I.Y. Vrublevsky\*, A.L. Bespalov

\*Academy of the Army named after Hetman Petro Sahaydachnyi,
Department of Engineering Mechanics
National University "Lviv Polytechnics",
Department of Descriptive Geometry and Graphics

## THE CALCULATION OF THE THREE-MASS VIBRATORY BOWL FEEDER THAT PROVIDES ELLIPTICAL OSCILLATION WITH AN AXIAL VIBRATORY DRIVE

© Vrublevsky I.Y., Bespalov A.L., 2013

Розроблено методику обчислення параметрів тримасового вібраційного бункерного живильника з осьовим електромагнітним приводом, які забезпечать оптимальні еліптичні коливання бункера: відношення амплітуд складових коливань, оптимальний за швидкістю кут зсуву фаз між ними. Наведено приклад розрахунку конструкції вібробункерного живильника з еліптичними коливаннями.

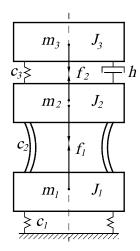
The method for calculation of the parameters of three-mass vibratory bowl feeder with an axial electromagnetic drive has been carried. These parameters must provide the optimum values of elliptical oscillations: the ratio of the bowl vibration component amplitudes, the phase difference angle. An example of calculation of three-mass vibratory bowl feeder with elliptical oscillations is shown.

Statement of the problem. The use of elliptical oscillations of the bowl in the vibratory feeders with electromagnetic drive, when the vertical component of the oscillations is behind the phase from the horizontal, is quite effective to increase the feed rate and the track angle of parts, especially in the modes of movement without jumping. Increase of conveying velocity of parts and the track angle compared with the simplest feeders with linear oscillations is possible only at a certain ratio of the vibration parameters: the amplitudes of the components of the oscillations and the phase difference angle between them [1]. Therefore, the actual problem is the development of methods of computation of the parameters values of vibratory drive, which will provide the optimal parameters of elliptical oscillation of the bowl.

The analysis of last researches and publications. Elliptical oscillations of vibratory bowl feeder most often get through the use of a tangential electromagnetic vibratory drive of the angular oscillations. [1]. But they have a very low coefficient of efficiency and their use leads to a significant power consumption, therefore, there design vibratory bowl feeder with axial vibratory drives was proposed, which provides the elliptical oscillations due to the presence among the bowl and intermediate mass a damper or additional axial vibratory drive [2], the dynamics of such vibratory feeder considered in [3,4]. Formulas for determining the natural frequencies and amplitudes of vibrations and the angle of phase difference of the components of three-mass oscillating system were obtained. Mathematical dependences, however, difficult enough for calculations of parameters of the feeder's drive, which would ensure the optimum values of parameters of elliptical oscillation to reach the maximum feed rate and the track angle of parts.

The wording of the objectives of the study. The aim of the research is to develop an engineering method for calculating the parameters of three-mass vibratory bowl feeder with an axial vibratory drive, which will provide the optimal speed parameter values of elliptical oscillations (amplitude of component vibrations, the angle of phase diffrence between them, the optimal track angle of the transported parts.

Statement of the basic material of research. Dynamic scheme of a vibratory bowl feeder can be presented in the form of a three-mass oscillating system (Fig. 1), in which the working body – the bowl of mass  $m_3$  and axial moment of inertia of  $J_3$  is connected by the elastic system consisting of the transverse flat springs with axial stiffness  $c_3$ , with the intermediate plate of mass  $m_2$  and axial moment of inertia  $J_2$ , which in turn is connected to the reactive frame of mass  $m_1$  and axial moment of inertia  $J_1$  with the help of elastic system in the form of hyperboloid torsion with the axial stiffness  $c_2$ , and the reactive frame set on the sit atop vibration insulators with the stiffness  $c_1$  [3, 4]. Vibratory axial electromagnetic driver is mounted between the reactive frame and intermediate plate, the force of which  $f_1 = F_1 \cos \omega t$ , where  $F_1$  — maximal force of driver,  $\omega = 2\pi \nu$  — circular frequency of oscillations,  $\nu$  — cyclic frequency of oscillations,  $\nu$  — time. Between the bowl and the intermediate plate — damper with a coefficient of viscous resistance  $\mu$  or additional axial electromagnetic driver with the force  $\mu$  =  $\mu$ 



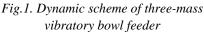




Fig.2. The vibratory bowl feeder with elliptical oscillations

The natural frequencies of the feeder oscillations [4]:

$$\omega_1 = \sqrt{\frac{c_2}{M_1 + \kappa^2 J_n}}, \ \omega_2 = \sqrt{\frac{c_3}{M_2} - (\frac{h}{2M_2})^2},$$
 (1)

where 
$$M_1 = \frac{m_1(m_2 + m_3)}{M_0}$$
,  $M_2 = \frac{m_3(m_1 + m_2)}{M_0}$ ,  $J_n = \frac{J_1(J_2 + J_3)}{J_0}$ ,  $M_0 = m_1 + m_2 + m_3$ ,  $J_0 = J_1 + J_2 + J_3$ ,

 $\kappa = (r \sin(\arccos l \sin \psi / 2r) \tan \psi)^{-1}$ , r – radius mounting springs of length l of the hyperboloid torsion, inclined at an angle  $\psi$  to the vertical (Fig. 1). Amplitudes  $\theta_3$  of torsional and  $A_3$  vertical oscillations of the bowl [4]:

$$\theta_3 = \frac{F_1 \kappa J_1}{c_2 J_0 (1 - \zeta^2)} \sqrt{1 + \delta \eta^2 \mu_1 k_1 (\delta \eta^2 \mu_1 k_1 + 2\cos(\varphi - \varphi_1))}, \qquad (2)$$

$$A_3 = \frac{F_1 m_1 \mu_2}{c_2 M_0 (1 - \zeta^2)} \sqrt{1 + \delta \eta^2 k_2 k_3 (\delta \eta^2 k_2 k_3 + 2\cos(\varphi - \varphi_1 + \varphi_2))} , \qquad (3)$$

where

$$\mu_1 = \frac{1}{\sqrt{(1-\eta^2)^2 + (\gamma \eta)^2}}, \qquad \mu_2 = \frac{\sqrt{(1-\eta^2 + \gamma^2 \eta^2)^2 + \gamma^2 \eta^6}}{(1-\eta^2)^2 + \gamma^2 \eta^2}, \quad \tan \varphi_2 = \frac{\gamma \eta^3}{1-\eta^2 + \eta^2 \gamma^2}, \quad (4)$$

$$\tan \varphi_1 = \frac{\gamma \eta}{1 - \eta^2}, \qquad k_1 = m_1 / m_1 + m_2, \qquad k_2 = \frac{(M_1 + \kappa^2 J_n) M_0 (1 - \zeta^2)}{m_1 m_3 \zeta^2}, \qquad k_3 = \mu_1 / \mu_2,$$

 $\zeta = \omega/\omega_1$  – resonance tuning on the first natural frequency,  $\eta = \omega/\omega_2$  – resonance tuning on the second natural frequency without resistance,  $\gamma = h/\sqrt{c_3 M_2}$  – the relative damping. The torsional oscillations with an amplitude  $\theta_3$  of the bowl screw track on an average radius R will provide horizontal oscillations with an amplitude  $A_\theta = \theta_3 R$ .

Conveying velocity *V* of parts can be defined by the formula [1]:

$$V = A_{x} \omega K \quad , \tag{5}$$

where  $A_x$  – amplitude of oscillations in the direction of conveying (longitudinal oscillations), K – a dimensionless coefficient of velocity, which depends on several dimensionless parameters, in particular relations  $\tan\alpha/f$ ,  $\alpha$  – track angle, f – friction coefficient. If conveying up without jumping of the conditions for lifting to a certain height in the shortest time  $\tan\alpha=0.35f$ , and for elliptical oscillations with optimum angle of phase difference K=0.37 [6]. Amplitude  $A_y$  of oscillations in the direction perpendicular to the direction of conveying (normal

oscillations) is limited for conveying with and without jumping by the overload parameter  $w = \frac{A_y \omega^2}{g \cos \alpha}$ , g – the

acceleration of gravity. In particular for conveying without jumping w=1, for jumping conveying of parts with the frequency 25 Hz w=1.2. When oscillations are elliptical the relationship between the amplitudes of horizontal  $A_{\theta}$  and vertical  $A_3$  oscillations with the amplitudes of longitudinal  $A_x$  and normal  $A_y$  oscillations [7]:

$$A_{\theta} = A_x \sqrt{\cos^2 \alpha + b^{-2} \sin^2 \alpha - \sin 2\alpha \cos \varepsilon / b} , A_3 = A_y \sqrt{\cos^2 \alpha + b^2 \sin^2 \alpha + b \sin 2\alpha \cos \varepsilon} , \tag{6}$$

where  $b = A_x/A_y$ . Moreover if for angles  $\alpha$  a few degrees, which are used in the bowl feeders, amplitude  $A_{\theta}$  and  $A_x$  almost do not differ, the angle  $\varepsilon$  of phase difference between longitudinal and normal oscillations significantly different from the angle  $\varepsilon_3$  of phase difference between horizontal and vertical oscillations. Therefore, determining the angle  $\varepsilon$ , it is necessary to calculate the angle  $\varepsilon_3$  [7]:

$$\varepsilon_3 = \arctan \frac{2b \sin \varepsilon}{2b \cos \varepsilon \cos \alpha + (b^2 - 1) \sin 2\alpha} \tag{7}$$

Dividing equation (2) to (3), we obtain the ratio of the amplitudes, which should be equal to the  $A_{\theta}/A_3$ :

$$b_0 = \frac{\kappa J_1 M_0 R}{J_0 m_1 \mu_2} \sqrt{\frac{1 + \delta \eta^2 \mu_1 k_1 (\delta \eta^2 \mu_1 k_1 + 2\cos(\varphi - \varphi_1))}{1 + \delta \eta^2 k_2 k_3 (\delta \eta^2 k_2 k_3 + 2\cos(\varphi - \varphi_1 + \varphi_2))}}$$
(8)

If only damper is used without an additional vibratory driver ( $\delta$ =0) radical expression (8) is equal to unity, and the angle of phase difference between horizontal (torsional) and vertical oscillations of the bowl will be equal  $\varphi$  and must be equal  $\epsilon_3$ . At the same time to provide the necessary optimal values of the angle  $\varphi$  and the ratio of amplitudes  $b_0$ , as will be shown below, it is not always possible. Therefore, to improve the stability of the feeder and to simplify the regulation of parameters it is suggested to use instead of damper the additional vibratory driver of small capacity with the force  $f_2$  [2]. When using additional vibratory driver without damper ( $\gamma$ =0) the desired angle of phase difference is given by

$$\tan \varepsilon_3 = \frac{\delta \eta^2 (k_2 k_3 - \mu_1 k_1) \sin \varphi}{1 + \delta \eta^2 ((\mu_1 k_1 + k_2 k_3) \cos \varphi + \delta \eta^2 \mu_1 k_1 k_2 k_3)}.$$
 (9)

Let's consider an example of the calculation of vibratory feeder with diameter of bowl 600 mm, developed and produced in the scientific-research laboratory of the National University «Lviv Polytechnic» and put into production (Fig. 2). Vibratory bowl feeder with an electromagnetic drive with the oscillation frequency of 25 Hz is designed for conveying without jumping and feed of brass parts on a steel screw track with a velocity of 150 mm/sec. We know its construction and mass-inertial parameters – mass and moments of inertia of three components:  $m_1$ =123.5 kg;  $J_1$ =7.35 kg m²;  $m_2$ =20.6 kg;  $J_2$ =0.54 kg m²;  $m_3$ =38.2 kg;  $J_3$ =2.88 kg m². Parameters of torsion: r=11.7 cm; l=24.5 cm;  $\psi$ =12°; R=28 cm. Determine the optimal values of track angle, component amplitude oscillations, the phase difference angle between them and the main parameters of vibratory drive.

A friction coefficient brass-steel determined by measuring the angle of friction: f = 0.25. Optimal screw track angle is  $\alpha = arctg(0.35f) = 4^{\circ}$ . With a frequency 25 Hz  $\omega = 157$  cer<sup>-1</sup> and with K = 0.37 from formula (5) we obtain the required amplitude of longitudinal oscillations  $A_x = 2.58$  mm. Amplitude of normal oscillation is determined from the condition of conveying without jumping w = 1:

 $A_y = \frac{wg \cos \alpha}{\omega^2} = 0.4$  mm. The ratio between the amplitudes of longitudinal and normal oscillations b = 6.45. The

optimal angle of phase difference depends on the parameters  $\tan \alpha/f$  i b/f [5]. From graphs [5] or approximated formulas, derived in [6], we define  $\varepsilon = 82^{\circ}$ . From formulas (6) we determine the amplitudes  $A_{\theta}$ =2.57 mm,  $A_{3}$  = 0.46 mm, their ratio  $b_{0}$ =5.59, and the formula (7) determines the phase difference angle between them  $\varepsilon_{3}$ =59.7°. As shown by the research and experience, for the stable operation of vibratory feeders, value of resonance tuning should be  $\zeta$ =0.92...0.95,  $\eta$ =0.8...0.9 [8]. Having their lower limit values, calculate according to the known method [1,8] stiffness  $c_{2}$  and  $c_{3}$  and dimensions of springs, which will ensure the necessary values of resonance tuning, as well as the force of drive  $F_{1}$ , necessary for providing the amplitude  $A_{\theta}$ . Substituting  $b_{0}$ =5.59 in (8) with  $\delta$ =0 we obtain  $\mu_{2}$ =2.07. Substituting this value in second equation (4), and  $\varphi$ = $\varepsilon_{3}$ =59.7° – in third equation (4), we obtain a system of equations with two unknowns, the solution of which will be  $\eta$ =0.99;  $\gamma$ =0.55. Thus, we need dampers with too large damping coefficient, and the value of tuning  $\eta$  sufficiently close to resonance, which reduces stability, so ensuring proper elliptical trajectory oscillations via damper is problematic.

Use instead of damper an additional vibratory driver and calculate its parameters  $\delta$  and  $\phi$ , necessary for providing the optimal elliptical oscillations of bowl. Having value  $\eta$  from the above range, with  $\gamma$ =0  $\phi_1$ = $\phi_2$ =0,  $\mu_1$ = $\mu_2$ =2.78, and considering all the known parameters, substituting them in formulas (8) and (9), we obtain a system of two equations with unknown  $\delta$  and  $\phi$ . Solving it, we get the required force of the additional driver in relation to the force of main driver  $\delta$ =0.03 and the angle of phase difference between them  $\phi$ =67°. While solving this system of equations is more complicated than in the previous case, in practice these parameters control of the vibratory driver (changing voltage applied and the angle of phase difference) is much simpler than the damper settings.

The findings of the study. The methods of engineering calculation of parameters of three-mass vibratory bowl feeder with axial vibratory drive (mass and moments of inertia, damper settings, the ratio of the forces of vibratory drivers and the angle of phase difference between them, the stiffness of elastic systems) have been developed, which provide the optimal speed parameter values of elliptical oscillation: the ratio of the amplitudes of the components and the angle of phase difference between them. A concrete example of the values of parameters calculation of vibratory bowl feeder with elliptical oscillations, introduced in the production, has been shown.

1. Повідайло В.О. Вібраційні процеси та обладнання — Львів: Нац. ун-т «Львівська політехніка», 2004. 2. А. с. № 1070090 СССР. Вибрационный бункерный питатель / В.А. Повидайло, И.И. Врублевский. Опубл. 30.01.1984. — Б.И. № 4, 1984. 3. Повидайло В.А., Беспалов А.Л., Врублевский И.И. Исследование динамики трехмассового вибробункерного питателя с эллиптическими колебаниями // Автоматизация производственных процессов в машиностроении и приборостроении. – Львов: Вища шк., 1984. – Вып. 23, с. 74-79. 4. Повидайло В.А., Врублевский И.И. Динамика вибробункерного питателя с осевыми вибровозбудителями // Машиноведение. – 1985. – № 6. - C. 22-25. 5. Врублевский И.И. Разработка и исследование вибрационных устройств, осуществляющих организацию рабочей среды роботосистем. – Автореферат диссертации к.т.н. – Каунас: Каунас. политех. ин-т. – 1986. б. Врублевський І.Й. Методика визначення параметрів еліптичних коливань під час швидкісного безвідривного вібротранспортування // Автоматизація виробничих процесів у машинобудуванні та приладобудуванні. – Вип. 45. – Львів: Нац. ун-ту "Львівська політехніка". – 2011. – С. 175–178. 7. Ефимов В.Г. Настройка двухкомпонентного электромагнитного привода // Вопросы динамики и прочности. — Рига: Зинатне, 1980. — Вып. 36. — С. 45-48. 8. Повидайло В.А., IЩигель В.А. Конструкция и расчёт вибрационных бункерных питателей з независимыми осевыми и крутильными колебаниями // Автоматизация производственных процессов в машиностроении и приборостроении. – Львів: Вища шк., 1973. – Вип. 13. – С. 112–115.