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PHYSICAL PRINCIPLES OF SMALL SPACE SETTLEMENTS ROADSIDE AREA POLLUTION FORECASTING

The methods of small space settlements roadside area pollution forecasting based on the results of theoretical and experimental researches concerning regular occurrence of contaminating matters distribution are considered.

Key words: methods of prognosis, motor transport, small settlements, differential equalization.

Розглянуто методи прогнозу забруднення придорожнього простору малих населених пунктів, що базуються на результатах теоретичних та експериментальних дослідженнях закономірностей розповсюдження забруднюючих речовин.

Ключові слова: методи прогнозу, автомобільний транспорт, малі населені пункти, диференційне рівняння.

Relevance of the problem

The development of methods of small space settlements roadside area pollution forecasting is based on the results of theoretical and experimental studies of the laws of pollutants distribution from sources, in particular from road transport. Such studies are carried out mainly in two directions. One of them is the development of the theory of atmospheric diffusion, based on the mathematical description of the propagation of impurities by solving the equations of turbulent diffusion. Another one is mainly associated with empirical-statistical analysis of the spread of pollutants in the atmosphere and using for this purpose the interpolation model of Gaussian type.

The first direction is more versatile because it allows exploring the spread of contaminants from different sources with different characteristics of the environment, taking into account the building. It makes it possible to use the parameters of the turbulent exchange used in meteorological problems of heat – and moisture exchange in the atmosphere.

Problem solving

In general, the problem of roadside space pollution forecasting can be mathematically defined as the solution for certain initial and boundary conditions of the differential equation [1]

$$u\frac{\partial q}{\partial x} - w\frac{\partial q}{\partial z} = \frac{\partial}{\partial z}k_z\frac{\partial q}{\partial z} + \frac{\partial}{\partial x}k_x\frac{\partial q}{\partial x} - \alpha q \qquad (1)$$

where t – time; x_i – coordinates; u_i and k_i – constituents of the average speed of impurities replacement and the exchange coefficient relating to the axial direction x_i (i = 1, 2, 3); α – the coefficient determining the change in concentration due to impurity conversion; q - the concentration of polluting substances, mg / m^3.

Equation (1) describes the spatial distribution of average concentrations, as well as their changes with time. In this connection it can be considered as a predictive equation.

Typically, in a Cartesian coordinate system axes x_1, x_2, arranged in a horizontal plane, are denoted by x and y, and the vertical axis x_3 – through z, respectively, $u_1 = u$, $u_2 = v$, $u_3 = w$, $k_1 = k_x$ $k_2 = k_y$, $k_3 = k_z$.

In general, the exchange coefficient in a turbulent flow is presented by a second order tensor. Equation (1) is written on the assumption that the axes of coordinates coincide with the principal axes of the tensor, at the same time its diagonal components disappear and only the diagonal components are distinct from zero.

In solving practical problems the form of equation (1) can be simplified. For example, if the *x*-axis is oriented in the direction of the mean wind speed, then $v = 0$. The vertical motion in the atmosphere above the horizontal homogeneous underlying surface are small and can almost be taken as $w=0$ in the case of light impurity without its own movement speed. If we consider a heavy admixture, that gradually settles, then *w* is the deposition rate (which is part of the equation with a negative sign). In the presence of wind the term k_x can be neglected which takes into account diffusion along axis *x*, as the impurity diffusion flow in this direction is much less than the convection one.

In case of prognostic problems solving in principle the retention in (1) of the non-stationary member $\frac{\partial q}{\partial t}$ is essential. However, during periods of time comparable to transfer time of impurities x/u from the source to the point under consideration, the diffusion process gets stationary [1].

Changes in the concentrations in the atmosphere over time are usually quasi-stationary in nature, and member $\frac{\partial q}{\partial t}$ can often be practically excluded, putting it equal to zero, and only assume that the coefficients of equation (1) are known functions of *t* time. The inclusion of this term, as it will be shown below, is essential only in certain cases, particularly in determination of private concentrations of impurities caused by the road transport in condition of gentle breeze and low intensity of the turbulent exchange.

Thus, the original predictive equation (1) reduces to the commonly used atmospheric diffusion equation:

$$u\frac{\partial q}{\partial x} - w\frac{\partial q}{\partial z} = \frac{\partial}{\partial z}k_z\frac{\partial q}{\partial z} + \frac{\partial}{\partial y}k_y\frac{\partial q}{\partial y} - \alpha q \ . \qquad (2)$$

In case of light impurity ($w = 0$) the second term in (2) vanishes, and at consideration of retaining impurities $(\alpha = 0)$ excludedthe last term on the right-hand side of the equation is excluded.

At presence of vertical currents in the atmosphere in the term $w\frac{\partial q}{\partial z}$ the value *w* also includes the vertical component of the air velocity. In a hilly terrain, when the direction is not horizontal and depends on distance x, it is also necessary to consider the member $\frac{\partial k}{\partial x}\ \frac{\partial q}{\partial x}$.

At presence of a point source with coordinates $x = 0, y = 0, z = H$ [2] is taken as the boundary condition.

$$uq = M\delta(y)\delta(z - H) \text{ when } x = 0 \qquad (3)$$

where *M* – release of substances from the source per unit of time, $\delta(\xi)$ - the delta function.

At prognostic predictions (taking into account quasi-stationarity of the process) *M* is generally considered as a function of time t.

The boundary conditions at an infinite distance from the source are accepted in accordance with the natural assumption that the concentration decreases to zero

$$q \to 0 \text{ при } |y| \to \infty, \qquad (4)$$

$$q \to 0 \text{ при } |z| \to \infty. \qquad (5)$$

At formulation of the boundary condition on the underlying surface they distinguish cases when the impurities are distributed over the water surface. Much of the water absorbs impurities so the concentration of pollutants directly at the water surface is equal to zero, i.e.

$$q = 0 \text{ for } z = 0 \qquad (6)$$

On the surface of the soil the contaminants are usually weakly interacting. Once on its surface, the impurities are not accumulated, but are carried away with turbulent vortexes into the atmosphere. Therefore, with sufficient accuracy it is assumed that the average impurity turbulent flow at the surface is small, i.e.

$$R_z \frac{\partial q}{\partial z} = 0 \text{ when } z = 0. \qquad (7)$$

Other boundary conditions will be specified at consideration of particular problems concerning the study of roadside area pollution.

Scientific novelty

Under the influence of the terrain, the character of movement and the air flow turbulent condition, causing a significant change in distribution of concentration from pollution sources (road transport) changes.

Description of turbulent diffusion of impurities in a hilly area is also performed with the help of equation (2), but written for the area with a curved boundary. Besides, the coefficients of this equation are complex functions of the coordinates. The problem is simplified by assuming that in the direction of the axis y (perpendicular to the wind direction), the underlying surface is uniform. This allows to use the ratio between R_y, R_z , and *u* and proceed to equation (2) for concentration from a line source. Further, it is convenient to introduce a change of variables,

$$z' = z - h(x), x' = x, \qquad (8)$$

where $z = h(x)$ describes the boundary of the underlying surface.

Then (2) for $\alpha = 0$ takes the form:

$$u\frac{\partial q'}{\partial x'} + \left(w - u\frac{\partial h}{\partial x'}\right)\frac{\partial q'}{\partial z'} = \frac{\partial}{\partial z'} R_z \frac{\partial q'}{\partial z} \ . \qquad (9)$$

The components of velocity *u* and *w* are related by the continuity equation.

$$\frac{\partial u}{\partial x'} + \frac{\partial \omega}{\partial z'} = 0 \ . \qquad (10)$$

In case of a slightly sloping terrain, when the angles of slopes inclination are small, the air flow almost completely wraps around the uneven terrain. In this case, *u* and R_z are only the functions of the height above the underlying surface: $u = u[z - h(x)]$, $R_z = R_z[z - h(x)]$. From (9) we get that $w = u\frac{\partial h(x)}{dx}$. Then (9) reduces to the diffusion equation for a gently sloping terrain in the absence of vertical velocities. This means that the flat topography does not have a significant impact on the spread of contaminants. This conclusion, made for the first time in [2], has a significant practical importance. It follows also that the change in the concentration field arises in those cases when the values *u* and R_z depend not only on $z - h(x)$, but also on *x*.

To determine the change of values of the maximum concentration ratio from sources with height *H*, and located over a hill (q_m) and over a flat surface $(\lambda' < 0)$ when the source is moving (transport stream) it is necessary to calculate the relative ratio of ingredient environmental pollution. The largest concentrations are attained at location of the source on the leeward slope it is especially noticeable when the height of the source *(H)* is small compared with the height of the hill (h_0). With increasing of *H* the influence of the hill on maximum concentration q_m decreases, and in case when $H/h_0 > 0.5$ it is relatively small. This makes it easier to find the solution of the problem by using an approximate method for potential flows, and assess the limits of its application [3].

The essence of the given method developed in the above stated scientific works concerning assessing the effect of topography on the distribution of impurities consists in introduction of an analytic function $\tau(t) = \varphi(x,z) + i\psi(x,z)$ derived from the complex argument $t = x + iz$ by which the conformal mapping of the study area of the flow with curvilinear grace on the half-plane is carried out. Functions φ and ψ represent the velocity potential and the current function.

The horizontal and vertical components of the potential flow velocity in the area under consideration are expressed by the following formulae:

$$u_n = \frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial z} \quad \omega_n = \frac{\partial\varphi}{\partial z} = \frac{\partial\psi}{\partial x}. \tag{11}$$

The curves $\psi(x,z) = \text{const}$ represent a streamline of the flow under study. In particular, the line $\psi(x,z) = 0$ is the boundary of the area. The inverse function $t(\tau) = x(\varphi,\psi) + iz(\varphi,\psi)$ performs a conformal mapping of the half-plane on the physical flow area. In the diffusion equation there is implemented a transition from the variables *x* and *z* to the “streaming" coordinates φ,ψ. In the resulting equation, we can neglect the terms describing the diffusive transfer along the stream, as it is small compared to the convective one. Then the turbulent diffusion equation (2) for concentration from the line source becomes

$$\frac{\partial q'}{\partial\varphi} = \frac{\partial k_z}{\partial\psi}\,\frac{\partial q'}{\partial\psi}, \tag{12}$$

initial and boundary conditions: $q = M\delta(\psi - \psi_H)$ if $\varphi = \varphi_H$,

$$k_\psi \frac{\partial q'}{\partial\psi} = 0 \quad \text{when} \quad \psi = 0, \tag{13}$$

$$q' \to 0 \ \text{when} \ \psi \to \infty,$$

where φ_H, ψ_H - the flux coordinates of the source located at the point $x = x_0$, $z = H + h(x_0)$, and $\psi = 0$ - the line that the border $z = h(x)$ passes into at mapping.

The resulting equation and boundary conditions are identical in form to those used in the problem in atmospheric diffusion over a uniform horizontal surface, the coordinate ψ playing the role of the height. On this basis it is assumed that the exchange ratio, i.e. that it is the function of the streamline, and therefore a natural generalization of a commonly used model for $k_z(z)$ is carried out. This solution allows to write solutions (13) and (14) in the form of

$$q' = q'(\varphi_m - \varphi_H, \psi, \psi_H). \tag{14}$$

The point φ_m, where they achieve the maximum of ground concentration, is determined from conditions $\frac{\partial q'}{\partial\varphi} = 0$, when $\psi = 0$. Then (14) implies that $\varphi_m - \varphi_H = f(\psi_H)$. This means that for the sources located on the same streamline, the maximum land concentration is the same.

The above formulas for q_m and φ_m in flux coordinates are written as

$$q_m = 0.3\frac{MK}{u_1}\left(\frac{V_\infty}{\psi_H}\right)^{2.3}. \tag{15}$$

$$\varphi_m = 0.4\frac{u_1}{k_1}\left(\frac{\psi_H}{V_\infty}\right)^{1.2}$$

Values q_m and φ_m in (15) may be associated with the corresponding values q_m and x_m for flat terrain, if we introduce corrections for the latter.

$$x=\left(\frac{HV_\infty}{\psi_H}\right)^{2.3},\quad \eta=\frac{x\left(\varphi_n+\varphi_m\right)}{x_m}. \tag{16}$$

Furthermore, it is possible to determine the ratio between the height of the sources in conditions of relief under study and planar areas, assuming that the maximum concentration from the sources take the same values. The height of the source when $\varphi=\varphi_H$ is determined by the distance $z\left(\varphi_H,\psi_H\right)-z\left(\varphi_H,0\right)$ of the streamline $\psi=\psi_H$ to the level of the underlying surface. Consequently, the ratio between these heights determines the correction factor p_1 to the height of the source *H*:

$$p_1=\frac{1}{H}\left[z\left(\varphi_H,\psi_H\right)-z\left(\varphi_H,0\right)\right]. \tag{17}$$

For certain forms of relief, such as a hill, for functions $\tau(t)$ and $t(\tau)$ it is possible to select relatively simple analytical expressions adequately describing the boundaries of the flow area.

In case of the boundary of the underlying surface of arbitrary shape the conformal mapping is given by the integral of the following type:

$$t=\tau+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{l(s)ds}{s-\tau}, \tag{18}$$

and is performed numerically. At the same time, the form of the function $l\left(s\right)$ determines the equation of the boundary of the flow area taking into account the effect of the wind speed change on the height of the initial rise ΔH.

For the cases of non-uniform heat relief there are solved generalized diffusion equations of motion, heat flux, and the balance of turbulence taking into account the curved boundaries of the underlying surface. In cases where the terrain is homogeneous in the direction perpendicular to the wind, these equations in the system of "flow" coordinates ξ_1 and ξ_3 take the following form:

$$q_s=u_1\frac{\partial}{\partial\xi_1}+u_3\frac{\partial}{\partial\xi_3}-\frac{\partial}{\partial\xi_3}kM\frac{\partial}{\partial\xi_3}. \tag{19}$$

$$q_v=\left(1-u_1^2\right)\frac{\partial V}{\partial\xi_1}-\frac{V_\infty^3}{V^2}\frac{\partial}{\partial\xi_3}k^0\frac{\partial u_1}{\partial\xi_3}+\frac{\partial}{\partial\xi_1}\int_0^\infty\left(1-u_1^2\right)V\frac{\partial V}{\partial\xi_3}\partial\xi_3+\frac{gL_1\Delta\Theta}{\Theta V^2}\int_0^\infty\frac{\partial\vartheta}{\partial\xi_1}\partial\xi_3 \quad . \tag{20}$$

$$q_T=\frac{\partial}{\partial\xi_3}\left(MKk-\frac{V_\infty^2}{V^2}k^0\right)\frac{\partial\Theta^0}{\partial\xi_3}-u_3\frac{\partial\Theta^0}{\partial\xi_3}. \tag{21}$$

$$q_b=Mk\sqrt{c_1}\left\{\left(\frac{V}{V_\infty}\right)^2\left[\left(\frac{\partial u_1}{\partial\xi_3}\right)^2=\frac{u_1L_1}{V}\frac{\partial V}{\partial\xi_3}\frac{\partial u_1}{\partial\xi_3}\right]-\frac{gL_1\Delta\Theta}{\Theta VV_\infty}\times\left(\frac{\partial\Theta^0}{\partial\xi_3}+\frac{\partial\vartheta}{\partial\xi_3}\right)-\left(\frac{V_\infty b}{V_k}\right)^2\right\}. \tag{22}$$

where $q_S;q_V;q_T;q_b$ – corresponds the concentration of pollutants, calculated taking into account the terrain, the speed of traffic flow (V), temperature in the atmospheric surface layer (T), the basic operating characteristics of roads (b).

Here, $\xi_1=\varphi/L_1V_\infty,\ \xi_3=\varphi/L_1V_\infty$, and φ as well as Ψ – real and imaginary parts of the function, conformably mapping the flow region on the half-plane, i.e., the velocity potential and stream function, V – velocity magnitude of the potential flow, V_∞ - the value of pollutants transportation from the source of pollution (traffic flow), L_1 - the characteristic scale of the relief u_1 and u_3 - components of the wind speed of the axes ξ_1 and ξ_3, k - exchange rate, k^0 – its value in the incoming flow, ϑ – potential temperature deviation from its value Θ^0, normalized by $\Delta\Theta$, i.e. on the temperature differential on the underlying surface.

In view of (19) – (22) the relative ratio of ingredient pollution will be represented as

$$P_{\text{ингред}} = \frac{q_{\text{факт}}}{q_{\text{норм}}} = \left(\frac{q_S}{q_{SH}}\right) \cdot \left(\frac{q_V}{q_{VH}}\right) \times \left(\frac{q_T}{q_{TH}}\right) \cdot \left(\frac{q_b}{q_{bH}}\right). \quad (23)$$

$$P_{\text{ингред}} = \frac{q_S \cdot q_V \cdot q_T \cdot q_b}{q_H}.$$

Thus, using the relations (19) – (23), given the terrain, it is possible to resolve issues associated with the formation of pollution of the roadside space of small communities.

Findings

Particular attention, in view of the above, must be given to the formation of pollution of the roadside area of small communities. In particular, it requires a more detailed consideration of the question of analysis and evaluation of the environmental performance of highways as well as calculate the relative rate of ingredient pollution.

In establishing the boundary conditions they take into account the fact that in the incoming flow there develops an internal boundary layer over the obstacle. At a sufficiently large distance from the underlying surface its influence fades, and the speed of the internal boundary layer continuously increases to reach the free stream velocity. It is also assumed that the turbulence energy fades to zero, since the generation occurs under the influence of the underlying surface.

The values found for the horizontal and vertical components of wind speed, as well as the exchange ratio, were used to calculate the diffusion equation (2) in case if the highways run through different parts of the hilly terrain.

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