

## THE MAIN DESIGN CONCEPTIONS OF CONCRETE STRUCTURES IN THE ADM MODEL CODE

© Mitrofanov V.P., Pinchuk N.M., Mitrofanov P.B., 2013

The System Analysis of the Eurocode 2 and project *fib* Model Code 2010 shows their radical disadvantage: the absence or incompleteness of links between partial and general design that in accordance with system approach means the insufficiently high level of those designs development, caused by the absence of the enough General Theory of Reinforced Concrete (RC). It is stated the essence of the General Theory of RC taken as a principle of the more perfect ADM model Code for projecting of RC Structures.

**Key words:** reinforced concrete structures, designs, disadvantages, links, General Theory.

Системний аналіз Eurocode 2 та проекту *fib* Model Code 2010 показує їх основний недолік: відсутність або неповноту зв'язків між окремими та загальними розрахунками, що за системного підходу означає недостатньо високий рівень розвитку цих розрахунків, обумовлений відсутністю загальної теорії залізобетону. Викладається сутність загальної теорії залізобетону, покладеної в основу досконалішої моделі ADM норм проектування залізобетонних конструкцій.

**Ключові слова:** залізобетонні конструкції, розрахунки, недоліки, зв'язки, загальна теорія.

### Problem setting

The increasing backwardness of Eurocode 2 [1] from the contemporary level of the RC knowledge and appearance of the improved *fib* Model Code 2010 [2] sharpen the necessity of impartial estimation of quality level of these documents and alternative offers. The present paper is limited by the analysis of the mentioned codes only part which concerns the designs of RC Structures (RCS) and RC elements (RCE). More than that it is considered here only the sides of RCE and RCS designs connected with their stress-strain states (SSS), deforming and destruction processes, these processes modelling, designs algorithms and their realization. The rest designs sides connected with securing of the RCS reliability under different actions are remained on the background.

According to the System Analysis [3], any system development level is evaluated as its systematicity level, that is as presence and quality of the system signs: elements composition, links between elements, organization and structure, integral qualities. The system signs must express the purpose demands to a system: 1) integral qualities – the satisfactory ratio of reliability and economy of projected structures, their ecology features; 2) combination of enough completeness (exactness) and simplicity of RCE and RCS design models; 3) explanatory ability of models which lightens the understanding by the designers and students the physical reality of designs and simplifies the mastering by them these designs; 4) enough simplicity of the RCE and RCS models design methods, guaranteeing the accessibility of these methods realization for the wide society of designers and students, mastered the usual university course of calculating mathematics and programming; 5) forcing out (if it is possible) of empirical designs and substitution of the ones by the designs based on the enough General Theory of RC, leading to the fulfillment of the mentioned above demands 1) ... 4).

One can see the code designs are man-made socio-technical system in which people play the decisive role: the researchers and specialists as code workers out, on the one hand, and the designers and students as code

consumers, on the other hand. In such systems consequent on people participation the most important links do not belong to the nature, as the ones are determined by the level of reached knowledge limited both objective (phenomena complication) and subjective (insufficient information) causes.

Considering the totality of code designs as system and determining its system properties, it is lightly revealed the elements composition – various designs on limit states. But determination of the second system property – links between elements (designs) – at once shows the absence or incompleteness these links, especially between designs under partial and general SSS. For example, the normal sections strength design under action one moment  $M$  must be derived as partial case of more general case of SSS with action forces  $M$  and  $Q$ . Nevertheless in codes designs of all world countries the link between mentioned designs is absent because the inclined sections strength design on action of the shear force  $Q$  is based on the empirical relationship, from which the known strength design of normal sections does not follow with  $Q=0$ , although this design must be obtained if the RCE failure model under joint action of forces  $Q$  and  $M$  would be enough perfect.

The elements design cases in which the forces  $M$ ,  $Q$ ,  $N$  are combined in various way with the torsional moment  $T$  are still less connected. For code designs of elements under two- and three-axial SSS (as rule empirical) the absence of common conception basis is especially visible. These designs are different cases of shear, crushing, punching shear, pressing out of concrete and mortar, pulling out of reinforcement and others. That is why the code designs do not form the internal system integrity that in accordance with the system approach means insufficiently high level of those designs development. The source of the unsatisfactory RCS designs level is the empiricism dominance, that is absence of the enough General Theory of RC.

Although the experiment is one of important scientific methods for knowledge obtaining, the empirical knowledge level is on the principle the lowest level, which (as the sciences history shows) is substituted in development result by the higher knowledge level – enough General Scientific Theory. Besides, experimental data are not only «truth criterion» but the limitation of empirical knowledge consequent on limitation of ones by the conditions of experiment conducting. Therefore experiments do not always reveal all most important factors and insufficient completeness of empirical knowledge limits the solving possibilities of problems, the conditions of which go out behind the limits of experiment conditions. Herewith the reliable structures optimization is impossible consequent on incompleteness of taken into account factors. The physical content of experimentally studied phenomena may be kept non-revealed, so the explanatory ability of empirical designs is decreased and the mastering of ones by the designers and students is made difficult. Therefore the empirical designs need forcing out from the codes if the reached theory level allows it. Nevertheless when the enough General Theory is absent, the experiment may be occurred only way for obtaining of practically useful relationships.

Thus the codes of all countries include the problem situation caused by the RCS designs development problem. As the RCS designs connections is secured by the enough General Theory of RC (GTRC) the latter just is the problem-solving system which must generalize, classify and explain the deformation and destruction processes of various RCE on the basis of certain general statements and models.

### **Analysis of investigations and publications**

The certain continuum model is always put into basis of the GTRC. Then it is necessary to base the concrete constitutive model for the triaxial stress-strain relationships that are highly complicated and for GTRC the ones are remained the main difficulty, which is not still overcome completely at present in spite of such huge number of suggestions that literature on this question is difficulty reviewed. At present it is known the different types of concrete constitutive models (relationships, theories), branching on the certain signs. There are deformation and incremental theories, isotropic and anisotropic models, elastic – plastic models with perfect plasticity or with strengthening [4, 5]. The last models got more wide application to concrete and RC and, especially, consequent on its simplicity, the model of rigid-perfect-plastic body connected with the theory of limit balance [6]. This model leads to satisfied proximity of theoretic strength

to experimental one not only for bar, slab and shell structures [7 – 9] but for massive elements under two- and three- axial SSS [10].

Besides the concrete constitutive model the important part of GTRS must be statements that generalize the experimental data of concrete and RC properties to the level of generalized statements and models permitting to reflex the various character of the SSS stages of RCS. Namely generalized statements and models form the united conception basis of the GTRS, needed for working out the different designs on various limit states under different RCE SSS.

**Paper purpose:** to state the essence of the offered GTRC version put into base of the ADM Model Code for RCS projecting that is worked out by the Center for Advanced Design Methods of Concrete Structures (Poltava, Ukraine, E-mail: vpm.admcs@mail.ru)

### Paper content

The present paper is limited by account of the «first necessity» designs (the GTRS core), which includes the designs on strength, crack-resistance and stiffness of RCS under short-time static loading. On the basis of mentioned designs it develops the taking into account of long-time processes, action of different dynamic loads, heat-moisture influences and others. The GTRC core includes: 1) generalized properties of concrete, reinforcement and RC; 2) generalized models of RCE and RCS; 3) problems solving methods on strength, crack-resistance and rigidity of the RCE and RCS models.

### Generalized concrete properties

**Property C1:** depending on sign and value of the mean (hydrostatic) stress (pressure)  $\sigma$  it can be different cases of the concrete deforming and failure (see [11], fig. 2):

**brittle** on mainly tensile  $\sigma$  with failure by the rupture macrocrack;

**pseudoplastic** on the mean compressive  $\sigma$  with failure by the surface having on the macrolevel the externally shear character, but on the microlevel the one combines microruptures and microshears; this deforming is accompanied by the concrete volume increasing (dilatation) consequent on development of the rupture microcracks system mutually combined with the shear microcracks system [12];

**perfect plastic** on the high compressive  $\sigma$ , when the rupture microcracks development is suppressed, the dilatation does not take place and the concrete and rocks plastic limit state is reached at the expense of microshears and dislocation movement [13].

The pseudoplastic and brittle cases of concrete behavior have the most practical importance.

**Property C2:** the concrete physical non-linearity and the necessity to take into account the descending branch of the relationship «stress  $\sigma_{ij}$  – strain  $\varepsilon_{ij}$ » under one-, two- and three-axial SSS. With increasing of compressive  $\sigma$  the maximum stress  $\sigma_{ij}$  enhances also, the descending branch becomes more gently sloping and turns out parallel to strain axis when the perfect plasticity comes.

**Property C3:** Extreme strength criterion (ESC) of concrete and RC elements under non-uniform SSS for the pseudoplastic behavior case. The ESC means the failure state coming in a point when the strict maximum of curve «load parameter F – characteristic element strain  $U_{ch}$ » is reached

$$F_u = F(U_{ch}) \Big|_{U_{ch}=U_{ch,u}} = \max,$$

where  $F_u$ ,  $U_{ch,u}$  – ultimate load parameter and corresponding characteristic element strain. The ESC is necessary consequent on the specific stress redistribution in concrete with non-uniform SSS what is confirmed by the tests of various RCE [14, 15].

### Reinforcement properties

The GTRC core worked out is limited by the usual steel reinforcement for which unlike recommendations [1] it is used more exact approximations of the tension diagrams  $\sigma_s - \varepsilon_s$  by the three-link piece – continuous function [14].

## Generalized properties of RC

**Property RC1:** the bond between reinforcement and concrete that promotes to their joint deformation.

**Property RC2:** dependence of the RCE SSS character and failure type from the work reinforcement quantity. This property is well known for partial SSS cases. For example, the bending RCE are divided on under-reinforced, balance-reinforced and over-reinforced. For eccentrically compressed RCE the cases large and small eccentricities are distinguished although this division depends on considerably still reinforcement quantity.

The Property RC2 was generalized on the case of joint forces M, Q, N action for which the RCE classification was ground depending on quantity both longitudinal  $A_s$  and lateral  $A_{sw}$  reinforcement [16] (see also [17], fig. 1, [18], fig. 2). The RCE groups of this classification are differed by the combination of complete, balance and incomplete use longitudinal  $A_s$  and lateral  $A_{sw}$  reinforcement resistance and corresponding singularity of SSS and failure type, described in detail in [16 – 18]. The RCE classification [16] revealed the conditions needed for plastic failure on the inclined dangerous crack under shear force Q action and the one showed the considerable influence of the shear force on the normal sections strength [19]. The RCE classification [16] together with other design statements led to the «Optimization Strength Theory of RCE (OSTRC) on the inclined sections under joint forces M, Q, N action» [17, 18], which passed the many-sided verification and showed its practical importance [19 – 21]. This theory is part of the GTRC core offered here.

The Property RC2 is also clear-cut under the torsion action in RCE [22]. Thus the dependence of SSS character and failure type from reinforcement quantity is also spread on the RCE under general case action of M, Q, N, T forces, for which the classification of SSS partial cases and failure types with various combination of M, Q, N, T forces are ground and the corresponding designs schemes are obtained, but consideration of them is impossible in this paper limits.

**Property RC3**, which allows to make more precise the notion «element of the bar RCS» as part on the length of which the forces M, Q, N, T are sign-constant and both one dangerous inclined and one dangerous normal crack are developed. The made more precise RCE notion lead to its unified design scheme including at one dangerous inclined and normal crack (see [17], fig. 2, [18], fig. 3). Then strength design of the bar RCS is reduced to design of its elements with their peculiar SSS and corresponding needed reinforcement  $A_s$  and  $A_{sw}$  on the strict determined length. The statically indeterminable bar RCS are previously designed by the method using the ESC (see above the Property C3) and securing the possible most complete redistribution of the forces (see below).

## Models of RCE and RCS

The Property C1 leads to the specific for RCE two cases of concrete deformation and destruction that determine two corresponding types of models – brittle and pseudoplastic – which are able to be basis for various designs of RCE and RCS.

The brittle model is connected with phenomena in which the concrete and reinforcement tension plays the key role and the one is accompanied by the rupture cracks development. In such phenomena the cracks spreading is most adequate described by the Brittle Fracture Mechanics (BFM) [23]. The BFM demands the knowledge of the Critical Coefficient of Stress Intensity  $K_{IC}$ , which may be adopted from experiments [24, 25] for usual concrete and from [24] for concrete with light coarse aggregate.

For pseudoplastic model it needs the non-linear physical relationships  $\sigma_{ij} - \varepsilon_{ij}$ , taking into account the descending branch, which with the hydrostatic pressure increasing transforms gradually into branch parallel to the strain axis. Such complicated relationships for two- and three-axial SSS remain still not reached at present even for the simplest deformation model. Therefore it is necessary to use the

approximate concrete physical models selecting from ones the combining enough simplicity with satisfactory accuracy. The known model of perfect plastic solid may be suitable for concrete and RC elements. This model is long ago applied in the limit balance theory [6] for the carrying capacity design with acceptable exactness of bar [7], slab [8] and shell [9] RCS. This model on base of the variational method [26] was spread on the massive two- and three-dimensional elements [10] and the one showed the satisfactory accuracy for various elements subjected to the shear [27, 28], crushing and punching shear [10, 29], compressed concrete confined by the steel tube [30] and others. This theory as «Elementary Mechanics of Concrete Pseudoplastic Ultimate State» [11] is part of GTRS core offered.

By present the concrete non-linear physical relationship with descending complete branch was more researched only under axial compression for which the great number of approximative functions  $\sigma_c - \varepsilon_c$  were suggested [31]. That is why the more improved in comparison with [1] «Deformation Model with the ESC» was worked out for strength design of the normal sections of bending and eccentrically compressed-tensile RCE [14, 15]. This model is component of the GTRC and ADM Model Code offered here. The one may be used for the statically indeterminable bar RCS design with taking into account the actual permissible degree of the forces redistribution.

Thus in the GTRC offered the two exactness levels of pseudoplastic model are used: 1) complete pseudoplastic model taking into account the descending  $\sigma_c - \varepsilon_c$  branch and ESC, which is effectively realized in strength designs of the RCE normal sections with concrete under axial compression; 2) the approximate pseudoplastic model as model of perfect plastic body used with two- and three-axial SSS of RCE.

Besides it is often needed model as the *combination of brittle and pseudoplastic models*. The necessity of above model is arisen if the RCE has the compressed and tensile with rupture cracks zones.

The table 1 shows the applicability realms of the above considered models in design on the limit states.

### **Methods of RCE design problems solution**

The realization of above models is connected with use the certain methods of practical problems solution which must be accessible for application by the wide society of designers and students. But Finite Element Method (FEM) does not is such because it demands the complicated, laborious and expensive computer software which are used by the comparatively narrow circle of professionals. Nevertheless there are methods acceptable by their simplicity and accuracy. So for problems connected with the brittle model, the *method of sections* [32] is quite suitable for RCE designs.

The pseudoplastic model design method depends on the its accuracy level used (see above). The strength problems of RCE on the normal sections, formulated on base of complete pseudoplastic model, are reduced to the optimization problems of non-linear mathematic programming and the ones are solved with help optimization programs, for example, contained in the table processor MS Excel.

As in the RCE strength problems under two- and three-axial SSS the pseudoplastic model is substituted by the approximate model of perfect plastic body, one may use the design methods from the known arsenal of plasticity theory. So, the boundary problems solutions with the interrupted velocity functions are especially attracted by their simplicity. Herewith the plastic strains are considered as localized on the surface  $S_l$  of the strong interruption velocity field, the values of which are different on the  $S_l$  both sides. Consequence on concrete dilatation the velocity interruption is possible as in tangential as in normal to  $S_l$  directions. The velocity interruption surfaces do not must considered as the mathematic abstractions because in tests it is observed the highly close to ones the shear failure surfaces. Moreover in work [33] it was shown the ability of the pseudoplastic materials, possessing by the internal friction and dilatation, to strains localization in the narrow layers of shear, widely observed in rocks and concretes [13, 33].

Conformity of RCE models types to the main designs groups

RCE model type	RCE DESIGNS GROUPS*					
	on serviceability limit states			on ultimate limit states		
				with failure		with loss stability
	on tensile zone	on compressed zone				
Brittle model	① on formation of normal and inclined cracks	–	–	④ minimum longitudinal and lateral reinforcement	–	–
Pseudoplastic (perfect plastic model)	–	–	–	–	⑤ ultimate load under one-, two- and three-axial SSS	–
Combination of brittle and pseudoplastic models	–	② on opening of normal and inclined cracks	③ on deformation (displacements)	–	–	⑥ critical load of compressed RCE and RCS

Note \*: designs on the equilibrium loss of structures as a rigid body (overturning, uplift, sliding) are omitted.

The plasticity theory solutions are especially simple when the ones are based on the direct variational calculus methods and corresponding variational principles. The variational strength design method of concrete and RC elements based on variational principle of virtual velocities and stresses [26] was worked out in [10]. There are design examples of various concrete and RCE in particular in [10, 27–30]. The variational method application is begun from selection of the shape, placing and character (shear, rupture) of the velocity interruption surface that is failure surface, dividing the considered element into parts at the destruction stage. Herewith it is revealed the important merit of the above method – its visuality, physical clearness of failure character and explanatory ability, making this method as compared to the FEM more accessible for mastering of the one by the designers and students. Variational method leads to the optimization problems which may successfully be solved with help of the table processor MS Excel programs.

The Table 2 unites the components of the offered GTRC core and shows their characteristic features.

Table 2

**Components of the GTRC offered and design groups  
in the ADM Model Code for RCS projecting**

points order	Components of GTRC and designs groups № in ADM Model Code, [references]	Characters of GTRC components and design groups			
		Dimension of considered elements	Elements SSS (forces)	Concrete constitutive model	Model design method
1.	⑤ Deformation model with Extreme Strength Criterion on normal sections of RCE, [14, 15]	bar elements, beam slabs	bending (M), eccentric compression-tension (M, N)	complete pseudoplastic model	non-linear mathematic programming
2.	⑤ Optimization Strength Theory of RCE on inclined sections (with torsion), [17, 18]	bar and slab elements	joint action of forces M, Q, N, T	perfect plastic body	non-linear mathematic programming
3.	⑤ Elementary mechanics of concrete pseudoplastic ultimate state, [10, 11, 35]	two- and three-dimensional elements	two- and three-axial SSS	perfect plastic body	direct methods of variational calculus on base of variational principles
4.	① Designs on formation of normal and inclined cracks, ④ Designs of minimum longitudinal and lateral reinforcement	bar and slab elements	bending (M, Q), eccentric compression-tension with torsion (M, Q, N, T)	brittle model	method of sections
5.	② Designs on opening of normal and inclined cracks ③ Designs on deformation (displacements)	bar and slab elements and structures	bending (M, Q), eccentric compression-tension with torsion (M, Q, N, T)	combination of brittle and pseudoplastic model	direct methods of variational calculus on base of variational principles
6.	⑥ Designs on critical load under stability loss	bar and slab elements and structures ○	axial (N) and eccentric compression-tension (M, N)	combination of brittle and pseudoplastic model	direct methods of variational calculus on base of variational principles

Note: The Numbers design groups № are coincide in the Tables 1 and 2.

## Conclusions

1. It is necessary to realize the radical demerit of designs by known codes for the RCS projecting: the insufficiently high development level, being revealed by the system analysis. This demerit essence is displayed as separateness of designs, absence or incompleteness of links between ones, that leads to the derivation impossibility of designs with partial SSS from designs (empirical usually) with more general SSS. The shown demerit reason is caused by the empiricism dominance, that is absence or insufficient development of the enough General Theory of RC, which ought to unite the separate designs into system of mutually linked designs what is possible on enough high development level of design models leading to the higher integral qualities of designs and codes on the whole.

The necessity of GTRC was realized in the former USSR by V.M. Bondarenko, V.N. Baikov and V.Y. Bachinski in 1978 – 1979, but its development was proceeded slowly because it not attach importance to the one.

2. It is suggested the GTRC core version, including the generalized properties of concrete, reinforcement and RC, the generalized RCE and RCS models and the models designs general methods.

3. The models and their design methods offered are in comparison with the FEM more accessible for use by the wide society of designers and students. The realms of preferable application, on the one hand, the suggested simple models and design methods and, on the second hand, the FEM, exact herewith:

for first – the multitude of comparatively simple problems often occurring in the practice for which the failure surface (scheme) is enough clear determined;

for second – the complex many-linked structures in which it is difficult to determine the shape and placing of the destruction field and surface.

4. The sudden failure of concrete and RC element during test does not mean the non-applicability of the perfect plasticity model for this element strength design. The great extent of conditional yielding plateau of concrete  $\sigma_{ij} - \varepsilon_{ij}$  curve does not is obligatory demand to the perfect plasticity model applicability, the conditions of which are spread on the concrete pseudoplastic behavior realm that are noted in the [11, 35].

5. The designers and students society ought to pass the development stage, connected with the more broad use in the RCS designs (including the massive RCE) the simplest perfect plasticity model in order to reach the mastering level of the Continuum Mechanics needed for passage to the use of worked out in future the Complete Pseudoplastic Model for three-axial SSS with its Extreme Strength Criterion.

6. The project «*fib* Model Code 2010» makes great progress in relation to the securing structures safety by means of more differential demands to the RCS safety level depending on the structural conditions for which the exactness level of safety control method [34] may be from 1 to 3. But the determining design models, needed for the probabilistic designs, are remained on the former enough low level close to the level of Eurocode 2.

7. The widened international collaboration for the working out of the more perfect RCS designs is worth-while.

1. *Eurocode 2: Design of concrete structures. Part 1: General rules and rules for buildings. Brussels: CEN, 2002. – 226 p.* 2. *fib Bulletins 55, 56. Model Code 2010. Final draft. vol. 1, vol. 2, 2012. – 350 p.* 3. *Перегудов Ф.И., Тарасенко Ф.П. Введение в системный анализ. – М.: Высш. шк., 1989. – 367 с.* 4. *Карпенко Н.И. Общие модели механики железобетона. М.: Стройиздат, 1996. – 416 с.* 5. *Nielsen M.P. Limit Analysis and Concrete Plasticity. 2-nd ed. – Boca Raton, Florida, USA: CPC Press, 1999. – 908 p.* 6. *Гвоздев А.А. Расчёт несущей способности конструкций по методу предельного равновесия. – М.: Госстройиздат, 1949. – 280 с.* 7. *Тихий М., Ракосник И. Расчёт железобетонных рамных конструкций в пластической стадии. Пер. с чешск. – М.: Стройиздат, 1976. – 198 с.* 8. *Ждахин Л.П. Расчёт железобетонных бункеров по предельным состояниям. – М.: Стройиздат, 1970. – 304 с.* 9. *Овечкин А.М. Расчёт железобетонных осесимметричных конструкций. – М. Госстройиздат, 1961. – 300с.* 10. *Митрофанов В.П. Вариационный метод в теории идеальной пластичности бетона // Строительная механика и расчёт сооружений. – 1990. – № 6. – С. 23–28.*



11. Митрофанов В.П. Теория идеальной пластичности как элементарная механика псевдопластического предельного состояния бетона: основы, ограничения, практические аспекты, совершенствование / Коммунальное хозяйство городов. Вып. 72, серия технические науки. – К.: Техника, 2006. – С. 6–26. 12. Зайцев Ю.В. Моделирование деформаций и прочности бетона методами механики разрушения. М.: Стройиздат, 1982. – 196 с. 13. Николаевский В.Н. Механика пористых и трещиноватых сред. – М.: Недра, 1984. – 232 с. 14. Митрофанов В.П. Практическое применение деформационной модели с экстремальным критерием прочности железобетонных элементов / Коммунальное хозяйство городов. Вып. 60, серия технические науки. – К.: Техника, 2004. – С. 29–48. 15. Mitrofanov V.P. Extreme strength criterion and design of RC Elements // Structural Concrete. Journal of the fib. – 2009. – 10. – № 4. – pp. 163–172. 16. Митрофанов В.П. Напряженно-деформированное состояние, прочность и трещинообразование железобетонных элементов при поперечном изгибе: Автореф. дис. ... канд. техн. наук. – М.: ВЗИСИ, 1982. – 42 с. 17. Mitrofanov V.P. Optimization strength theory of reinforced concrete bar elements and structures with practical aspects of its use // Byggningsstatiska Meddelelser. – Copenhagen: Danish Society for Structural Science and Engineering. vol. 71, № 4, Dec. 2000. – pp. 73–125. 18. Митрофанов В.П. Оптимизационная теория прочности железобетонных элементов по наклонным и нормальным сечениям при совместном действии изгибающих моментов, поперечных и продольных сил / Будівельні конструкції. Вип. 67. – К.: НДІБК, 2007. – С. 231–243. 19. Воскобойник П.П. Сложное напряженное состояние бетона зоны разрушения и его учет в расчете прочности нормальных сечений железобетонных элементов: Автореф. дис. ... канд. техн. наук. – Одесса: ОИСИ, 1985. – 22 с. 20. Котляров В.А. Прочность железобетонных элементов при совместном действии изгибающих моментов, продольных сжимающих и поперечных сил: Автореф. дис. ... канд. техн. наук. – Полтава: ПолтИСИ, 1992. – 20 с. 21. Микитенко С.М. Міцність при згині залізобетонних елементів з повним використанням опору поперечної і високоміцної поздовжньої арматури: Автореф. дис. ... канд. техн. наук. – Полтава: ПолтТУ, 1995. – 24 с. 22. Ernst G.C. Ultimate Torsional Properties of Rectangular Reinforced Concrete Beams // ACI Journal, v. 29, № 4, Oct. 1957. – pp. 341–356. 23. Черепанов Г.П. Механика хрупкого разрушения. – М.: Наука, 1974. – 640 с. 24. Митрофанов В.П., Жовнир А.С., Быкова Л.В. Трещиностойкость некоторых бетонов и условия автомодельности зоны предразрушения опытных образцов // Известия вузов. Строительство и архитектура. – 1982. – № 11. – С. 4–8. 25. Ягуст В.И. Сопротивление развитию трещин в бетонных конструкциях с учетом влияния макроструктуры материала: Автореф. дис. ... канд. техн. наук. – М.: НИИЖБ, 1982. – 24 с. 26. Колмогоров В.Л. Механика обработки металлов давлением. – М.: Металлургия, 1986. – 688 с. 27. Mitrofanov V.P. Investigation of Destruction Zone Resistance of HSC of Beams under Shear Forces Action. 5th Int. Symp. on Utilization of HS/HP Concrete, 20–24 June 1999, Sandefjord, Norway, proc. vol. 1. – pp. 461–468. 28. Mitrofanov V., Pogrebnoy V., Dovzhenko O. Strength of Concrete Elements Under Shear Action According to the Theory of Plasticity and Tests. Proc. of the 2nd fib Congress, June 5–8, 2006, Naples, Italy. – paper ID3–61. 29. Довженко О.О. Міцність бетонних та залізобетонних елементів при місцевому прикладанні стискаючого навантаження: Автореф. дис. ... канд. техн. наук. – Полтава: ПолтИСИ, 1993. – 20 с. 30. Митрофанов В.П., Дергам Али Н. Пособие по расчёту прочности трубобетонных элементов при осевом сжатии. – Полтава: ПолтНТУ им. Ю. Кондратюка, 2008. – 91 с. 31. Тур В.В., Рак Н.А. Прочность и деформации бетона в расчетах конструкций. – Брест: Изд-во БГТУ, 2003. – 252 с. 32. Морозов Е.М. Метод сечений в теории трещин // Известия вузов. Строительство и архитектура. – 1969. – № 12. – С. 5–9. 33. Райс Дж. Механика очага землетрясения. Пер. с англ. – М.: Мир, 1982. – 217 с. 34. Аугусту Г., Баратта А., Кашиати Ф. Вероятностные методы в строительном проектировании. Пер. с англ. – М.: Стройиздат, 1988. – 584 с. 35. Mitrofanov V. The theory of Perfect Plasticity as the Elementary Mechanics of a Pseudo-plastic Ultimate State of Concrete: Bases, Limitations, Practical Aspects, Improving. Proc. of the 2<sup>nd</sup> fib Congress, June 5–8, 2006, Naples, Italy. – paper ID7–6.