УДК 621.391

В. М. Лозинська
Донецький національний технічний університет

ВИКОРИСТАННЯ ЛІНІЙНИХ MAX-PLUS МОДЕЛЕЙ У ЗАДАЧАХ УПРАВЛІННЯ ТРАФІКОМ

Запропоновано лінійні математичні моделі Max-plus механізмів та методів управління трафіком, як для мережевого вузла окремо (планувальник обслуговування черг буферів маршрутизатору), так і для усієї мережі (алгоритми маршрутизації). Моделі, що подані, дають змогу підвищити ефективність оцінки параметрів якості, оскільки в них враховано основні параметри трафіку, що надходить. Також запропоновані математичні моделі відтворюють динаміку роботи планувальника в просторі станів, а перехід до базису Max-plus алгебри дає змогу отримати задані оцінки з урахуванням необхідних параметрів якості обслуговування.

Ключові слова: параметри якості обслуговування, механізми та методи управління трафіком, планувальник обслуговування черг буферів маршрутизатора, алгоритми маршрутизації, Max-plus алгебра, дискретно-подієва система, простір станів.

V. M. Lozynska
Donetsk national technical university

LINEARY MAX-PLUS MODELS APPLYNG IN TRAFFIC MANAGEMENT TASKS

Development of next generation networks concept let to reconfigure the most of existing traffic controls at telecommunication node. Necessity of its modification causes of low efficient network resources usage. Among of frequently used traffic controls at telecommunication node are service queues schedulers. This is due to traffic "conditioning" feature and as result opportunity of decreasing data lost level. For modern telecommunication devices most common are schedulers with fair and hybrid queue servicing. Limitation of existing algorithms can be explained by it static character at the same time with dynamic changing character of incoming traffic and it non-controllability. For modification of these algorithms, formalized an optimization problem, limitations of which are quality of service parameters. Also being of more than one queue at one network device suggests packet service synchronization. All these tasks can be solved with two ways, presented in the paper: discrete-event models and Max-plus algebra linear models. For every model was find state space equations by conversion into Max-plus algebra basis. Control functions in represented models execute vectors of incoming request delays and requests' service time. The solving of the given optimization problem is in finding of adjustments vector, which in fact shows the required queue buffer space size for every computing cycle. Thus, in represented article for the first time the max-plus algebra applications is showed for the solving traffic management tasks.

Key words: traffic controls, queue service discipline, Max-plus algebra, discrete-event system, state space.

Introduction

Because of traffic management mechanisms and methods that take into consideration quality of service (QoS) requirements to different application the next generation modern conception is carried out. Wherein the most full-featured and developed technology in what this conception involved is Multy-protocol Label switching technology (MPLS). In particular it has an extension that directs to the flexible traffic management − MPLS Traffic Engineering (MPLS-TE). However despite of all function possibilities that were initially laid in MPLS-TE mechanisms, its potential is not realized completely. This due to mathematic models and methods restrictions on which basic traffic management mechanisms are based on such as routing protocols and queue schedulers' mechanisms [1].

Shortest pass graph models in modern routing protocols are not take into consideration all features of traffic that is not allowed to find out routes along which all quality of service requirements are provided. Queue schedulers' mechanisms are into static models which are not taken into consideration dynamic traffic feature that does not let to have the original traffic estimates. Hence the difficulty of basic management traffic mechanisms description for the MPLS networks and development new ones are connected with some presenting of traffic dynamic and with the necessity of finding tasks solutions which are relating to different open system model levels.

Because of traffic management tasks in view of network resources usage performance with the required quality of service became the most basic in line with telecommunicate networks' recourses management then mathematic models' designing of effective traffic management algorithm is the actual researching problem.

The aim of this work is to present the usage of different linear max-plus algebra models for an increasing the quality of modeling compared with existing mathematical models on what traffic management models and mechanisms are based on.

1. Presentation of Max-plus (minus) linear algebra

Mathematical apparatus and algebras analysis is showed that the idempotent algebras are the universal tool for the describing traffic management mechanisms in general and its' discrete-event features in particularly [2, 3].

Idempotent semifield is a set of M real numbers, endowed with the operation $\oplus$ (associative addition), and it has neutral element 0: $0 \oplus a = a$ for any $a \in M$ and it is idempotent if $a \oplus a = a$ for any $a \in M$. We say that the semifield is idempotent if there is one more operation $\otimes$ (commutative) has neutral element 1 and connected with $\oplus$ and distributive with both sides:

$$a \otimes (b \oplus c) = a \otimes b \oplus a \otimes c, \tag{1}$$

$$(b \oplus c) \otimes a = b \otimes a \oplus c \otimes a. \tag{2}$$

The set $\underline{\mathbf{R}}$, with adding $e = -\infty$, endowed with the operations $\oplus$ and $\otimes$, that is for all $x, y \in \underline{\mathbf{R}}$ define next:

$$x \oplus y = \max(x, y), \tag{3}$$

$$x \otimes y = x + y, \tag{4}$$

with

$$x \otimes e = e. \tag{5}$$

There are two kinds of linear systems in $\mathbf{R}_{\max}$ for any two matrices $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ size $l \times n$ and $n \times m$ for which are able to compute solutions:

$$\{\mathbf{A} \oplus \mathbf{B}\}_{ij} = a_{ij} \oplus b_{ij}, \tag{6}$$

$$\{\mathbf{A} \otimes \mathbf{B}\}_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes b_{kj} \tag{7}$$

Multiplying and adding operations with matrices and scalars are defined also. So, for any $l \in \underline{\mathbf{R}}$ and matrices $\mathbf{A} = (a_{ij})$ size $l \times n$.

$$\{l \oplus \mathbf{A}\}_{ij} = l \oplus a_{ij}, \tag{8}$$

$$\{l \otimes \mathbf{A}\}_{ij} = l \otimes a_{ij}. \tag{9}$$

Matrix e , with all e elements is zero matrix. Matrix $\mathbf{I}$, with e elements on main diagonal and all another e elements is neutral matrix.

If $\mathbf{A} \in \underline{\mathbf{R}}$ then getting matrix $\mathbf{A}^n$ with $n > 0$ can be defined as follows:

$$\mathbf{A}^n = \mathbf{A} \otimes \mathbf{K} \otimes \mathbf{A} = \bigotimes_{i=1}^{n} \mathbf{A} . \tag{10}$$

The most searching interest has matrix operation $\mathbf{A}^*$, which is used for the solving space-state equations with Max-plus algebra. If G(A) has no circuit with positive weight, then:

$$\mathbf{A}^* = \mathbf{A} \oplus \mathbf{A}^2 \oplus \mathbf{K} \oplus \mathbf{A}^{n-1}, \tag{11}$$

where n is the dimension of matrix $\mathbf{A}$.

It is possible to derive the min operation ($R_{\min} = (R_e, \oplus', \cdot')$, $R_e = R \cup \{+\infty\}$), from the two operations $\oplus'$ and $\cdot'$ as follows [4]:

$$a \oplus' b = \min(a, b) , \tag{12}$$

$$a \cdot' b = a + b , \tag{13}$$

for all $a, b \in \mathrm{R}_e$.

Besides of features that were given this algebra can be used for the optimization tasks solving, formalizing, getting into and solutions of the space-state equations, for the making solutings about process controllability.

1.1. Shortest path task solving with the Max-minus algebra

Consider the following a digraph, $D(V, E)$ (fig. 1), of telecommunication network, where the set of vertices E represents channels and the weight d_{ij} of each arc actually represents the average time it takes to get through the pass $(i, j) \in E$. It would like to find the smallest time, that takes packet propagation between any two intersections.

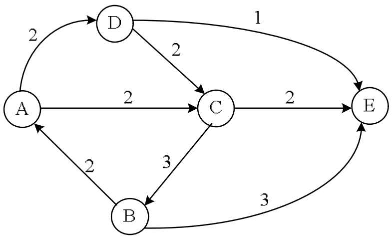


Fig. 1. Network digraph

Network digraph can be presented as follow matrix:

$$\mathbf{D} = \begin{pmatrix} \infty & \infty & 2 & 2 & \infty \\ 2 & \infty & \infty & \infty & 3 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & 2 & \infty & 1 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix}.$$

Based on Dejkstra's algorithm the shortest path between any of nodes can be find. As a result, there are the following passes between node A and all another ones with weights: $D(R_{AB}) = 5$, $D(R_{AC}) = 2$, $D(R_{AD}) = 2$, $D(R_{AB}) = 3$. The same way all another passes founded. For the simplicity get all results into follow matrix $D(R_{ij})$:

$$D(R_{ij}) = \begin{pmatrix} \infty & \underline{2} & 5 & 7 & \infty \\ 5 & \infty & 3 & 5 & \infty \\ \underline{2} & 4 & \infty & 2 & \infty \\ 2 & 4 & 7 & \infty & \infty \\ 3 & 3 & \underline{2} & \underline{1} & \infty \end{pmatrix}.$$

The same task can be solved with Min-plus algebra. For this next equation can be solved [5]:

$$\mathbf{D}^{+} = \underset{l \in (n-1)}{\oplus'} (\mathbf{D}^{l}), \tag{14}$$

$\mathbf{D}^{l}$ – delay matrix $n \times n$; n – network nodes.

Solving this task the element i, j of matrix $\mathbf{D}^2$ can be counted as:

$$d_{ij} = \min(d_{i1} + d_{1j}, d_{i2} + d_{2j}, \mathbf{K}, d_{in} + d_{nj}). \tag{15}$$

For the matrix with delays which contains k arc passes is necessary to count $\mathbf{D}^k$. Thus for the n-node network matrix $\mathbf{D}^{n-1}$ has to be counted. So for digraph (fig.1) has to be counted matrices $\mathbf{D}^2$, $\mathbf{D}^3$, $\mathbf{D}^4$ and then find the minimum of all matrices:

$$\mathbf{D}^{+} = \mathbf{D} \oplus' \mathbf{D}^2 \oplus' \mathbf{D}^3 \oplus' \mathbf{D}^4, \tag{16}$$

$$\mathbf{D}^{+} = \begin{pmatrix} \infty & 5 & 2 & 2 & 3 \\ 2 & \infty & 4 & 4 & 3 \\ 5 & 3 & \infty & 7 & 2 \\ 7 & 5 & 2 & \infty & 1 \\ \infty & \infty & \infty & \infty & \infty \end{pmatrix}.$$

Thus all elements are minimum of delays along all network passes from i to j.

1.2. Processes representation in terms of Max-plus algebra

Another way of applying max-plus algebra is to present processes in telecommunication devices, for example the process of So let t_{ik} – time service kth packet at ith network device transmitter; a_{ik} – incoming time of kth packet to ith transmitter queue; $x_i(k)$ – start time of kth packet transmission to network from ith transmitter. t_{ik}, a_{ik} – set of real random non-negative values. At start time there are no packets at the transmitters. Thus the process of coming and servicing packets can be presented as follows:

$$x_i(k) = t_{ik} \otimes (a_{ik} \oplus x_i(k-1)) = t_{ik} \otimes a_{ik} \oplus t_{ik} \otimes x_i(k-1). \tag{17}$$

Same concept can be laid into process of burst gathering and serving in optical networks. Let t_{ik} – reaction time of electronic switch communication matrix, $x_i(k)$ – time of burst coming to the carrier wave from ith interface. g_{ik} – kth burst gathering time, a_{ik} – container incoming time to the interface. Then dynamic of the process can be described as follows:

$$x_i(k) = t_{ik} + g_{ik} + \max[a_{ik}, x_i(k-1)]. \tag{18}$$

And in Max-plus algebra basi:

$$x_i(k) = t_{ik} \otimes g_{ik} \otimes (a_{ik} \oplus x_i(k-1)) = t_{ik} \otimes g_{ik} \otimes a_{ik} \oplus t_{ik} \otimes g_{ik} \otimes x_i(k-1). \tag{19}$$

The formula (19) is good for the 1 switch interface but for the n interfaces it transforms into follows:

$$x_i(k) = t_{ik} \otimes g_{ik} \otimes \left(a_k \left[\overset{n}{\underset{i=1}{\oplus}} x_i^{-1}(k - r_i) \right]^{-1} \right), \tag{20}$$

r_i – state index. Thus this model lets to research optical switch working process according to burst delays getting into it.

2. Discrete-event systems, Time event graphs and Max-plus algebra

Petri nets are often used to represent phenomena like synchronization, parallelism and concurrency. Generally speaking, the more their structure and semantics are elaborated the more complex is their analysis. The relatively simple class of Petri nets called Timed Event Graphs (TEGs) is likely to be the most investigated one. Indeed, TEGs are easily represented in the form of linear equations in (Max,+) algebra, provided that the places and the transitions be overtaking free. This linear (Max,+) form being very similar to the state representation of the classical discrete linear systems.

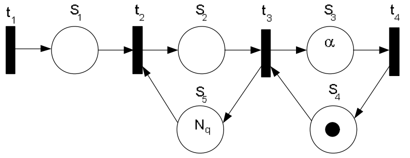


Fig. 2. TEG with autonomous transition

Next figure represents TEG for the network device working process.

On figure, t_1 – packet admission; t_2 – packet gets into buffer; N_q – buffer size; t_3 – packet is served with specified rule of servicing a ; t_4 – getting packet to the network. To get dynamic equation makes next designations. The autonomous transition burning of what does not depend of Petry net marking, designed as t_1. Time moments of burning t_1 are made incoming set $\{u(k)\}_{k\geq 1}$. t_4 transmission burning is output and according time moment of kth burning designs as $y(k)$, $k = 1,2,\ldots$. Let $x_1(k), x_2(k)$ being time moments of k th burning t_2 and t_3 transmission. Then:

$$\begin{aligned} x_1(k+1) &= x_2(k-N_q+1) \oplus u(k+1), \\ x_2(k+1) &= x_1(k+1) \oplus a \cdot x_2(k), \\ y(k) &= a \cdot x_2(k), \end{aligned} \tag{21}$$

or in vector-matrix sign

$$\mathbf{x}(k+1) = \underbrace{\begin{pmatrix} e & e \\ e & e \end{pmatrix}}_{=\mathbf{A}_0} \cdot \mathbf{x}(k+1) \oplus \underbrace{\begin{pmatrix} e & e \\ e & a \end{pmatrix}}_{=\mathbf{A}_1} \cdot \mathbf{x}(k) \oplus \ldots \oplus \underbrace{\begin{pmatrix} e & e \\ e & e \end{pmatrix}}_{=\mathbf{A}_{N_q}} \cdot \mathbf{x}(k-N_q+1) \oplus \underbrace{\begin{pmatrix} e \\ e \end{pmatrix}}_{=\mathbf{B}_0} \cdot u(k+1)$$

$$y(k) = \underbrace{\begin{pmatrix} e & a \end{pmatrix}}_{=\mathbf{C}_1} \cdot \mathbf{x}(k), \tag{22}$$

where $\mathbf{x}(k) = (x_1(k) \;\; x_2(k))^T$, and all matrices $\mathbf{A}_i = \mathbf{N},\; i = \overline{2, N_q - 1}$.

The first equation in (22) can be written next way:

$$\mathbf{x}(k+1) = \mathbf{A}_0^* \cdot (\mathbf{A}_1 \cdot x(k) \oplus \mathbf{A}_2 \cdot x(k-1) \oplus \ldots \oplus \mathbf{B}_0 \cdot u(k+1)), \tag{23}$$

$$\mathbf{x}(k+1) = \underbrace{\begin{pmatrix} e & e \\ e & a \end{pmatrix}}_{=\overline{\mathbf{A}}_1} \cdot \mathbf{x}(k) \oplus \ldots \oplus \underbrace{\begin{pmatrix} e & e \\ e & e \end{pmatrix}}_{=\overline{\mathbf{A}}_{N_q}} \cdot \mathbf{x}(k-N_q+1) \oplus \underbrace{\begin{pmatrix} e \\ e \end{pmatrix}}_{=\overline{\mathbf{B}}_0} \cdot u(k+1).$$

Lets: $\tilde{\mathbf{x}}(k) = (\mathbf{x}(k)^T \;\; \mathbf{x}(k-1)^T \;\; \ldots \;\; \mathbf{x}(k-N_q+1)^T)^T$ and $\tilde{u}(k) = u(k+1)$, then (23):

$$\begin{pmatrix} \mathbf{x}(k+1) \\ \mathbf{x}(k) \\ \mathbf{x}(k-1) \\ \ldots \\ \mathbf{x}(k-N_q+3) \\ \mathbf{x}(k-N_q+2) \end{pmatrix} = \underbrace{\begin{pmatrix} \overline{\mathbf{A}}_1 & \mathbf{N} & \mathbf{N} & \ldots & \mathbf{N} & \overline{\mathbf{A}}_{N_q} \\ \mathbf{E} & \mathbf{N} & \mathbf{N} & \ldots & \mathbf{N} & \mathbf{N} \\ \mathbf{N} & \mathbf{E} & \mathbf{N} & \ldots & \mathbf{N} & \mathbf{N} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \mathbf{N} & \mathbf{N} & \mathbf{N} & \ldots & \mathbf{N} & \mathbf{N} \\ \mathbf{N} & \mathbf{N} & \mathbf{N} & \ldots & \mathbf{E} & \mathbf{N} \end{pmatrix}}_{=\mathbf{A}} \cdot \begin{pmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \\ \mathbf{x}(k-2) \\ \ldots \\ \mathbf{x}(k-N_q+2) \\ \mathbf{x}(k-N_q+1) \end{pmatrix} \oplus \underbrace{\begin{pmatrix} \overline{\mathbf{B}}_0 \\ \mathbf{N} \\ \mathbf{N} \\ \ldots \\ \mathbf{N} \\ \mathbf{N} \end{pmatrix}}_{=\mathbf{B}} \cdot \tilde{u}(k), \tag{24}$$

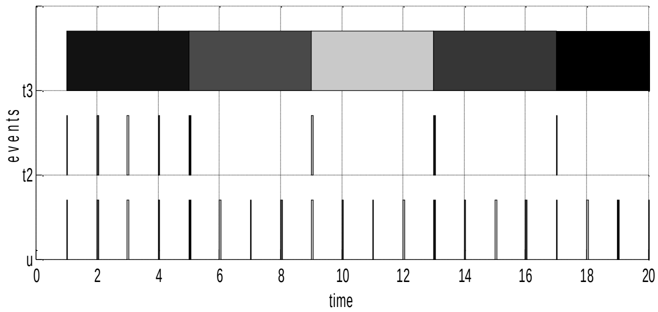
$$y(k) = \underbrace{\begin{pmatrix} \mathbf{C}_1 & \mathbf{N} & \ldots & \mathbf{N} \end{pmatrix}}_{=\mathbf{C}} \cdot \left(\mathbf{x}(k)^T \;\; \mathbf{x}(k-1)^T \;\; \ldots \;\; \mathbf{x}(k-N_q+1)^T\right)^T,$$

this let possibility to rewrite equation (22) in space-state equations.

$$\tilde{\mathbf{x}}(k+1) = \mathbf{A} \cdot \tilde{\mathbf{x}}(k) \oplus \mathbf{B} \cdot u(k),$$
$$y(k) = \mathbf{C} \cdot \tilde{\mathbf{x}}(k).$$

For this mathematical model was made Gantt diagram (fig. 3) to show if the model works right.

Fig. 3. Gantt diagram of service process



From the figure noticeable time moments of packets come u, and time moments of burning transmissions t_2 and t_3

Conclusion

In this paper, there was showed that many characteristics of the max-plus algebraic structure are similar to those in more familiar mathematical structures. It can be used matrix operations, solve systems of max-plus equations. Through applications, was showed that max-plus and min-plus algebras provide interesting tools that can be used to formulate and solve many problems of optimization.

In this paper there were showed that max-plus models can accept more features of the original telecommunication process then existing mathematical models. One more advantage of max-plus algebra is its linearity. This algebra is provided another possibilities in telecommunication processes modeling then existing queue theory and routing algorithms models.

1. Лозинская В. Н. Способ описания процессов в телекоммуникационных сетях с использованием аппарата Max-plus алгебра / В. Н. Лозинская // Збірник наукових праць Донецького інституту залізничного транспорту. – Донецьк, 2013. – Вип. 33. – С. 92–96. 2. Бессараб В. І. Аналіз частотних характеристик дискретно-безперервних систем при застосуванні критичного графа динаміки / В. І. Бессараб, А. О. Воропаєва, В. М. Лозинська // Наукові праці Донецького національного технічного університету: зб. наук. праць. – Донецьк: ДонНТУ, 2011. – Вип. 20(182). – С 96 – 101. 3. Baccelli Francois, Synchronization and Linearity An Algebra for Discrete Event Systems [Електронний ресурс] / Francois Baccelli, Guy Cohen, Geert Jan Olsder Jean-Pierre Quadrat. – Англия, 1992. – Режим доступу: https://www.rocq.inria.fr/metalau/cohen/SED/SED1-book.html 4. Кривулин Н. К. Методы идемпотентной алгебры в задачах моделирования и анализа сложных систем / Н. К. Кривулин. – СПб.: Изд-во С.-Петерб. ун-та, 2009. – 256 с. 5. Andersen H. Maria, Max_plus_algebras_thesis. – University of Copenhagen Keynote, 2002. – Режим доступу: http://www.busynessgirl.com/files/pdf/max_plus_algebras_thesis_2002.pdf. 6. Lozinskaya V. N. (2013), Mathematical models of telecommunication network components // Naukovi pratsi Donetskogo natsionalnogo tehnichnogo univercitety. Ser.:Obchysljuvalna technika ta avtomatyzatsija, vol.1, no. 24. P. 121–126. 7. Питерсон Дж. Теория сетей Петри и моделирование систем / Дж. Питерсон: пер. с англ. – М.: Мир, 1984. – 264 с.: ил.