\Omega)]^n)$.

Theorem 2 (uniqueness)**.** *Let conditions* **(A)**–**(E)** *hold. Then, problem* (1)–(5) *cannot have more than one weak solution.*

3. Auxiliary statements

For Banach spaces X and Y the notation $X \circlearrowleft Y$ means the continuous embedding; the notation $X \overline{\circlearrowleft} Y$ means a continuous and dense embedding; the notation $X \overset{K}{\subset} Y$ means a compact embedding.

3.1. **Projection operator.** Let $\mathcal{H}$ be a Hilbert space with a scalar product $(\cdot,\cdot)_{\mathcal{H}}$, $\mathcal{V}$ be a reflexive separable Banach space, $\mathcal{V} \overline{\circlearrowleft} \mathcal{H} \cong \mathcal{H}^* \overline{\circlearrowleft} \mathcal{V}^*$, $\{w^j\}_{j\in\mathbb{N}}$ be an orthonormal basis for the space $\mathcal{H}$, $m \in \mathbb{N}$ be a fixed number, and $\mathfrak{M}$ be the set of all linear combinations of the elements from $\{w^1,\ldots,w^m\}$. Define a unique orthogonal projection $P_m : \mathcal{H} \to \mathfrak{M}$ by the rule (see [12, p. 527])

$$P_m h := \sum_{j=1}^{m} (h, w^j)_{\mathcal{H}}\, w^j, \quad h \in \mathcal{H}. \tag{16}$$

This is a linear self-adjoint continuous operator (see Theorem 7.3.6 [12, p. 515]). If $\{w^j\}_{j\in\mathbb{N}} \subset \mathcal{V}$, then let us define an operator $\widehat{P}_m : \mathcal{V} \to \mathcal{V}$ (not necessarily self-adjoint) by the rule

$$\widehat{P}_m v := P_m v \quad \text{for every} \quad v \in \mathcal{V}. \tag{17}$$

For a conjugate operator $\widehat{P}_m^* : \mathcal{V}^* \to \mathcal{V}^*$ we have $\widehat{P}_m^*(\mathcal{V}^*) \subset \mathcal{V}$ (see [13, p. 865]).

Proposition 1 (Lemma 3.9 [13, p. 865-866])**.** *Assume that $\{w^j\}_{j\in\mathbb{N}}$ is an orthonormal basis for the space $\mathcal{H}$ such that $\{w^j\}_{j\in\mathbb{N}} \subset \mathcal{V}$, $\psi_1^m,\ldots,\psi_m^m \in \mathbb{R}$ are some numbers, and $F \in \mathcal{V}^*$. Then $z^m := \sum_{s=1}^{m} \psi_s^m w^s \in \mathcal{V}$ satisfies*

$$\begin{cases} \langle z^m, w^1\rangle_{\mathcal{V}} = \langle F, w^1\rangle_{\mathcal{V}}, \\ \quad\vdots \\ \langle z^m, w^m\rangle_{\mathcal{V}} = \langle F, w^m\rangle_{\mathcal{V}}, \end{cases} \tag{18}$$

if the following equality holds

$$z^m = \widehat{P}_m^* F \quad \text{in} \quad \mathcal{V}^*. \tag{19}$$

Suppose that H and Z_s are determined from (8) and (9) respectively, where $s \in \mathbb{N}$. From [14, Ch. 1, §6.1], we obtain the embeddings

$$Z_s \overline{\circlearrowleft} Z_1 \overline{\circlearrowleft} H \cong H^* \overline{\circlearrowleft} Z_1^* \overline{\circlearrowleft} Z_s^*.$$

Moreover, $Z_s \subset [H_0^s(\Omega)]^n$. Let $\{w^\mu\}_{\mu\in\mathbb{N}}$ be a set of all eigenfunctions of the problem

$$((w,v))_s = \lambda\,(w,v)_H \quad \forall\, v \in Z_s, \tag{20}$$

$\{\lambda_\mu\}_{\mu\in\mathbb{N}} \subset \mathbb{R}_{>0} := \{\lambda \in \mathbb{R} \mid \lambda > 0\}$ be a set of the corresponding eigenvalues. For the sake of convenience we have assumed that $\{w^\mu\}_{\mu\in\mathbb{N}}$ is an orthonormal set in H.

Proposition 2 (see [14, Ch. 1, §6.3])**.** *If $s \in \mathbb{N}$ and $s \geq \frac{n}{2}$, then the set $\{w^\mu\}_{\mu\in\mathbb{N}}$ of all eigenfunction of problem (20) is a basis for the space Z_s.*

The following Lemma is needed for the sequel.

Lemma 1. *Suppose that* P_m *and* $\widehat{P}_m$ *are determined from* (16) *and* (17) *respectively, where* $\mathcal{H} = H$, $\mathcal{V} = Z_s$, $s \in \mathbb{N}$, *and* $\{w^\mu\}_{\mu\in\mathbb{N}}$ *is an orthonormal basis for the space* H *that consists of all eigenfunctions of problem* (20)*. Then, for every* $w \in L^r(0,T;Z_s^*)$ *and* $r > 1$*, we have the inequality*

$$||\widehat{P}_m^* w; L^r(0,T;Z_s^*)|| \le ||w; L^r(0,T;Z_s^*)||. \qquad (21)$$

Proof. From [14, Ch. 1, §6.4.3], we get that

$$||\widehat{P}_m z||_{Z_s} \le ||z||_{Z_s}, \quad z \in Z_s. \qquad (22)$$

Since $||D^*||_{\mathcal{L}(B^*,A^*)} = ||D||_{\mathcal{L}(A,B)}$ for every $D \in \mathcal{L}(A,B)$ (see [15, p. 231]), using (22), we have

$$||\widehat{P}_m^* v||_{Z_s^*} \le ||v||_{Z_s^*}, \quad v \in Z_s^*. \qquad (23)$$

Hence, $\int_0^T ||\widehat{P}_m^* w(t)||_{Z_s^*}^r\, dt \le \int_0^T ||w(t)||_{Z_s^*}^r\, dt$ and so inequality (21) holds. □

0.1. Cauchy's problem for system of ordinary differential equations. Take $\ell \in \mathbb{N}$ and $Q = (0,T)\times\mathbb{R}^\ell$. In this section we seek a weak solution $\varphi : [0,T] \to \mathbb{R}^\ell$ of the problem

$$\varphi'(t) + L(t,\varphi(t)) = M(t), \quad t \in [0,T], \qquad \varphi(0) = \varphi^0, \qquad (24)$$

where $M : [0,T] \to \mathbb{R}^\ell$ and $L : Q \to \mathbb{R}^\ell$ are some functions (for the sake of convenience we have assumed that $L(t,0) = 0$ for every $t \in [0,T]$), and $\varphi^0 = (\varphi_1^0,\dots,\varphi_\ell^0) \in \mathbb{R}^\ell$.

Proposition 3 (the Carathéodory-LaSalle Theorem, see Theorem 3.24 [13, p. 872])**.** *Suppose that* $p \ge 2$*, the function* $L : Q \to \mathbb{R}^\ell$ *satisfies* L^p*-Carathéodory condition,* $M \in L^p(0,T;\mathbb{R}^\ell)$*, and* $\varphi^0 \in \mathbb{R}^\ell$*. If there exist nonnegative functions* $\alpha,\beta \in L^1(0,T)$ *such that for every* $\xi \in \mathbb{R}^\ell$ *and for a.e.* $t \in [0,T]$ *the inequality*

$$(L(t,\xi),\xi)_{\mathbb{R}^\ell} \ge -\alpha(t)|\xi|^2 - \beta(t) \qquad (25)$$

holds, then problem (24) *has a global weak solution* $\varphi \in W^{1,p}(0,T;\mathbb{R}^\ell)$*.*

3.2. **Additional statements.** Let $\mathbb{Z}_{\ge -1} := \{s \in \mathbb{Z} \mid s \ge -1\}$. The following Propositions are needed for the sequel.

Proposition 4 (the generalized De Rham Theorem, see Theorem 4.1 [16], Remark 4.3 [16], and Lemma 2 [17])**.** *Suppose that* Ω *be an open bounded connected and Lipschitz subset of* $\mathbb{R}^n$*,* $T > 0$*,* $s_1, s_2 \in \mathbb{Z}_{\ge -1}$*,* $h_1, h_2 \in [1,\infty]$*, and* $\mathcal{F} \in W^{s_1,h_1}(0,T;[W^{s_2,h_2}(\Omega)]^n)$*. Then, if*

$$\langle \mathcal{F}(\cdot), v\rangle_{[D(\Omega)]^n} = 0 \quad in \quad D^*(0,T) \qquad (26)$$

for all $v \in \mathcal{V} = \{v \in [C_0^\infty(\Omega)]^n \mid \operatorname{div} v = 0\}$*, then there exists a unique*

$$\pi \in W^{s_1,h_1}(0,T;W^{s_2+1,h_2}(\Omega)) \qquad (27)$$

such that

$$\nabla\pi = \mathcal{F} \quad in \quad [D^*(Q_{0,T})]^n, \qquad (28)$$

$$\int\limits_\Omega \pi(\cdot)\, dx = 0 \quad in \quad D^*(0,T). \qquad (29)$$

Moreover, there exists a positive number C_1 *(independent of* $\mathcal{F},\pi$*) such that*

$$||\pi; W^{s_1,h_1}(0,T;W^{s_2+1,h_2}(\Omega))|| \le C_1||\mathcal{F}; W^{s_1,h_1}(0,T;[W^{s_2,h_2}(\Omega)]^n)||. \qquad (30)$$

Proposition 5 (the Aubin theorem, see [18] and [19, p. 393])**.** *If $s, h \in (1, \infty)$ are fixed numbers, $\mathcal{W}, \mathcal{L}, \mathcal{B}$ are Banach spaces, and $\mathcal{W} \overset{K}{\subset} \mathcal{L} \circlearrowleft \mathcal{B}$, then*

$$\{u \in L^s(0,T;\mathcal{W}) \mid u_t \in L^h(0,T;\mathcal{B})\} \overset{K}{\subset} [L^s(0,T;\mathcal{L}) \cap C([0,T];\mathcal{B})].$$

Proposition 6 (Lemma 1.18 [20, p. 39])**.** *If $u^m \underset{m\to\infty}{\longrightarrow} u$ in $L^p(Q_{0,T})$ $(1 \le p \le \infty)$, then there exists a subsequence (we call it $\{u^m\}_{m\in\mathbb{N}}$ again) such that $u^m \underset{m\to\infty}{\longrightarrow} u$ a.e. in $Q_{0,T}$.*

It is clear that if $u = (u_1, \ldots, u_n) \in [L^2(\mathcal{O})]^n$, where $\mathcal{O} = \Omega$ or $\mathcal{O} = Q_{0,T}$, then

$$|| \, |u|; L^2(\mathcal{O})||^2 = \int\limits_{\mathcal{O}} |u|^2 \, dy = \sum_{l=1}^{n} ||u_l; L^2(\mathcal{O})||^2 \le n||u; [L^2(\mathcal{O})]^n||^2,$$

and so

$$|| \, |u|; L^2(\mathcal{O})|| \le \sqrt{n}||u; [L^2(\mathcal{O})]^n||. \tag{31}$$

Lemma 2. *If condition* **(E)** *holds, then the operators $\mathtt{E} : [L^2(Q_{0,T})]^n \to [L^2(Q_{0,T})]^n$ and $E(t) : [L^2(\Omega)]^n \to [L^2(\Omega)]^n$, where $t \in (0,T)$, are linear bounded and continuous. Moreover, there exists a constant $E^0 > 0$ such that for every $z \in [L^2(\Omega)]^n$, $t \in (0,T)$, $u \in [L^2(Q_{0,T})]^n$, and $\tau \in (0,T]$, the following estimates are true:*

$$|| \, |E(t)z|; L^2(\Omega)|| \le E^0|| \, |z|; L^2(\Omega)|| \le \sqrt{n}E^0||z; [L^2(\Omega)]^n||; \tag{32}$$

$$|| \, |\mathtt{E}u|; L^2(Q_{0,\tau})|| \le E^0|| \, |u|; L^2(Q_{0,\tau})|| \le \sqrt{n}E^0||u; [L^2(Q_{0,\tau})]^n||. \tag{33}$$

Proof. It follows from the Cauchy-Bunyakowski-Schwarz inequality and **(E)** that

$$|| \, |E(t)z|; L^2(\Omega)||^2 = \int\limits_{\Omega} |(E(t)z)(x)|^2 \, dx = \int\limits_{\Omega} \left| \int\limits_{\Omega} \mathfrak{Z}(x,t,y)z(y) \, dy \right|^2 dx \le$$

$$\le \int\limits_{\Omega} \left| \int\limits_{\Omega} ||\mathfrak{Z}(x,t,y)||_n \cdot |z(y)| \, dy \right|^2 dx \le \int\limits_{\Omega} \left(\int\limits_{\Omega} ||\mathfrak{Z}(x,t,y)||_n^2 \, dy \right) \left(\int\limits_{\Omega} |z(y)|^2 \, dy \right) dx \le$$

$$\le |E^0|^2 \int\limits_{\Omega} |z(y)|^2 \, dy = |E^0|^2|| \, |z|; L^2(\Omega)||^2,$$

where $E^0 = \operatorname*{ess\,sup}\limits_{t\in(0,T)} \left(\int\limits_{\Omega} dx \int\limits_{\Omega} ||\mathfrak{Z}(x,t,y)||_n^2 \, dy \right)^{1/2}$ and $|| \cdot ||_n$ means a norm of the square matrix. Thus, using (31), we get (32). Estimate (33) is proved in a similar way. □

Lemma 3. *Let conditions* **(A)**–**(E)** *hold, $\{w^j\}_{j\in\mathbb{N}} \subset V$, $m \in \mathbb{N}$, $L = (L_1, L_2, \ldots, L_m)$,*

$$L_\mu(t,\xi) = \langle A(t)z^m, w^\mu\rangle_V + (E(t)z^m, w^\mu)_\Omega, \quad \mu = \overline{1,m}, \quad t \in (0,T), \quad \xi \in \mathbb{R}^m,$$

and $z^m(x) = \sum\limits_{\mu=1}^{m} \xi_\mu w^\mu(x)$ for $x \in \Omega$. Then

$$\Big(L(t,\xi), \xi\Big)_{\mathbb{R}^m} \ge \int\limits_{\Omega} \left[a_0 \sum_{i=1}^{n} |z^m_{x_i}|^2 + g_0|z^m|^q - E^0|z^m|^2 \right] dx, \quad t \in (0,T), \quad \xi \in \mathbb{R}^m. \tag{34}$$

Proof. It is clear that

$$(L(t,\xi),\xi)_{\mathbb{R}^m} = \langle A(t)z^m, z^m\rangle_V + (E(t)z^m, z^m)_\Omega. \tag{35}$$

If we use conditions **(A)** and **(G)**, then we get

$$\langle A(t)z^m, z^m\rangle_V = \int\limits_\Omega \Bigg[\sum_{i,j=1}^n \Big(A_{ij}(x,t)z^m_{x_i}(x), z^m_{x_j}(x)\Big)_{\mathbb{R}^n} + $$
$$+\Big(G(x,t)|z^m(x)|^{q-2}z^m(x), z^m(x)\Big)_{\mathbb{R}^n}\Bigg]\,dx \ge \int\limits_\Omega \Bigg[a_0\sum_{i=1}^n |z^m_{x_i}|^2 + g_0|z^m|^q\Bigg]\,dx. \tag{36}$$

Using (31) and (32), we obtain

$$\Big|(E(t)z^m, z^m)_\Omega\Big| = \Bigg|\int\limits_\Omega (E(t)z^m, z^m)_{\mathbb{R}^n}\,dx\Bigg| \le \int\limits_\Omega |E(t)z^m|\cdot|z^m|\,dx \le$$
$$\le ||\,|Ez^m|; L^2(\Omega)||\cdot||\,|z^m|; L^2(\Omega)|| \le E^0||\,|z^m|; L^2(\Omega)||^2 = E^0\int\limits_\Omega |z^m|^2\,dx. \tag{37}$$

Thus, (35)-(37) imply that (34) holds. □

4. Proofs of main results

Proof of Theorem 1. The solution will be constructed via Faedo-Galerkin's method.

Step 1 (construction of approximation). Let $\{w^\mu\}_{\mu\in\mathbb{N}}$ and Z_s be taken from Proposition 2, $s\in\mathbb{N}$ satisfies (14). By definition, put

$$u^m(x,t) := \sum_{\mu=1}^m \varphi^m_\mu(t)w^\mu(x), \quad (x,t)\in Q_{0,T}, \quad m\in\mathbb{N},$$

where the unknown function $\varphi := (\varphi^m_1,\ldots,\varphi^m_m)$ satisfies

$$(u^m_t(t), w^\mu)_\Omega + \langle A(t)u^m(t), w^\mu\rangle_V + (E(t)u(t), w^\mu)_\Omega = (F(t), w^\mu)_\Omega,\ t\in(0,T),\ \mu=\overline{1,m}, \tag{38}$$

$$\varphi^m_1(0) = \alpha^m_1, \quad \ldots, \quad \varphi^m_m(0) = \alpha^m_m. \tag{39}$$

Here the numbers $\alpha^m_1,\ldots,\alpha^m_m\in\mathbb{R}$ are chosen so that, $u^m_0 \underset{m\to\infty}{\longrightarrow} u_0$ strongly in H, where $u^m_0(x) := \sum\limits_{j=1}^m \alpha^m_j w^j(x), \quad x\in\Omega$. It is clear that the condition

$$u^m(0) = u^m_0 \tag{40}$$

holds. Let us show that the mentioned function φ exists. Let L be a vector-valued function from Lemma 3. Then Cauchy problem (38)-(39) takes form (24) if $M(t) = (\,(F(t), w^1)_\Omega, \ldots, (F(t), w^m)_\Omega)$, $\quad t\in(0,T)$. It follows from condition **(F)** that $M\in L^2(0,T;\mathbb{R}^m)$. Conditions **(A)**-**(E)** yield that the function L satisfies L^∞-Carathéodory condition.

Using estimate (34), conditions $a_0>0$ and $g_0>0$, and the orthogonality of the basis $\{w^\mu\}_{\mu\in\mathbb{N}}$ in H, we receive:

$$\Big(L(t,\varphi^m),\varphi^m\Big)_{\mathbb{R}^m} \ge -E^0\int\limits_{\Omega_t} |u^m|^2\,dx = -E^0\int\limits_{\Omega_t}\sum_{\mu=1}^m |\varphi^m_\mu|^2|w^\mu|^2\,dx \ge -C_2|\varphi^m|^2,$$

where $C_2 > 0$ is independent of t, φ^m. Then estimate (25) with $\alpha(t) \equiv C_2$ and $\beta(t) \equiv 0$ is performed, and from the Carathéodory-LaSalle theorem (see Proposition 3) we have that $\varphi \in H^1(0,T;\mathbb{R}^m)$ is a solution of problem (24) and therefore problem (38)-(39).

Step 2 (getting of estimates). Multipling the μ-th equation of (38) by $\varphi^m_\mu(t)$ and summing $\mu = \overline{1,m}$, we get:

$$\sum_{\mu=1}^m \Big(u^m_t(t), w^\mu \varphi^m_\mu(t)\Big)_\Omega + \sum_{\mu=1}^m \langle A(t)u^m(t), w^\mu \varphi^m_\mu(t)\rangle_V + \sum_{\mu=1}^m \Big(E(t)u^m(t), w^\mu \varphi^m_\mu(t)\Big)_\Omega = \\ = \sum_{\mu=1}^m \Big(F(t), w^\mu \varphi^m_\mu(t)\Big)_\Omega, \quad t \in (0,T).$$

After integrating for $t \in (0,\tau) \subset (0,T)$ and some transformation, we receive:

$$\int\limits_{Q_{0,\tau}} \left[(u^m_t, u^m)_{\mathbb{R}^n} + \sum_{i,j=1}^n (A_{ij} u^m_{x_i}, u^m_{x_j})_{\mathbb{R}^n} + (G|u^m|^{q-2}u^m, u^m)_{\mathbb{R}^n} + (\mathtt{E}u^m, u^m)_{\mathbb{R}^n} \right] dxdt = \\ = \int\limits_{Q_{0,\tau}} (F, u^m)_{\mathbb{R}^n}\, dxdt, \quad \tau \in (0,T]. \tag{41}$$

Clearly, using (40), we obtain:

$$\int\limits_{Q_{0,\tau}} (u^m_t, u^m)_{\mathbb{R}^n}\, dxdt = \int\limits_{Q_{0,\tau}} \frac{1}{2}\frac{\partial}{\partial t}(|u^m|^2)\, dxdt = \frac{1}{2}\int\limits_{\Omega_\tau} |u^m|^2\, dx - \frac{1}{2}\int\limits_{\Omega} |u^m_0|^2\, dx. \tag{42}$$

Using condition **(A)**, we get the following estimate:

$$\sum_{i,j=1}^n \Big(A_{ij} u^m_{x_i}, u^m_{x_j}\Big)_{\mathbb{R}^n} \geq a_0 \sum_{i=1}^n |u^m_{x_i}|^2. \tag{43}$$

It follows from condition **(G)** that

$$\Big(G|u^m|^{q-2}u^m, u^m\Big)_{\mathbb{R}^n} = \sum_{l=1}^n g_l(x,t)|u^m|^{q-2}|u^m_l|^2 \geq g_0 \sum_{l=1}^n |u^m|^{q-2}|u^m_l|^2 = g_0|u^m|^q. \tag{44}$$

Using the Cauchy-Bunyakowski-Schwarz inequality and (33), we obtain:

$$\left| \int\limits_{Q_{0,\tau}} (\mathtt{E}u^m, u^m)_{\mathbb{R}^n}\, dxdt \right| \leq \int\limits_{Q_{0,\tau}} |\mathtt{E}u^m|\cdot|u^m|\, dxdt \leq ||\,|\mathtt{E}u^m|; L^2(Q_{0,\tau})||\cdot||\,|u^m|; L^2(Q_{0,\tau})|| \leq \\ \leq E^0||\,|u^m|; L^2(Q_{0,\tau})||^2 = E^0 \int\limits_{Q_{0,\tau}} |u^m|^2\, dxdt. \tag{45}$$

Clearly,

$$|(F, u^m)_{\mathbb{R}^n}| \leq |F|\cdot|u^m| \leq \frac{|F|^2}{2} + \frac{|u^m|^2}{2}. \tag{46}$$

Using (42)-(46), from equality (41), we obtain the following estimate:

$$\frac{1}{2}\int\limits_{\Omega} |u^m(x,\tau)|^2\, dx + \int\limits_{Q_{0,\tau}} \left[a_0 \sum_{l=1}^n |u^m_{x_l}|^2 + g_0|u^m|^q \right] dxdt \leq$$

$$\leq \frac{1}{2}\int\limits_{\Omega} |u_0^m|^2\, dx + \frac{1}{2}\int\limits_{Q_{0,\tau}} |F|^2\, dxdt + \int\limits_{Q_{0,\tau}} \left(\frac{1}{2} + E^0\right)|u^m|^2\, dxdt. \tag{47}$$

Take $y(t) := \int_\Omega |u^m(x,t)|^2\, dx, \quad t \in [0,T]$. Then, from (47), we get an estimate:

$$\frac{1}{2}\, y(\tau) \leq C_3 + \left(\frac{1}{2} + E^0\right)\int\limits_0^\tau y(t)\, dt, \quad \tau \in [0,T].$$

Therefore, the Gronwall lemma implies that $y(\tau) \leq C_4$, and so

$$\int\limits_\Omega |u^m(x,\tau)|^2\, dx \leq C_4, \quad \tau \in (0,T]. \tag{48}$$

It follows from (47) and (48) that

$$\int\limits_{Q_{0,\tau}} \left[\sum_{i=1}^n |u_{x_i}^m|^2 + |u|^2 + |u|^q\right] dxdt \leq C_5, \quad \tau \in (0,T], \tag{49}$$

This estimate yields that

$$\int\limits_{Q_{0,\tau}} \left|G|u^m|^{q-2}u^m\right|^{q'} dxdt \leq C_6 \int\limits_{Q_{0,\tau}} |u^m|^q\, dxdt \leq C_7. \tag{50}$$

From (33), (48), and (49) it follows the estimates

$$||u^m; L^\infty(0,T,H)|| + ||u^m; U(Q_{0,T})|| \leq C_8, \tag{51}$$

$$||\mathtt{E}u^m; L^2(0,T;H)|| \leq C_9, \qquad ||\mathtt{E}u^m; [L^2(Q_{0,T})]^n|| \leq C_8, \tag{52}$$

Here the constants $C_3, \ldots, C_8$ are independent of m.

By (50)-(52) we have existence of the subsequence $\{u^{m_k}\}_{k\in\mathbb{N}} \subset \{u^m\}_{m\in\mathbb{N}}$ such that

$$u^{m_k} \underset{k\to\infty}{\longrightarrow} u \quad * -\text{weakly in} \quad L^\infty(0,T;H) \quad \text{and weakly in} \quad U(Q_{0,T}),$$

$$G|u^m|^{q-2}u^m \underset{k\to\infty}{\longrightarrow} \chi_1 \quad \text{weakly in} \quad [L^{q'}(Q_{0,T})]^n,$$

$$\mathtt{E}u^m \underset{k\to\infty}{\longrightarrow} \chi_2 \quad \text{weakly in} \quad [L^2(Q_{0,T})]^n.$$

Step 3 (additional estimates). Estimate (49) implies the inequality

$$||\mathcal{A}u^m; [U(Q_{0,T})]^*|| \leq C_{10}. \tag{53}$$

Since s satisfies (14), from the construction of the space $U(Q_{0,T})$, we obtain:

$$U(Q_{0,T}) \mathrel{\bar{\circlearrowleft}} L^2(0,T;H) \mathrel{\bar{\circlearrowleft}} [U(Q_{0,T})]^*, \tag{54}$$

$$L^{\max\{2,q\}}(0,T;Z_s) \mathrel{\bar{\circlearrowleft}} L^{\max\{2,q\}}(0,T;V) \mathrel{\bar{\circlearrowleft}} U(Q_{0,T}) \mathrel{\bar{\circlearrowleft}} L^{\min\{2,q\}}(0,T;V). \tag{55}$$

Therefore,

$$[U(Q_{0,T})]^* \mathrel{\bar{\circlearrowleft}} L^r(0,T;V^*) \mathrel{\bar{\circlearrowleft}} L^r(0,T;Z_s^*), \qquad r = \frac{\max\{2,q\}}{\max\{2,q\}-1}. \tag{56}$$

Using (55) and (51), we obtain:

$$||u^m; L^{\min\{2,q\}}(0,T;V)|| \leq C_{11}||u; U(Q_{0,T})|| \leq C_{12}. \tag{57}$$

Using Proposition 1 and notation (10)-(13), (16), and (17), in same way as in [14, Ch. 1, §5.3], we rewrite (38) as

$$u_t^m = \widehat{P}_m^*(F - \mathcal{A}u^m - \mathtt{E}u^m). \tag{58}$$

Thus, from (58), estimate (21), embeddings (56) and (54), and estimates (52)-(53), we get:

$$||u_t^m; L^r(0,T; Z_s^*)|| = ||\widehat{P}_m^*(F - \mathcal{A}u^m - \mathtt{E}u^m); L^r(0,T; Z_s^*)|| \leq$$

$$\leq ||F - \mathcal{A}u^m - \mathtt{E}u^m; L^r(0,T; Z_s^*)|| \leq C_{13}||F - \mathcal{A}u^m - \mathtt{E}u^m; [U(Q_{0,T})]^*|| \leq$$

$$\leq C_{14}\Big(||F; L^2(0,T;H)|| + ||\mathcal{A}u^m; [U(Q_{0,T})]^*|| + ||\mathtt{E}u^m; L^2(0,T;H)||\Big) \leq C_{15}. \tag{59}$$

Here the constants $C_{10}, \ldots, C_{15} > 0$ are independent of m.

Since $V \overset{K}{\subset} H \circlearrowleft Z_s^*$, from (57), (59), the Aubin theorem (see Proposition 5), and Proposition 6, we obtain:

$$u^{m_k} \underset{k\to\infty}{\longrightarrow} u \quad \text{in} \quad L^{\min\{2,q\}}(0,T;H) \cap C([0,T]; Z_s^*),$$

$$u^{m_k} \underset{k\to\infty}{\longrightarrow} u \quad \text{almost everywhere in} \quad Q_{0,T}.$$

Therefore, (5) holds and $\chi_1 = G|u|^{q-2}u$. Since $\mathtt{E}$ is a linear operator, we get $\chi_2 = \mathtt{E}u$.

Step 4 (passing to the limit). Take $\psi \in C^1([0,T])$ such that $\psi(T) = 0$. When we multiply equality (38) by $\psi(t)$, integrate for $t \in (0,T)$, and integrate the first term by parts, we obtain the following:

$$\int\limits_{Q_{0,T}} \Bigg[-\Big(u^m, w^\mu\Big)_{\mathbb{R}^n}\psi_t + \sum_{i,j=1}^n \Big(A_{ij}u_{x_i}^m, w_{x_j}^\mu\Big)_{\mathbb{R}^n}\psi + \Big(G|u^m|^{q-2}u^m, w^\mu\Big)_{\mathbb{R}^n}\psi +$$

$$+\Big(\mathtt{E}u^m, w^\mu\Big)_{\mathbb{R}^n}\psi\Bigg]dxdt = \int\limits_\Omega \Big(u_0^m, w^\mu\Big)_{\mathbb{R}^n}\psi(0)\,dx + \int\limits_{Q_{0,T}} \Big(F, w^\mu\Big)_{\mathbb{R}^n}\psi\,dxdt.$$

Taking $m = m_k$ and letting $k \to \infty$, due to arbitrariness of ψ, we get (15) and

$$\langle \mathcal{F}, z\rangle_{U(Q_{0,T})} = 0 \qquad \forall\, z \in U(Q_{0,T}), \tag{60}$$

where $\mathcal{F} := F - u_t - \mathcal{A}u - \mathtt{E}u$. Hence, $u_t \in [U(Q_{0,T})]^*$. Taking $z(x,t) = w(x)\varphi(t)$, $x \in \Omega$, $t \in (0,T)$, from (60), we obtain:

$$\int\limits_0^T \langle \mathcal{F}(t), w\rangle_{[D(\Omega)]^n}\,\varphi(t)\,dt = 0, \qquad w \in [D(\Omega)]^n, \quad \varphi \in D(0,T),$$

and so (26) holds. Clearly,

$$\mathcal{F} \in L^2(0,T; [H^{-1}(\Omega)]^n) + [L^{\frac{q}{q-1}}(Q_{0,T})]^n \subset W^{0,h}(0,T; [W^{-1,h}(\Omega)]^n),$$

where h is taken from (14). Then, the generalized De Rham theorem (see Proposition 4) yields that there exists $\pi \in W^{0,h}(0,T; W^{0,h}(\Omega)) = L^h(Q_{0,T})$ such that (28)-(29) hold. Thus, π satisfies (1) in $[D^*(Q_{0,T})]^n$ and (3) in $D^*(0,T)$. Theorem 1 is proved. $\square$

Proof of Theorem 2. Let $\{u_1, \pi_1\}$ and $\{u_2, \pi_2\}$ be weak solutions of problem (1)-(5). Set $u := u_1 - u_2$. Take (15) for u_1:

$$\langle u_{1\,t}(t), v\rangle_V + \langle A(t)u_1(t), v\rangle_V + (E(t)u_1(t), v)_\Omega = (F(t), v)_\Omega. \tag{61}$$

Take (15) for u_2:

$$\langle u_{2\,t}(t), v\rangle_V + \langle A(t)u_2(t), v\rangle_V + (E(t)u_2(t), v)_\Omega = (F(t), v)_\Omega. \tag{62}$$

Subtracting (62) from (61), setting $v = u(t)$, and integrating for $t \in (0, \tau) \subset (0, T)$, we obtain:

$$\int_0^\tau \Big[\langle u_t(t), u(t)\rangle_V + \langle A(t)u_1(t) - A(t)u_2(t), u_1(t) - u_2(t)\rangle_V + (E(t)u(t), u(t))_\Omega\Big]\, dt =$$
$$= \int_0^\tau (F(t), v)_\Omega\, dt, \quad \tau \in (0, T].$$

After the simple transformations, in the same way as (47), from this equality, we get:

$$\frac{1}{2}\int_{\Omega_\tau} |u|^2\, dx + \int_{Q_{0,\tau}} \left[a_0 \sum_{i=1}^n |u_{x_i}|^2 + (G|u_1|^{q-2}u_1 - G|u_2|^{q-2}u_2, u_1 - u_2)_{\mathbb{R}^n}\right] dxdt \le$$
$$\le C_{16} \int_{Q_{0,\tau}} |u|^2\, dxdt, \quad \tau \in (0, T]. \tag{63}$$

Let $y(\tau) := \int_{\Omega_\tau} |u|^2\, dx$, $\tau \in (0, T]$. Then, from (63) it follows that $\frac{1}{2}y(\tau) \le C_{16}\int_0^\tau y(t)\, dt$, $\tau \in (0, T]$. Using the Gronwall lemma, we see that $y(\tau) \le 0$ for $\tau \in [0, T]$, and so $u_1 = u_2$.

Since $\{u_1, \pi_1\}$ and $\{u_2, \pi_2\}$ satisfy (1) in $D^*(Q_{0,T})$, we obtain:

$$(u_1 - u_2)_t + \mathcal{A}u_1 - \mathcal{A}u_2 + \mathtt{E}u_1 - \mathtt{E}u_2 + \nabla(\pi_1 - \pi_2) = 0.$$

Then the equality $u_1 = u_2$ yields that $\nabla(\pi_1 - \pi_2) = 0$. Therefore, for $t \in (0, T)$ we have that $\pi_1(t) - \pi_2(t) = C(t)$. It follows from condition (3) with π_1 and π_2 that $C(t) = 0$. Thus, $\pi_1 = \pi_2$ and Theorem 2 is proved. □

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МІШАНА ЗАДАЧА ДЛЯ НЕЛІНІЙНОЇ ІНТЕГРО-ДИФЕРЕНЦІАЛЬНОЇ СИСТЕМИ СТОКСА

Олег БУГРІЙ, Мар'яна ХОМА

Львівський національний університет імені Івана Франка,
вул. Університетська, 1, Львів, 79000
e-mail: oleh.buhrii@lnu.edu.ua, marianna.khoma88@gmail.com

Розглянуто нелінійну інтегро-диференціальну систему рівнянь Стокса. Доведено існування та єдиність узагальненого розв'язку мішаної задачі для цієї системи.

Ключові слова: еволюційна система Стокса, інтегро-диференціальне рівняння, мішана задача, узагальнений розв'язок.