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NUMERICAL SOLUTION OF INVERSE SPECTRAL PROBLEMS FOR DIRAC OPERATORS ON A FINITE INTERVALS IN SOME SPECIAL CASES

Puyda D. V.

Ivan Franko National University of Lviv
1, Universytetska str., Lviv, 79000, Ukraine

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In this note, we provide a Maple implementation to solve the inverse spectral problem of reconstructing the self-adjoint Dirac operators on $(0,1)$ from eigenvalues and specially defined norming matrices in the simplest case when only a finite number of eigenvalues and norming matrices are perturbed.

Key words: Dirac operators, inverse spectral problems

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Introduction

The role of Dirac and Sturm–Liouville operators in modern physics and mathematics can hardly be overrated. The inverse spectral problems for such operators, which are of practical importance in microelectronics, nanotechnology etc., consist in finding the spectral characteristics which determine the operator uniquely and providing efficient methods of reconstructing the operator from these characteristics. The study of inverse spectral problems for Dirac and Sturm–Liouville operators has a rather long history. We refer the reader to the references cited in [5, 6, 7, 8] for some known results on the subject.

Inverse spectral problems for Dirac operators with matrix-valued potentials were recently treated in the author's papers [5, 7, 8]. Namely, using the technique that was suggested in [6], the inverse spectral problem of reconstructing the self-adjoint Dirac operators on $(0,1)$ with square-integrable matrix-valued potentials and some separated boundary conditions from eigenvalues and specially defined norming matrices was solved in [5]. Therein, a complete description of the class of the spectral data was given and a procedure of reconstructing the operator from its spectral data was suggested. The more general case of the operators with summable matrix-valued potentials was treated in [7]. The results of [7] were further extended to solve the inverse spectral problem for the operators with general (especially, non-separated) boundary conditions in [8].

In this note, we provide a Maple implementation to solve the inverse spectral problem of reconstructing the self-adjoint Dirac operators on $(0,1)$ from eigenvalues and norming matrices in the simplest but nevertheless practically important case when only a finite number of eigenvalues and norming matrices are perturbed. The algorithm is based on the results which were obtained in [5, 7].

The paper is organized as follows. In the reminder of this Introduction, we introduce some notations which are used in this paper. In Sect. I, we introduce the setting of the problem which is considered in this paper. In Sect. II, we review the results which were obtained in [5, 7] to solve the problem under consideration. In Sect. III, we solve the problem numerically under assumption that only a finite number of eigenvalues and norming matrices are perturbed. In Appendix, we provide a Maple implementation of the suggested algorithm.

Notations. We write $\mathcal{M}_r$ for the set of all $r\times r$ matrices $A=(a_{ij})_{i,j=1}^r$ with complex entries and $\mathcal{M}_r^+$ for the set of all self-adjoint and non-negative matrices $A\in\mathcal{M}_r$, i.e. such that $a_{ij}=\overline{a_{ji}}$ and $(Av\mid v)\ge 0$ for any non-zero $v\in\mathbb{C}^r$, $(\cdot\mid\cdot)$ denoting the standard inner product in $\mathbb{C}^r$. We endow $\mathcal{M}_r$ and $\mathcal{M}_r^+$ with the operator norm. We write I for the identity $r\times r$ matrix.

We say that a measurable function $f=f(x)$, $x\in(a,b)$, belongs to $L_2(a,b)$ if

$$\int_a^b |f(x)|^2\mathrm{d}x<\infty,$$

where the integral is understood in the Lebesgue sense. We refer the reader to [3] for further details on the theory of L_2-spaces. We denote by $L_2((a,b),\mathbb{C}^r)$ and $L_2((a,b),\mathcal{M}_r)$ the sets of all r-component vectors and $r\times r$ matrices composed of functions from $L_2(a,b)$, respectively.

We write $W_2^1((a,b),\mathbb{C}^r)$ for the set of all r-component vectors composed of functions from the Sobolev space $W_2^1(a,b)$. Each function $f\in W_2^1(a,b)$ has the derivative f' belonging to $L_2(a,b)$. We take the derivatives of vector- and matrix-valued functions componentwise.

I. Setting of the problem

In this section, we introduce the setting of the problem which is considered in this paper.

Let $\mathfrak{Q}_2 := L_2((0,1),\mathcal{M}_r)$. For an arbitrary $q \in \mathfrak{Q}_2$, we consider the differential expression $\mathfrak{t}_q$ given by the formula

$$\mathfrak{t}_q(f) := \frac{1}{\mathrm{i}}\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\frac{\mathrm{d}}{\mathrm{d}x}f + \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix} f$$

on the domain

$$D(\mathfrak{t}_q) := \left\{ f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \,\middle|\, f_1, f_2 \in W_2^1((0,1),\mathbb{C}^r) \right\}.$$

Here, $q^* := (\overline{q_{ji}})_{i,j=1}^r$ is the adjoint function to $q := (q_{ij})_{i,j=1}^r$.

In the Hilbert space

$$\mathbb{H} := L_2((0,1),\mathbb{C}^r) \times L_2((0,1),\mathbb{C}^r),$$

we introduce the self-adjoint Dirac operator T_q given by the formula $T_q f := \mathfrak{t}_q(f)$ on the domain

$$D(T_q) := \{ f \in D(\mathfrak{t}_q) \mid f_1(0) = f_2(0),\ f_1(1) = f_2(1) \}.$$

The function q will be called the *potential* of the operator T_q.

A number $\lambda \in \mathbb{C}$ is called an eigenvalue of the operator T_q if there exists a non-zero $f \in D(T_q)$ such that $T_q f = \lambda f$. The spectrum $\sigma(T_q)$ of the operator T_q is the set of its eigenvalues – it consists of countably many isolated real points λ_j, $j \in \mathbb{Z}$, accumulating only at $+\infty$ and $-\infty$. For definiteness, we assume that $(\lambda_j)_{j\in\mathbb{Z}}$ is a strictly increasing sequence such that $\lambda_0 \le 0 < \lambda_1$.

Let m_q denote the Weyl–Titchmarsh function of the operator T_q (see [2]). The function m_q is an $r\times r$ matrix-valued function and $\{\lambda_j\}_{j\in\mathbb{Z}}$ is the set of its poles. For each $j \in \mathbb{Z}$, we set

$$\alpha_j := -\operatorname*{res}_{\lambda=\lambda_j} m_q(\lambda).$$

Each α_j is a non-zero matrix in $\mathcal{M}_r^+$. We call α_j the *norming matrix* of the operator T_q corresponding to the eigenvalue λ_j. In the scalar case $r = 1$, α_j will be called the norming constant corresponding to λ_j.

Note that in the free case $q = 0$ one has $\sigma(T_0) = \{\pi n\}_{n\in\mathbb{Z}}$. In this case, the norming matrix corresponding to each eigenvalue πn, $n \in \mathbb{Z}$, is the identity $r \times r$ matrix.

The sequence $\mathfrak{a}_q := ((\lambda_j,\alpha_j))_{j\in\mathbb{Z}}$ composed of eigenvalues and norming matrices of the operator T_q will be called the *spectral data* of the operator T_q. The operator T_q is uniquely determined by its spectral data (see [5, 7]). The inverse spectral problem for the operator T_q then consists in:

- providing a complete description of the class of the spectral data, i.e. providing the necessary and sufficient conditions in order that a sequence

  $$\mathfrak{a} := ((\lambda_j,\alpha_j))_{j\in\mathbb{Z}},$$

  where $(\lambda_j)_{j\in\mathbb{Z}}$ is a strictly increasing sequence of real numbers such that $\lambda_0 \le 0 < \lambda_1$ and α_j, $j \in \mathbb{Z}$, are non-zero matrices in $\mathcal{M}_r^+$, is the spectral of some operator T_q with $q \in \mathfrak{Q}_2$;
- providing an efficient method of reconstructing the operator T_q from its spectral data.

II. Solution of the inverse spectral problem for the operator T_q

Here we summarize the results which were obtained in [5] to solve the inverse spectral problem for the operator T_q.

Let $\mathfrak{a} := ((\lambda_j,\alpha_j))_{j\in\mathbb{Z}}$ be an arbitrary sequence, where $(\lambda_j)_{j\in\mathbb{Z}}$ is a strictly increasing sequence of real numbers such that $\lambda_0 \le 0 < \lambda_1$ and α_j, $j \in \mathbb{Z}$, are non-zero matrices in $\mathcal{M}_r^+$. We partition the real axis into the pairwise disjoint intervals

$$\Delta_n := \left(\pi n - \frac{\pi}{2}, \pi n + \frac{\pi}{2}\right], \qquad n \in \mathbb{Z}.$$

Then the following theorem gives a complete description of the class of the spectral data for the operator T_q:

Theorem 1. *A sequence $\mathfrak{a} := ((\lambda_j,\alpha_j))_{j\in\mathbb{Z}}$ is the spectral data of some operator T_q with $q \in \mathfrak{Q}_2$ if and only if it satisfies the following three conditions:*

(A_1) $\sup_{n\in\mathbb{Z}} \sum_{\lambda_j\in\Delta_n} 1 < \infty$, $\sum_{n\in\mathbb{Z}}\sum_{\lambda_j\in\Delta_n} |\lambda_j - \pi n|^2 < \infty$ *and*

$$\sum_{n\in\mathbb{Z}} \left\| \sum_{\lambda_j\in\Delta_n} \alpha_j - I \right\|^2 < \infty;$$

(A_2) *there exists $N_0 \in \mathbb{N}$ such that for any natural $N > N_0$ it holds*

$$\sum_{n\in\mathbb{Z}}\sum_{\lambda_j\in\Delta_n} \operatorname{rank}\alpha_j = (2N+1)r;$$

(A_3) *the system of functions*

$$\mathcal{S} := \{ \mathrm{e}^{\mathrm{i}\lambda_j x} v \mid v \in \operatorname{Ran}\alpha_j,\ j \in \mathbb{Z} \}$$

is complete in $L_2((-1,1),\mathcal{M}_r)$.

Here, $\operatorname{rank}\alpha_j$ and $\operatorname{Ran}\alpha_j$ denote the rank and the range of α_j, respectively. Note that conditions (A_1) and (A_2) are easy to verify. From the practical point of view, the most complicated condition is (A_3).

Next, it is proved in [5] that there is a one-to-one correspondence between the operators T_q with $q \in \mathfrak{Q}_2$ and their spectral data. Therefore, the operator T_q can be reconstructed from its spectral data. The procedure of reconstructing the operator T_q from its spectral data can proceed as follows:

Step 1. *Given a sequence $\mathfrak{a} := ((\lambda_j,\alpha_j))_{j\in\mathbb{Z}}$ satisfying the conditions (A_1) – (A_3) (i.e. the one which is the* *spectral data of some operator T_q with $q \in \mathfrak{Q}_2$), construct the function*

$$h(x) := \lim_{N\to\infty} \sum_{n=-N}^{N} \left\{ \left(\sum_{\lambda_j \in \Delta_n} \mathrm{e}^{2\mathrm{i}\lambda_j x} \alpha_j \right) - \mathrm{e}^{2\mathrm{i}\pi n x} I \right\}, \qquad (1)$$

$x \in (-1,1)$, where the limit is understood in the topology of the space $L_2((-1,1),\mathcal{M}_r)$. The function h will be called the accelerant of the operator T_q.

Step 2. *Solve the Krein equation*

$$r(x,t) + h(x-t) + \int_0^x r(x,s)h(s-t)\,\mathrm{d}s = 0, \qquad (2)$$

$0 \le t \le x \le 1$. Conditions (A_1) – (A_3) imply that this equation has a unique solution $r = r_h$ in a special class of functions denoted by $G_2^+(\mathcal{M}_r)$ (see [5, 6]). Then find a potential q by the formula

$$q(x) := \mathrm{i} r_h(x,0), \qquad x \in (0,1). \qquad (3)$$

It follows from the results of [5] that $\mathfrak{a}$ is the spectral data of the operator T_q.

Taking into account the results which were obtained in [7], the similar procedure can be written also to solve the inverse spectral problem for the operator T_q in the more general case when $q \in L_1((0,1),\mathcal{M}_r)$. However, it would be more complicated from a practical point of view because the description of the spectral data involves the theory of distributions in this case.

So, one observes that solving the inverse spectral problem for the operator T_q is actually reduced to solving the integral equation (2). We refer the reader, e.g., to [4, 9] for some results on solving integral equations of this kind.

III. On the numerical solution of inverse spectral problem for the operator T_q

In this section, we solve the inverse spectral problem for the operator T_q numerically in the simplest case when only a finite number of eigenvalues and norming matrices are perturbed, i.e. differ from eigenvalues and norming matrices of the free operator with $q = 0$. For the simplicity of exposition, we concentrate ourselves only on the case of the scalar potential. The case of matrix-valued one can be treated similarly.

We say that a collection

$$\mathfrak{b} := ((\lambda_n, \alpha_n))_{n=-N}^{N}, \qquad N \in \mathbb{N}, \qquad (4)$$

where $(\lambda_n)_{n=-N}^{N}$ is an increasing collection of real numbers and $\alpha_n > 0$ for each $n = -N,\dots,N$, is the spectral data of the operator T_q with $q \in L_2(0,1)$ if

$$\sigma(T_q) = \{\lambda_n\}_{n=-N}^{N} \cup \{\pi n\}_{|n|>N},$$

the norming constant of the operator T_q corresponding to the eigenvalue λ_n, $n = -N,\dots,N$, is α_n and the norming constant of the operator T_q corresponding to the eigenvalue πn, $|n| > N$, is 1. Then the following proposition follows directly from Theorem 1 and Kadec's 1/4-theorem (see, e.g., [10]):

Proposition 1. *If a collection $\mathfrak{b}$ of (4) is such that*

$$|\lambda_n - \pi n| < \pi/4, \qquad n = -N,\dots,N, \qquad (5)$$

then $\mathfrak{b}$ is the spectral data of some operator T_q with $q \in L_2(0,1)$.

Theorem C.4 in [7], which is a vector analogue of Kadec's 1/4-theorem is some sense, can be used to obtain the analogue of Proposition 1 in the case of matrix-valued potential q.

So, let a collection $\mathfrak{b}$ of (4) satisfy the condition (5). It then follows from Proposition 1 that $\mathfrak{b}$ is the spectral data of some operator T_q with $q \in L_2(0,1)$. Formula (1) for the accelerant of the operator T_q then reads

$$h(x) = \sum_{n=-N}^{N} \left(\alpha_n \mathrm{e}^{2\mathrm{i}\lambda_n x} - \mathrm{e}^{2\mathrm{i}\pi n x} \right). \qquad (6)$$

Since the accelerant h of (6) is continuous, it follows from the results of [1] that the corresponding potential q is continuous on $[0,1]$, i.e. $q \in C[0,1]$.

In view of formula (6), it is then easy to solve the Krein equation (2). Indeed, let $r = r(x,t)$ be a solution of (2). Fix an arbitrary $x \in (0,1)$ and set $r_x(t) := r(x,t)$, $t \in (0,x)$. It then follows from (2) that

$$r_x(t) = \sum_{n=-N}^{N} a_n(x)\mathrm{e}^{-2\mathrm{i}\pi n t} - \sum_{n=-N}^{N} \alpha_n b_n(x) \mathrm{e}^{-2\mathrm{i}\lambda_n t} - h(x-t), \quad (7)$$

where

$$a_n(x) := \int_0^x \mathrm{e}^{2\mathrm{i}\pi n s} r_x(s)\mathrm{d}s,$$
$$b_n(x) := \int_0^x \mathrm{e}^{2\mathrm{i}\lambda_n s} r_x(s)\mathrm{d}s,$$

$n = -N\dots N$. We then obtain from (7) that for each $j = -N,\dots,N$,

$$a_j(x) = \sum_{n=-N}^{N} a_n(x) \int_0^x \mathrm{e}^{2\mathrm{i}\pi(j-n)t}\,\mathrm{d}t - \sum_{n=-N}^{N} \alpha_n b_n(x) \int_0^x \mathrm{e}^{2\mathrm{i}(\pi j - \lambda_n)t}\,\mathrm{d}t - \int_0^x h(x-t)\mathrm{e}^{2\mathrm{i}\pi j t}\,\mathrm{d}t \qquad (8)$$

and

$$b_j(x) = \sum_{n=-N}^{N} a_n(x) \int_0^x \mathrm{e}^{2\mathrm{i}(\lambda_j - \pi n)t}\,\mathrm{d}t - \sum_{n=-N}^{N} \alpha_n b_n(x) \int_0^x \mathrm{e}^{2\mathrm{i}(\lambda_j - \lambda_n)t}\,\mathrm{d}t - \int_0^x h(x-t)\mathrm{e}^{2\mathrm{i}\lambda_j t}\,\mathrm{d}t. \qquad (9)$$

Note that (8) – (9) is a system of linear equations with respect to $a_n(x)$ and $b_n(x)$, $n = -N, \dots, N$. Solving this system and substituting the coefficients $a_n(x)$ and $b_n(x)$ into formula (7) one obtains the value of $r_x(t)$. Taking into account that $q \in C[0,1]$, it then follows from formula (3) that $q(x) = r_x(0)$, $x \in (0,1)$, where

$$r_x(0) = \sum_{n=-N}^{N} (a_n(x) - \alpha_n b_n(x)) - h(x), \quad x \in (0,1).$$

Appendix. A Maple implementation

Here we provide a Maple implementation to solve the inverse spectral problem for the operator T_q in the case when only a finite number of eigenvalues and norming constants are perturbed. The algorithm uses the scheme which was described in the previous section.

So, let $\mathfrak{b} := ((\lambda_n, \alpha_n))_{n=-N}^{N}$ satisfy the condition (5) and thus be the spectral data of some operator T_q with $q \in C[0,1]$. We represent $\mathfrak{b}$ by the value of N and two arrays indexed from $-N$ to N, e.g.,

```
N := 1:
lambda := Array(−N .. N, [-Pi, 0, Pi]):
alpha := Array(−N .. N, [1, 1, 1]):
```

We then suggest the following procedure to find the potential q for which $\mathfrak{b}$ is the spectral data of the operator T_q:

```
q := proc(x)
  local a, b, var, sys, j, h, r:
  var := {seq(a[n], n = −N .. N)}
    union {seq(b[n], n = −N .. N)}:
  sys := { }:
  h := x → add(alpha[n]·exp(2·I·lambda[n]·x)
     − exp(2·I·Pi·n · x), n = −N .. N):
  for j from −N to N do
  sys := sys union {a[j] = add(a[n]·int(exp(2·I·Pi·(j
     −n)·t), t = 0 .. x), n = −N .. N)
     − add(alpha[n]·b[n]·int(exp(2·I·(Pi·j−lambda[n])·t),
     t = 0 .. x), n = −N .. N)
     −(int(h(x − t)·exp((2·I)·Pi·j·t), t = 0 .. x))}:
  end do:
  for j from −N to N do
  sys := sys union {b[j]
     = add(a[n]·int(exp(2·I·(lambda[j]−Pi·n)·t),
     t = 0 .. x), n = −N .. N)
     − add(alpha[n]·b[n]·int(exp(2·I·(lambda[j]
     −lambda[n])·t), t = 0 .. x), n = −N .. N)
     − int(h(x − t)·exp(2·I·lambda[j]·t), t = 0 .. x)}:
  end do:
  sol := solve(sys, var):
  assign(sol):
  r := x → add(a[n] − alpha[n]·b[n], n = −N .. N)
     − h(x):
  return r(x):
end proc:
```

Finally, we provide some examples:

Example 1. $N = 0$; $\lambda_0 = 0.1$, $\alpha_0 = 1$.

```
#Example 1
N := 0:
lambda := Array(−N .. N, [0.1]):
alpha := Array(−N .. N, [1]):
plot(Re(q(x)), x = 0 .. 1, color = black,
  font = ["ROMAN", 16], caption = "Re q(x)");
```

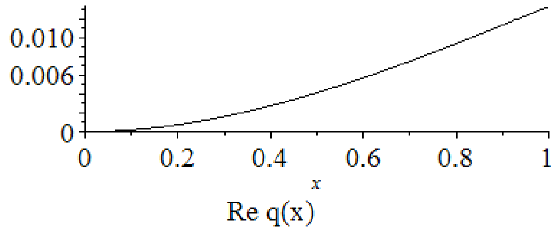

Fig. 1. Example 1, the real part of $q(x)$

```
plot(Im(q(x)), x = 0 .. 1, color = black,
  font = ["ROMAN", 16], caption = "Im q(x)");
```

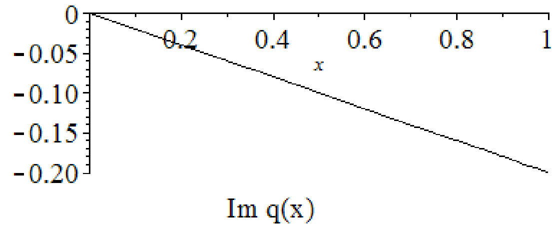

Fig. 2. Example 1, the imaginary part of $q(x)$

Example 2. $N = 1$; $\lambda_{-1} = -\pi + 0.1$, $\alpha_{-1} = 0.9$; $\lambda_0 = 0$, $\alpha_0 = 1.1$; $\lambda_1 = \pi - 0.1$, $\alpha_1 = 1$.

```
#Example 2
N := 1:
lambda := Array(−N .. N, [−Pi + 0.1, 0, Pi − 0.1]):
alpha := Array(−N .. N, [0.9, 1.1, 1]):
qRe := { }: qIm := { }:
for j from 0 to 1 by 0.05 do
    qRe := qRe union {[j, Re(q(j))]}:
    qIm := qIm union {[j, Im(q(j))]}:
end do:
with(plots):
listplot(qRe, color = black,
    font = ["ROMAN", 16], caption = "Re q(x)");
```

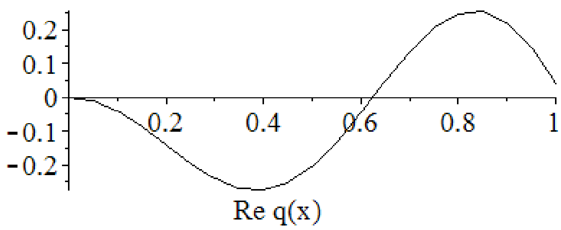

Fig. 3. Example 2, the real part of $q(x)$

```
listplot(qIm, color = black,
    font = ["ROMAN", 16], caption = "Im q(x)");
```

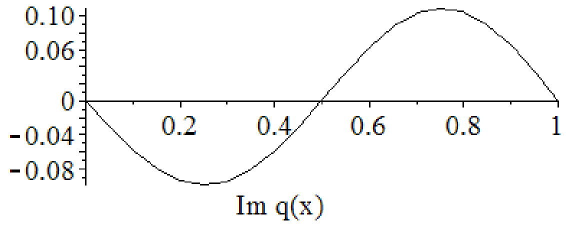

Fig. 4. Example 2, the imaginary part of $q(x)$

Example 3. Note that the potential q depends continuously on the accelerant h considered in the appropriate metric spaces (see [7, Theorem 1.5]). Therefore, if $h \to 0$ in $L_1(-1,1)$, then $q \to 0$ in $L_1(0,1)$. This can be illustrated by the following examples: $N = 0$; $\alpha_0 = 1$;
(a) $\lambda_0 = 0.1$; (b) $\lambda_0 = 0.075$; (c) $\lambda_0 = 0.05$.

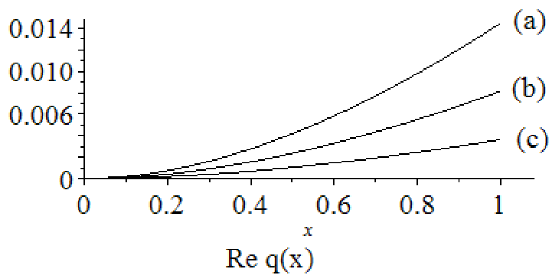

Fig. 5. Example 3, the real part of $q(x)$

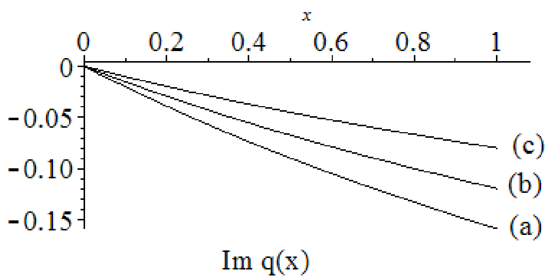

Fig. 6. Example 3, the imaginary part of $q(x)$

Example 4. $N = 1$; $\lambda_{-1} = -\pi$; $\alpha_{-1} = 1$; (a) $\lambda_0 = 0.1$, $\alpha_0 = 1.1$; $\lambda_1 = \pi - 0.1$, $\alpha_1 = 0.9$;
(b) $\lambda_0 = 0.05$, $\alpha_0 = 1.05$; $\lambda_1 = \pi - 0.05$, $\alpha_1 = 0.95$.

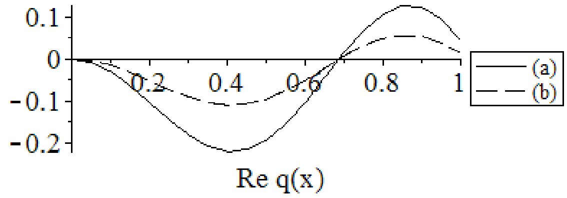

Fig. 7. Example 4, the real part of $q(x)$

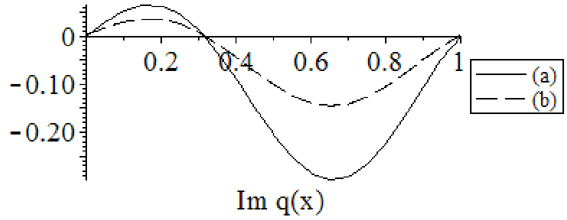

Fig. 8. Example 4, the imaginary part of $q(x)$

Conclusions

In this note, a numerical solution of the inverse problem of reconstructing the self-adjoint Dirac operators on $(0,1)$ in the simplest but practically important case when only a finite number of eigenvalues and norming constants are perturbed is provided. We give a Maple implementation of the suggested algorithm and provide some examples. The suggested procedure can be used in practical applications where the inverse spectral problems for Dirac operators on a finite intervals arise.

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ЧИСЛОВОЙ СПОСОБ РЕШЕНИЯ ОБРАТНОЙ СПЕКТРАЛЬНОЙ ЗАДАЧИ ДЛЯ ОПЕРАТОРА ДИРАКА НА КОНЕЧНОМ ИНТЕРВАЛЕ В НЕКОТОРЫХ ЧАСТНЫХ СЛУЧАЯХ

Пуйда Д. В.

Львовский национальный университет имени Ивана Франко,
ул. Университетськая, 1, Львов, 79000, Украина

Предлагается решение в системе Maple обратной спектральной задачи восстановления самосопряженного оператора Дирака на интервале $(0, 1)$ по собственным значениям и специально определенным нормировочным матрицам в простейшем случае, когда возмущено лишь конечное количество собственных значений и нормировочных матриц.

Ключевые слова: оператор Дирака, обратные спектральные задачи.

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ЧИСЛОВИЙ МЕТОД РОЗВ'ЯЗУВАННЯ ОБЕРНЕНОЇ СПЕКТРАЛЬНОЇ ЗАДАЧІ ДЛЯ ОПЕРАТОРА ДІРАКА НА СКІНЧЕННОМУ ПРОМІЖКУ В ДЕЯКИХ ЧАСТКОВИХ ВИПАДКАХ

Пуйда Д. В.

Львівський національний університет імені Івана Франка
вул. Університетська 1, Львів, 79000, Україна

Наводиться реалізація в системі Maple розв'язування оберненої спектральної задачі відновлення самоспряженого оператора Дірака на проміжку $(0, 1)$ за власними значеннями і спеціально означеними нормівними матрицями у найпростішому випадку, коли збурено лише скінченну кількість власних значень та нормівних матриць.

Ключові слова: оператор Дірака, обернені спектральні задачі.

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