УДК 004.942

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MATHEMATICAL MODELING AND ANALYSIS OF WOOD STRENGTH IN THE CONTEXT OF BIAXIAL STRESS STATE

A mathematical model of the anisotropic materials strength under biaxial stress state that was developed by earlier authors is specified considering the known dependence of the elasticity modulus of the manner of loading characteristics and limits asymmetry of strength in the areas of structural symmetry. Also there is a new method of developed model parameter identification that is based on passport specification related to wood characteristics of different species. The numerical experiments were conducted and on the analysis of the results basis the adequacy of the model is substantiated.

Keywords: mathematical model, strength, strain, warp, volumetric strain, potential energy density.

МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТА АНАЛІЗ МІЦНОСТІ ДЕРЕВИНИ В УМОВАХ ДВОВІСНОГО НАПРУЖЕНОГО СТАНУ

З урахуванням відомих залежностей модулів пружності від способу навантаження та особливостей асиметрії меж міцності у напрямках структурної симетрії уточнено раніше розроблену авторами математичну модель міцності анізотропних матеріалів в умовах двовісного напруженого стану. Запропоновано новий спосіб ідентифікації параметрів розробленої моделі на основі паспортних фізико-механічних характеристик деревини різних порід. Проведено числові експерименти та на основі аналізу отриманих результатів обґрунтовано адекватність зазначеної моделі.

Ключові слова: математична модель, міцність, напруження, деформації, відносна об’ємна деформація, густина потенціальної енергії.

Rationale. The rise of quality products and the reduce of energy costs at its production is one of the main conditions for the successful development of woodworking enterprises. The decisive role in solving this problem belongs to the study of short-term strength of wood with a complex stress state, because strength is one of the factors that significantly limits the intensification of the dehydration timber processes. The duration of the processes cannot be arbitrarily small, its value should be like the one that at all points of dryable material stress tensor components do not exceed the limit values. Otherwise there will be a residual voltage which are major factors in reducing the material quality indicators. To determine the limit stresses in the wood with the uniaxial stress state experimental methods of investigation are used and the method of mathematical modeling is applied to the wood with a complex state of stress.

The results of mathematical modeling of materials anisotropic strength are provided in the works of R. Mises, Mises – Hill, E. K. Ashkenazi, K. V. Zaharov, O. K. Malmeystra, Holdenblata – Kopnovaetc. However, the application of these criteria for the modelling of curves and surfaces with a two-axial strength of timber, flat and volumetric stress state is not sufficiently substantiated. None of these models are not adapted and not tested on wood of coniferous and hardwoods. The complexity of this problem solution is that the input data to these models (criteria) is the persistable strength (for instance, boundary displacement orthotropic material along diagonal planes of structural symmetry), which currently are not subjects to experimental determination. Therefore, the task of modeling the strength of the wood is present-day as of today there are no technical solutions to identify and control voltages in dried lumber and the existing mechanical strength theory do not describe fully the strength of wood in a complex stress state.

Mathematical model of wood strength. The lack of empirical data to determine the limits of pure shear strength along the main diagonal and the structural symmetry planes of wood of different species does not give the opportunity to confirm or deny the authenticity of the presently known strength criteria for the test material. Therefore, in order not to carry out additional highly complex experimental trialing of methods and techniques which are insufficiently substantiated, we will continue the research, purpose, objectives and partial results of which are covered in the work [1]. In particular, we adapt the proposed in this paper mathematical model of the strength of anisotropic materials to wood of coniferous and hardwood. For this we add its dependence on material elasticity modules on the method of intensity and condition of determination the constants of the model. The result is a mathematical model for determining and predicting the strength of wood with a biaxial stress state. The components of this model are:

- mathematical model of the strength of anisotropic materials under biaxial stress state [1]

$$\frac{1}{2\sqrt{E_{11}E_{22}}}\left((1-\mu_{23}\mu_{32})\sqrt{\frac{E_{22}}{E_{11}}}\sigma_{11}^2+(\mu_{13}\mu_{32}+\mu_{23}\mu_{31}-\mu_{12}\mu_{21})\sigma_{11}\sigma_{22}+(1-\mu_{13}\mu_{31})\sqrt{\frac{E_{11}}{E_{22}}}\sigma_{22}^2\right)- \quad (1)$$
$$-\frac{k}{E_{11}}\left(1-\mu_{23}\mu_{32}+(\mu_{23}\mu_{31}-\mu_{21})\sqrt{\frac{E_{11}}{E_{22}}}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{11}}{E_{33}}}\right)(\sigma_{11}-a)-$$
$$-\frac{k}{E_{22}}\left(1-\mu_{13}\mu_{31}+(\mu_{13}\mu_{32}-\mu_{12})\sqrt{\frac{E_{22}}{E_{11}}}+(\mu_{31}\mu_{12}-\mu_{32})\sqrt{\frac{E_{22}}{E_{33}}}\right)(\sigma_{22}-b)-c=0\,;$$

- dependency of the modules of wood elasticity on the method of intensity [2]

$$\mathrm{E_{ii}}=\begin{cases}E_{ii}^{P}\ \textit{для}\ \sigma_{ii}>0\ i\ \sigma_{jj}-\textit{будь-якого},\ i\neq j;\\ E_{ii}^{C}\ \textit{для}\ \sigma_{ii}<0\ i\ \sigma_{jj}-\textit{будь-якого},\ i\neq j;\end{cases} \qquad (2)$$

- constraint for the determine of *a, b* constants and k model (1): the amount of traction strength σ_{ii}^{P-} and σ_{ii}^{P+}, and under biaxial states of stress $(\sigma_{ii}>0,\ \sigma_{jj}\to-0,\ i\neq j)$ and $(\sigma_{ii}>0,\ \sigma_{jj}\to+0,\ i\neq j)$ is identically equal

$$\sigma_{ii}^{P-}=\sigma_{ii}^{P+}=\sigma_{ii}^{P}\,. \qquad (3)$$

Where

$$c=\frac{1}{2\sqrt{E_{11}^{P}E_{22}^{P}}}\left((1-\mu_{23}\mu_{32})\sqrt{\frac{E_{22}^{P}}{E_{11}^{P}}}a^2+(\mu_{13}\mu_{32}+\mu_{23}\mu_{31}-\mu_{12}\mu_{21})ab+(1-\mu_{13}\mu_{31})\sqrt{\frac{E_{11}^{P}}{E_{22}^{P}}}b^2\right); \qquad (4)$$

μ_{ij} – the Poisson ratio; E_{ii} – modulus of elasticity; σ_{ii} – the components of the stress tensor; *a*, b, c i k – constant coefficients; $E_{\mathrm{ii}}^{\mathrm{P}}$ $E_{\mathrm{ii}}^{\mathrm{C}}$ – wood modulus of elasticity the values in uniaxial tension and compression in the direction of i – than isotropy.

Determination of the *a, b* constants and k model (1). In order to solve this problem we investigate the curve (1) in points $(\sigma_{11}=\sigma_{11}^{P};\ \sigma_{22}=0;\ \sigma_{33}=0)$ and $(\sigma_{11}=0;\ \sigma_{22}=\sigma_{22}^{P};\ \sigma_{33}=0)$. For this we consider such cases: $(\sigma_{11}>0;\ \sigma_{22}\to-0;\ \sigma_{33}\to+0)$, $(\sigma_{11}>0;\ \sigma_{22}\to+0;\ \sigma_{33}\to+0)$, $(\sigma_{11}\to-0;\ \sigma_{22}>0;\ \sigma_{33}\to+0)$, $(\sigma_{11}\to+0;\ \sigma_{22}>0;\ \sigma_{33}\to+0)$.

If the $\sigma_{11}>0$, $\sigma_{22}\to-0$, and $\sigma_{33}\to+0$ elastic modulus values E_{11}, E_{22} and E_{33}, according to relations (2), are equal to E_{11}^{P}, E_{22}^{C} and E_{33}^{P}, respectively strength and the equation (1) is following:

$$\frac{1-\mu_{23}\mu_{32}}{2E_{11}^{P}}\left(\sigma_{11}^{P}\right)^2+\frac{k}{E_{11}^{P}}\left(1-\mu_{23}\mu_{32}+(\mu_{23}\mu_{31}-\mu_{21})\sqrt{\frac{E_{11}^{P}}{E_{22}^{C}}}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{11}^{P}}{E_{33}^{P}}}\right)\left(\sigma_{11}^{P}-a\right)- \qquad (5)$$
$$-\frac{k}{E_{22}^{C}}\left(1-\mu_{13}\mu_{31}+(\mu_{13}\mu_{32}-\mu_{12})\sqrt{\frac{E_{22}^{C}}{E_{11}^{P}}}+(\mu_{31}\mu_{12}-\mu_{32})\sqrt{\frac{E_{22}^{C}}{E_{33}^{P}}}\right)b-c=0\,.$$

In case $\sigma_{11} > 0$, $\sigma_{22} \to +0$, and $\sigma_{33} \to +0$ – $E_{11} = E_{11}^P$, $E_{22} = E_{22}^P$, $E_{33} = E_{33}^P$, and the strength equation (1) with provision for constraint (3) can be written as:

$$\frac{1-\mu_{23}\mu_{32}}{2E_{11}^P}\left(\sigma_{11}^P\right)^2 + \frac{k}{E_{11}^P}\left(1-\mu_{23}\mu_{32}+\left(\mu_{23}\mu_{31}-\mu_{21}\right)\sqrt{\frac{E_{11}^P}{E_{22}^P}}+\left(\mu_{32}\mu_{21}-\mu_{31}\right)\sqrt{\frac{E_{11}^P}{E_{33}^P}}\right)\left(\sigma_{11}^P - a\right) - \tag{6}$$

$$-\frac{k}{E_{22}^P}\left(1-\mu_{13}\mu_{31}+\left(\mu_{13}\mu_{32}-\mu_{12}\right)\sqrt{\frac{E_{22}^P}{E_{11}^P}}+\left(\mu_{31}\mu_{12}-\mu_{32}\right)\sqrt{\frac{E_{22}^P}{E_{33}^P}}\right)b - c = 0.$$

If $\sigma_{11} \to -0$, $\sigma_{22} > 0$, and $\sigma_{33} \to +0$, then in such case $E_{11} = E_{11}^C$, $E_{22} = E_{22}^P$, $E_{33} = E_{33}^P$, and the strength equation (1) is the following:

$$\frac{1-\mu_{13}\mu_{31}}{2E_{11}^P}\left(\sigma_{22}^P\right)^2 - \frac{k}{E_{11}^C}\left(1-\mu_{23}\mu_{32}+\left(\mu_{23}\mu_{31}-\mu_{21}\right)\sqrt{\frac{E_{11}^C}{E_{22}^P}}+\left(\mu_{32}\mu_{21}-\mu_{31}\right)\sqrt{\frac{E_{11}^C}{E_{33}^P}}\right)a + \tag{7}$$

$$+\frac{k}{E_{22}^P}\left(1-\mu_{13}\mu_{31}+\left(\mu_{13}\mu_{32}-\mu_{12}\right)\sqrt{\frac{E_{22}^P}{E_{11}^C}}+\left(\mu_{31}\mu_{12}-\mu_{32}\right)\sqrt{\frac{E_{22}^P}{E_{33}^P}}\right)\left(\sigma_{22}^P - b\right) - c = 0.$$

If $\sigma_{11} \to +0$, $\sigma_{22} > 0$, and $\sigma_{33} \to +0$, then according to (2), $E_{11} = E_{11}^P$, $E_{22} = E_{22}^P$, $E_{33} = E_{33}^P$, and the strength equation (1) with provision for constraint (2) is:

$$\frac{1-\mu_{13}\mu_{31}}{2E_{11}^P}\left(\sigma_{22}^P\right)^2 - \frac{k}{E_{11}^P}\left(1-\mu_{23}\mu_{32}+\left(\mu_{23}\mu_{31}-\mu_{21}\right)\sqrt{\frac{E_{11}^P}{E_{22}^P}}+\left(\mu_{32}\mu_{21}-\mu_{31}\right)\sqrt{\frac{E_{11}^P}{E_{33}^P}}\right)a + \tag{8}$$

$$+\frac{k}{E_{22}^P}\left(1-\mu_{13}\mu_{31}+\left(\mu_{13}\mu_{32}-\mu_{12}\right)\sqrt{\frac{E_{22}^P}{E_{11}^P}}+\left(\mu_{31}\mu_{12}-\mu_{32}\right)\sqrt{\frac{E_{22}^P}{E_{33}^P}}\right)\left(\sigma_{22}^P - b\right) - c = 0.$$

In case we subtract equation (6) from the equation (5), and equation(8) from the equation (7), as a result, after some simple mathematical transformations we obtain a system of equations:

$$\begin{cases} \left(\left(1-\mu_{13}\mu_{31}\right)\sqrt{E_{11}^P}\left(\frac{1}{E_{22}^C}-\frac{1}{E_{22}^P}\right)+\left(\mu_{13}\mu_{32}-\mu_{12}+\left(\mu_{31}\mu_{12}-\mu_{32}\right)\sqrt{\frac{E_{11}^P}{E_{33}^P}}\right)\left(\frac{1}{\sqrt{E_{22}^C}}-\frac{1}{\sqrt{E_{22}^P}}\right)\right)b + \\ +\left(\mu_{23}\mu_{31}-\mu_{21}\right)\left(\frac{1}{\sqrt{E_{22}^C}}-\frac{1}{\sqrt{E_{22}^P}}\right)a = \left(\mu_{23}\mu_{31}-\mu_{21}\right)\left(\frac{1}{\sqrt{E_{22}^C}}-\frac{1}{\sqrt{E_{22}^P}}\right)\sigma_{11}^P; \\ \left(\mu_{13}\mu_{32}-\mu_{12}\right)\left(\frac{1}{\sqrt{E_{11}^C}}-\frac{1}{\sqrt{E_{11}^P}}\right)b + \left(\left(\mu_{23}\mu_{31}-\mu_{21}+\left(\mu_{32}\mu_{21}-\mu_{31}\right)\sqrt{\frac{E_{22}^P}{E_{33}^P}}\right)\left(\frac{1}{\sqrt{E_{22}^C}}-\frac{1}{\sqrt{E_{22}^P}}\right)+\right. \\ \left.+\left(1-\mu_{23}\mu_{32}\right)\sqrt{E_{22}^P}\left(\frac{1}{E_{11}^C}-\frac{1}{E_{11}^P}\right)\right)a = \left(\mu_{13}\mu_{32}-\mu_{12}\right)\left(\frac{1}{\sqrt{E_{11}^C}}-\frac{1}{\sqrt{E_{11}^P}}\right)\sigma_{22}^P. \end{cases} \tag{9}$$

We simplify the resulting system of equations. To do this, the first equation of the system is divided by $\frac{1}{\sqrt{E_{22}^C}}-\frac{1}{\sqrt{E_{22}^P}}$, and the second by $\frac{1}{\sqrt{E_{11}^C}}-\frac{1}{\sqrt{E_{11}^P}}$, then (9) the following result is received:

$$
\begin{cases}
\left(\mu_{23}\mu_{31}-\mu_{21}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{22}^P}{E_{33}^P}}+(1-\mu_{23}\mu_{32})\left(\sqrt{\frac{E_{22}^P}{E_{11}^P}}+\sqrt{\frac{E_{22}^P}{E_{11}^C}}\right)\right)a+(\mu_{13}\mu_{32}-\mu_{12})b= \\
\quad =(\mu_{13}\mu_{32}-\mu_{12})\sigma_{22}^P; \\
(\mu_{23}\mu_{31}-\mu_{21})a+\left(\mu_{13}\mu_{32}-\mu_{12}+(\mu_{31}\mu_{12}-\mu_{32})\sqrt{\frac{E_{11}^P}{E_{33}^P}}+(1-\mu_{13}\mu_{31})\left(\sqrt{\frac{E_{11}^P}{E_{22}^P}}+\sqrt{\frac{E_{11}^P}{E_{22}^C}}\right)\right)b= \\
\quad =(\mu_{23}\mu_{31}-\mu_{21})\sigma_{11}^P.
\end{cases} \tag{10}
$$

The solution of system (10) is formulae for determining the values of the a and b constants of the mathematical model (1):

$$a=B_2\frac{B_1\sigma_{22}^P-A_1\sigma_{11}^P}{A_2B_2-A_1B_2}; \tag{11}$$

$$b=A_1\frac{A_2\sigma_{11}^P-B_2\sigma_{22}^P}{A_2B_1-A_1B_2}, \tag{12}$$

where

$$A_1=\mu_{23}\mu_{31}-\mu_{21}; \tag{13}$$

$$A_2=\mu_{23}\mu_{31}-\mu_{21}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{22}^P}{E_{33}^P}}+(1-\mu_{23}\mu_{32})\left(\sqrt{\frac{E_{22}^P}{E_{11}^P}}+\sqrt{\frac{E_{22}^P}{E_{11}^C}}\right); \tag{14}$$

$$B_1=\mu_{13}\mu_{32}-\mu_{12}+(\mu_{31}\mu_{12}-\mu_{32})\sqrt{\frac{E_{11}^P}{E_{33}^P}}+(1-\mu_{13}\mu_{31})\left(\sqrt{\frac{E_{11}^P}{E_{22}^P}}+\sqrt{\frac{E_{11}^P}{E_{22}^C}}\right); \tag{15}$$

$$B_2=\mu_{13}\mu_{32}-\mu_{12}. \tag{16}$$

The k constant could be taken from the formulae below [1]:

$$k=\frac{f(\tilde{\sigma}_{11};\tilde{\sigma}_{22})-f(\hat{\sigma}_{11};\hat{\sigma}_{22})}{\varepsilon_V(\tilde{\sigma}_{11};\tilde{\sigma}_{22})-\varepsilon_V(\hat{\sigma}_{11};\hat{\sigma}_{22})}; \tag{17}$$

$$f(\sigma_{11};\sigma_{22})=\frac{1-\mu_{23}\mu_{32}}{2E_{11}}\sigma_{11}^2+\frac{\mu_{13}\mu_{32}+\mu_{23}\mu_{31}-\mu_{12}\mu_{21}}{2\sqrt{E_{11}E_{22}}}\sigma_{11}\sigma_{22}+\frac{1-\mu_{13}\mu_{31}}{2E_{22}}\sigma_{22}^2; \tag{18}$$

$$\varepsilon_V(\sigma_{11};\sigma_{22})=\frac{1}{E_{11}}\left(1-\mu_{23}\mu_{32}+(\mu_{23}\mu_{31}-\mu_{21})\sqrt{\frac{E_{11}}{E_{22}}}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{11}}{E_{33}}}\right)\sigma_{11}+ \tag{19}$$

$$+\frac{1}{E_{22}}\left(1-\mu_{13}\mu_{31}+(\mu_{13}\mu_{32}-\mu_{12})\sqrt{\frac{E_{22}}{E_{11}}}+(\mu_{31}\mu_{12}-\mu_{32})\sqrt{\frac{E_{22}}{E_{33}}}\right)\sigma_{22},$$

where $(\tilde{\sigma}_{11};\tilde{\sigma}_{22})$ and $(\hat{\sigma}_{11};\hat{\sigma}_{22})$ – the coordinates of randomly selected points on the strength curve of the researched material.

In order to do this, select the strength of the desired curve point $(\sigma_{11}^P;0)$ and $(0;\sigma_{22}^P)$. Then the formulae (17) for determining the k constant are:

$$k=\frac{f(\sigma_{11}^P;0)-f(0;\sigma_{22}^P)}{\varepsilon_V(\sigma_{11}^P;0)-\varepsilon_V(0;\sigma_{22}^P)}; \tag{20}$$

$$f(\sigma_{11}^P;0)-f(0;\sigma_{22}^P)=\frac{1-\mu_{23}\mu_{32}}{2E_{11}^P}(\sigma_{11}^P)^2-\frac{1-\mu_{13}\mu_{31}}{2E_{22}^P}(\sigma_{22}^P)^2; \tag{21}$$

$$\varepsilon_V(\sigma_{11}^P;0)-\varepsilon_V(0;\sigma_{22}^P)=\frac{1}{E_{11}^P}\left(1-\mu_{23}\mu_{32}+(\mu_{23}\mu_{31}-\mu_{21})\sqrt{\frac{E_{11}^P}{E_{22}^P}}+(\mu_{32}\mu_{21}-\mu_{31})\sqrt{\frac{E_{11}^P}{E_{33}^P}}\right)\sigma_{11}^P- \tag{22}$$

$$-\frac{1}{E_{22}^{P}}\left(1-\mu_{13}\mu_{31}+\left(\mu_{13}\mu_{32}-\mu_{12}\right)\sqrt{\frac{E_{22}^{P}}{E_{11}^{P}}}+\left(\mu_{31}\mu_{12}-\mu_{32}\right)\sqrt{\frac{E_{22}^{P}}{E_{33}^{P}}}\right)\sigma_{22}^{P}.$$

The results of mathematical modeling of wood boundaries stressed state and their analysis. In order to find the numerical calculations of the (1)–(4) model algorithm [3] is used, the practical implementation of which lies in: 1) the tabulation of functions $\varphi(\sigma_{11},\sigma_{22})$ with the steps of tabulation $\Delta\sigma_{11}$ and $\Delta\sigma_{22}$; 2) finding the argument σ_{11} for fixed values of the argument σ_{22} σ_1 so that at least one of the following conditions is fulfilled:

$$\varphi(\sigma_{11},\sigma_{22})>0,\ \text{and}\ \varphi(\sigma_{11},\sigma_{22}+\Delta\sigma_{22})<0\ \text{or}\ \varphi(\sigma_{11},\sigma_{22})<0,\ \text{and}\ \varphi(\sigma_{11},\sigma_{22}+\Delta\sigma_{22})>0, \quad (23)$$

where $\varphi(\sigma_{11},\sigma_{22})$ – function, which value is identically equal to the values of expression in the left-hand side of equation (1); 3) the choice from σ_{22} and $\sigma_{22}+\Delta\sigma_{22}$ numbers where the absolute value of the function $\varphi(\sigma_{11},\sigma_{22})$ is minimal; 4) construction of strengths curves of these points $(\sigma_{11},\sigma_{22})$.

With this algorithm boundary states of stress of pine and oak are calculated, the value of the physical and mechanical characteristics are provided in table 1 below. In particular, the strengths curves of these materials are plotted under biaxial stress state in the radial-tangential level of structure symmetry (Fig. 1).

Table 1

Stress-related characteristics of pine and oak with a temperature T=20 °C and a relative moisture of W = 12 %: the numerator – the value of traction, and the denominator – the value of compression

Wood Species	Modulus of elasticity. GPa			Yields value. MPa		The Poisson ratio					
	E_a	E_r	E_t	σ_r	σ_t	μ_{ar}	μ_{at}	μ_{rt}	μ_{ra}	μ_{ta}	μ_{tr}
Pine	11.9 / 11.9	0.54 / 0.67	0.47 / 0.55	3.23 / 5.10	2.63 / 7.50	0.03	0.037	0.38	0.49	0.41	0.79
Oak	14.2 / 14.2	1.18 / 1.40	0.91 / 1.01	8.0 / -	6.5 / -	0.07	0.09	0.34	0.43	0.41	0.83

We make the analysis of the component origin of plotted curves that characterize such a boundary stress material state, which characteristics are empirically determined. These points are the points of intersection of these curves with the coordinate axes A_C, C_C, B_C, D_C and $A_Д$, $C_Д$, $B_Д$, $D_Д$. According to Fig. 1, the value of A_C and C_C, $A_Д$ and $C_Д$ – points is the value of the limit strength traction and pine, oak wood compression respectively to the radial direction of orthotropy and the ordinate B_C and D_C, $B_Д$ and $D_Д$ – points is the value of strength boundaries traction and compression of mentioned above timber species in a tangential direction, that is currently known and given in the reference literature [2]. The analysis of these points coordinates showed that the mathematical model (1)–(4) adequately reflects the strength of anisotropy and asymmetry of wood in question along the main lines of symmetry. Indeed, as the abscissa of the A_C point is greater than ordinate B_C, point and the absolute value of the abscissa C_C point is less than the absolute value of the ordinate of D_C, point, the limit of short-term strength of pine traction in the radial direction is greater than the tensile strength traction in the tangential direction. In the case of compression it is vice versa: the absolute value of the tensile strength in the radial direction is smaller than the absolute value of the tensile strength in the tangential direction that is confirmed by experimental results [2]. The ratio of ultimate strength traction in the radial and tangential directions of the anisotropy for the pine that is true for the oak as well is found.

Thus, taking into account the results of the presently known experimental studies of short-term wood strength with a uniaxial stress state in the anisotropy directions a mathematical model (1)–(4) adequately describes the boundary state of pine and oak stress in a biaxial-stretching traction, compression-compression and traction-compression. In addition, we would like to note that the characteristic of plotted curves (Fig. 1) meet the basic requirements of building a heuristic known phenomenological theories of mechanical solids strength [4, 5]: 1) the curves limiting stressed state (curves of short-term strength) should be smooth and convex; 2) the curves of short-term strength of the material should cover the origin of the Cartesian frame of axis.

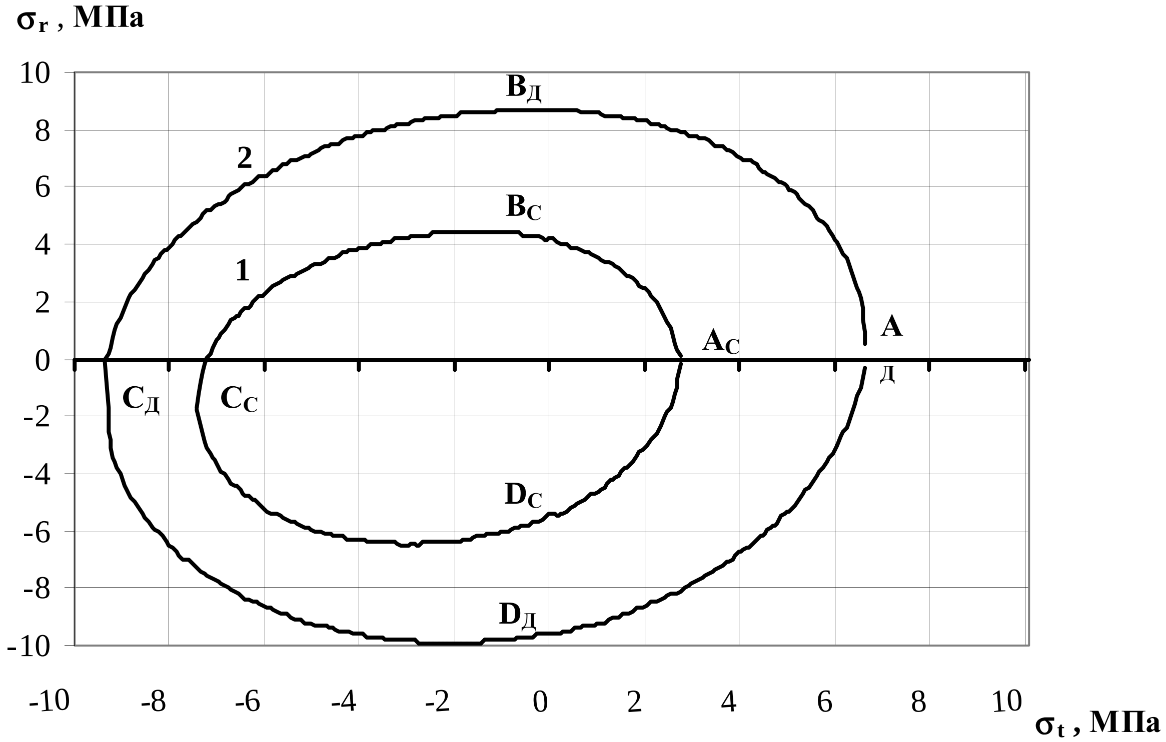

Fig. 1. Curves of short-term strength of wood with a temperatureT = 20 °C ,
relative moisture of W = 12 % and biaxial stress state in the radial – tangential structural symmetry plane:
1 – pine; 2 – oak

Conclusions

It is shown that the calculated curves of short-term pine and oak strength with a biaxial stress-strain state in the radial-tangential level of the structural symmetry describe satisfactorily the experimental tests on the material strength of the uniaxial traction and compression in the radial and tangential directions of anisotropy. In particular, it is sustained that: a) tensile strength of pine traction in the radial direction is greater than tensile strength in tangential direction, and it is conversely in the case of compression: the absolute value of the tensile strength in the radial direction is smaller than the absolute value of the tensile strength in the tangential direction; б) the absolute values of the maximum tensile strength in the radial and tangential directions of the pine wood anisotropy are less than the corresponding absolute values of the maximum compressive strength.

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