UDC 66.047

C. Bacotiu, F. Domnita, A. Hotupan, P. Kapalo
Technical University of Cluj-Napoca, Faculty of Building Services Engineering, Romania,
Technical University of Kosice, Institute of Architectural Engineering, Slovakia

VENTILATION DUCT SIZING TOOLS – TRADITION AND MODERNITY

Однією з найважливіших проблем при гідравлічному розрахунку систем опалення, вентиляції, водопостачання є розрахунок коефіцієнта тертя, який бере участь у рівнянні Дарсі–Вейсбаха. Коефіцієнт тертя є функцією числа Рейнольдса, відносної шорсткості і режиму течії. Крім графічного представлення в діаграмі Муді, ці змінні об'єднуються у відомому рівнянні Колебрука–Вайта, яке широко відоме серед інженерів і вчених. На жаль, це рівняння неоднозначне і повинне бути розв'язане за допомогою числових методів. Це є головним недоліком для інженера, який часто хоче швидкий результат, якщо це можливо, використовуючи просте, відоме рівняння. Отже, при традиційних гідравлічних розрахунках інженерам у довідниках запропоновано діаграму (номограму), в якій безпосередньо наведено перепад тиску на одиницю довжини (Па/м), тим самим приховуючи складність знаходження коефіцієнта тертя. Пізніше, коли персональні комп'ютери стали доступні, тактика змінилася: необхідно знайти простий розв'язок рівняння Колебрука–Вайта. Так, протягом останніх двох десятиліть багато авторів запропонували свої власні рівняння різної складності, роблячи вибір молодих інженерів ще важчим, ніж раніше. У цій статті зроблено спробу зробити огляд найчастіше використовуваних альтернатив рівняння Колебрука–Вайта, аналізуючи їх складність і математичну точність для різних чисел Рейнольдса і відносних шорсткостей. Крім того, деякі сучасні інструменти програмного забезпечення для вентиляційних каналів були досліджені.

Ключові слова: повітропровід, коефіцієнт тертя, рівняння Колебрука-Вайта, відносна шорсткість, число Рейнольдса.

One of the most important problems in the hydraulic design of various building services systems is the calculation of the friction factor involved in Darcy-Weisbach equation. Ventilation duct sizing is a good case study, showing how classic, old-school design tools collide with modern instruments of the digital era. The friction factor is a function of Reynolds number, relative roughness and flow regime. Apart from the graphical representation in Moody's chart, those variables are packed in the famous Colebrook-White equation, widely accepted by engineers and scientists. Unfortunately, this equation is an implicit one and must be solved using numerical methods. This is a major disadvantage for the average engineer, who often wants a quick result, if possible using a simple, explicit equation. Therefore, the traditional hydraulic design tool offered to engineers in handbooks was a chart (nomograph), giving directly the pressure drop per unit length (Pa/m), thus hiding the complexity of finding the friction factor. Later, when personal computers became available, the tactics have changed: Colebrook-White equation needed to be replaced by a simpler one. So, during the last two decades, many authors proposed their own explicit equations, more or less complicated, making the choice of young engineers even more difficult than before. The present paper tries to make an overview of the most used alternatives to Colebrook-White equation, analyzing their complexity and mathematical accuracy for different Reynolds numbers and relative roughnesses. Also, some modern software instruments for ventilation duct sizing were investigated.

Key words: duct sizing, friction factor, Colebrook-White, relative roughness, Reynolds.

Introduction. Linear friction losses generated by air flow in ventilation ducts are calculated by the Darcy-Weisbach equation:

$$\Delta p = l \cdot \frac{L}{D} \cdot \frac{r \cdot V^2}{2} \tag{1}$$

where Δp – linear friction losses, in terms of pressure, [Pa]; l – Darcy friction factor, dimensionless; D – hydraulic diameter of the duct, [m]; L – duct length, [m]; V – average air velocity across the duct section, [m/s]; r – air density, [kg/m^3].

The most complicated issue is the friction factor λ, which is a function of Reynolds number, relative roughness and flow regime:

$$l = f\left(\mathrm{Re}, k/D\right) \tag{2}$$

where k/D – relative roughness of the duct, dimensionless; Re – Reynolds number, dimensionless;

$$\mathrm{Re} = \frac{V \cdot D}{n} \tag{3}$$

where n – kinematic viscosity of air, [m^2/s].

The friction factor was historically first presented in the form of diagrams (Moody, Nikuradse). As those diagrams were not very convenient for quickly obtaining large amounts of λ values, a mathematical link between those variables was needed. The Colebrook-White equation was widely accepted by engineers and scientists as the most appropriate mathematical illustration of the hydraulic phenomenon:

$$\frac{1}{\sqrt{l}} = -2 \cdot \log_{10}\left(\frac{k/D}{3.7} + \frac{2.51}{\mathrm{Re}} \cdot \frac{1}{\sqrt{l}}\right) \tag{4}$$

It may be used for the entire domain of turbulent flows, covering the whole range of Reynolds numbers and relative roughnesses, though in some hydraulics books it is said to be most appropriate for the semi-rough (transitionally rough) turbulent flow regime. Unfortunately, this equation is an implicit one and must be solved using numerical methods. This is a major disadvantage for the average engineer, who often wants a quick result, if possible using a simple, explicit equation.

Traditionally, Romanian hydraulics literature recommended Altshul's equation as an explicit, simple way to compute the friction factor:

$$l = 0.11 \cdot \left(k/D + \frac{68}{\mathrm{Re}}\right)^{0.25} \tag{5}$$

Is it the only alternative? Definitely not.

During the last decades, many authors proposed their own explicit equations, more or less complicated, trying to obtain good approximations for Colebrook-White equation over the whole range of Reynolds numbers and relative roughnesses:

- Wood (1966) - for $4000<\mathrm{Re}<5\cdot10^7$ and $0.00001<k/D<0.04$

$$l = 0.53 \cdot (k/D) + 0.094 \cdot (k/D)^{0.225} + 88 \cdot (k/D)^{0.44} \cdot \mathrm{Re}^{-1.62 \cdot (k/D)^{0.134}} \tag{6}$$

- Swamee-Jain (1976) - for $5000<\mathrm{Re}<10^8$ and $0.000001<k/D<0.05$

$$l = \left[-2 \cdot \log_{10}\left(\frac{k/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^{-2} \tag{7}$$

- Chen (1979) - for $4000<\mathrm{Re}<4\cdot10^8$

$$l = \left\{-2 \cdot \log_{10}\left[\frac{k/D}{3.7065} - \frac{5.0452}{\mathrm{Re}} \cdot \log_{10}\left(\frac{(k/D)^{1.1098}}{2.8257} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right)\right]\right\}^{-2} \tag{8}$$

- Zigrang-Sylvester (1982) - for 4000<Re<10^8 and 0.00004<k/D<0.05

$$l=\left\{-2\cdot\log_{10}\left[\frac{k/D}{3.7}-\frac{5.02}{\mathrm{Re}}\cdot\log_{10}\left((k/D)-\frac{5.02}{\mathrm{Re}}\cdot\log_{10}\left(\frac{k/D}{3.7}+\frac{13}{\mathrm{Re}}\right)\right)\right]\right\}^{-2} \tag{9}$$

- Haaland (1983) - for 4000<Re<10^8 and 0.000001<k/D<0.05

$$l=\left\{-1.8\cdot\log_{10}\left[\left(\frac{k/D}{3.7}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right]\right\}^{-2} \tag{10}$$

- Manadilli (1997) - for 4000<Re<10^8 and 0<k/D<0.05

$$l=\left[-2\cdot\log_{10}\left(\frac{k/D}{3.7}+\frac{95}{\mathrm{Re}^{0.983}}-\frac{96.82}{\mathrm{Re}}\right)\right]^{-2} \tag{11}$$

- Fang (2011) - for 3000<Re<10^8 and 0<k/D<0.05

$$l=1.613\cdot\left[\ln\left(0.234\cdot(k/D)^{1.1007}-\frac{60.525}{\mathrm{Re}^{1.1105}}+\frac{56.291}{\mathrm{Re}^{1.0712}}\right)\right]^{-2} \tag{12}$$

- Papaevangelou, Evangelides, Tzimopoulos (2010) - for 4000<Re<10^8 and 0.000001<k/D<0.05

$$l=\frac{0.2479-0.0000947\cdot\left(7-\log_{10}\mathrm{Re}\right)^4}{\left[\log_{10}\left(\frac{k/D}{3.615}+\frac{7.366}{\mathrm{Re}^{0.9142}}\right)\right]^2} \tag{13}$$

Apart from these relatively simple equations, there are more complicated approaches (Serghides, 1984; Sonnad and Goudar, 2007), involving more than one equation, thus providing improved accuracy.
It may be very frustrating for a young engineer having such a wealth of formulas to choose just one best substitute to Colebrook-White equation, as simple as possible and very accurate.

What is the degree of accuracy for those equations ?

Which work best for a building services engineer, in his real-life duct sizing calculations?

Methods. In order to answer these questions, we need first to investigate Eq.(5) to Eq.(13) in comparison to Eq.(4), for a wide range of Reynolds numbers and relative roughnesses. Therefore, the relative error of all these approximate formulas with respect to Colebrook-White equation will be calculated as follows:

$$rel\_err=d=\frac{l_{C-W}-l}{l_{C-W}}\cdot 100 \qquad [\,\%] \tag{14}$$

In this paper we will use a number of 140 testing points, generated by 14 relative roughness values combined with 10 Reynolds numbers. The chosen ventilation duct material was a smooth galvanized steel sheet, having the absolute roughness k = 0.09 mm, according to [1]. Only round ducts were analysed, selecting 14 most usual diameters. 10 Reynolds numbers were calculated for each relative roughness, by choosing appropriate flows in order to respect 2 conditions: the transitionally rough turbulent zone and the maximum velocity allowed in HVAC ducts. Table 1 shows the range of diameters and the range of flows used to determine 10 Reynolds numbers for each relative roughness. The kinematic viscosity was chosen 15.1E-6 m^2/s, corresponding to air temperature of 20 ^oC.

For each of these 140 testing points, a λ value was calculated by each of the 9 equations in discussion, and then the relative error to Colebrook-White formula was determined, using Eq.(14).

The whole computing process was developed in a MS Excel spreadsheet (Fig. 1), allowing us to quickly manage this important volume of data and draw the conclusions. A VBA (Visual Basic for Applications) macro was written in order to solve the implicit Colebrook-White equation, using the Newton-Raphson numerical method.

Table 1

Input data for testing the 9 explicit equations

Usual duct diameters [mm]	Flow range for calculating Reynolds numbers [l/s]
63	12…25
80	20…40
100	31…63
125	48…98
160	78…160
200	130…250
250	200…390
315	310…620
400	500…1000
500	780…1550
630	1250…2500
800	2000…4000
1000	3100…6300
1250	5000…9800

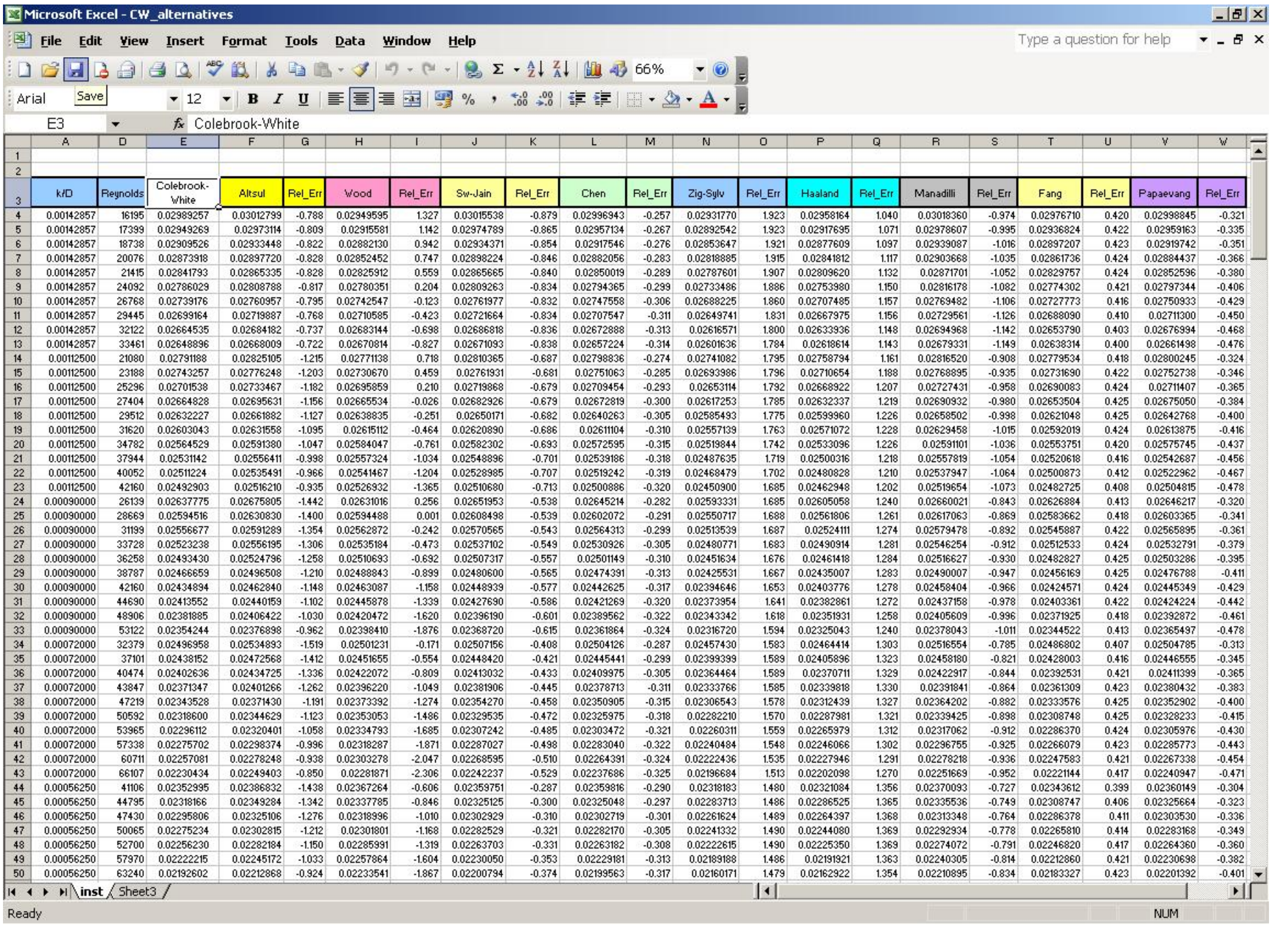

Fig. 1. MS Excel spreadsheet used to compare 9 explicit equations with the implicit Colebrook-White

Results and discussion. The obtained results are very interesting and are subject to raise more questions and investigation. Table 2 shows the results in a concise form, based on maximum (positive and negative) relative error of 9 approximations for Colebrook-White equation. It can be seen that Altshul and Wood equations have obtained poor results and can be eliminated from competition.

Chen, Manadilli and Papaevangelou formulas are constantly overestimating Colebrook-White (the relative error is never positive), whereas Zigrang-Sylvester, Haaland and Fang are constantly underestimating Colebrook-White (the relative error is never negative). Chen has surprisingly good results for such an old equation, Fang is performing well too, Haaland is not too bad for such a simple equation.

Engineers prefer a little bit of overestimation for safety reasons, so the “winner” seems to be Eq.(8), closely followed by Eq.(13) and Eq.(7), at least based on our 140 testing points. If simplicity is paramount, the best choice is Eq.(7).

Table 2

Maximum (+/-) relative error for 9 explicit equations in comparison to Colebrook-White formula, in 140 points

Number/name of the explicit equation	Max. positive relative error [%]	Max. negative relative error [%]
Eq.(5) Altshul	7.138	-1.519
Eq.(6) Wood	1.327	-4.045
Eq.(7) Swamee-Jain	0.062	-0.879
Eq.(8) Chen	-	-0.325
Eq.(9) Zigrang-Sylvester	1.923	-
Eq.(10) Haaland	1.422	-
Eq.(11) Manadilli	-	-1.149
Eq.(12) Fang	0.425	-
Eq.(13) Papaevangelou et al.	-	-0.478

Why is this result arguable ?

Because it seems that each equation has a “soft spot” where things can go wrong, locally the relative error can increase rapidly (present a spike), but for the rest of the range the results remain good. Relative error analysis shows that the “soft spot” is found for some equations at high Re numbers and small k/D, for others in the middle range, so there is no general rule.

However, the good news is that overall, the majority of explicit equations have an absolute relative error under 2 % for the duct sizing friction factor. Therefore, building services engineers may use them instead of implicit Colebrook-White equation, for spreadsheet-based simple design calculations.

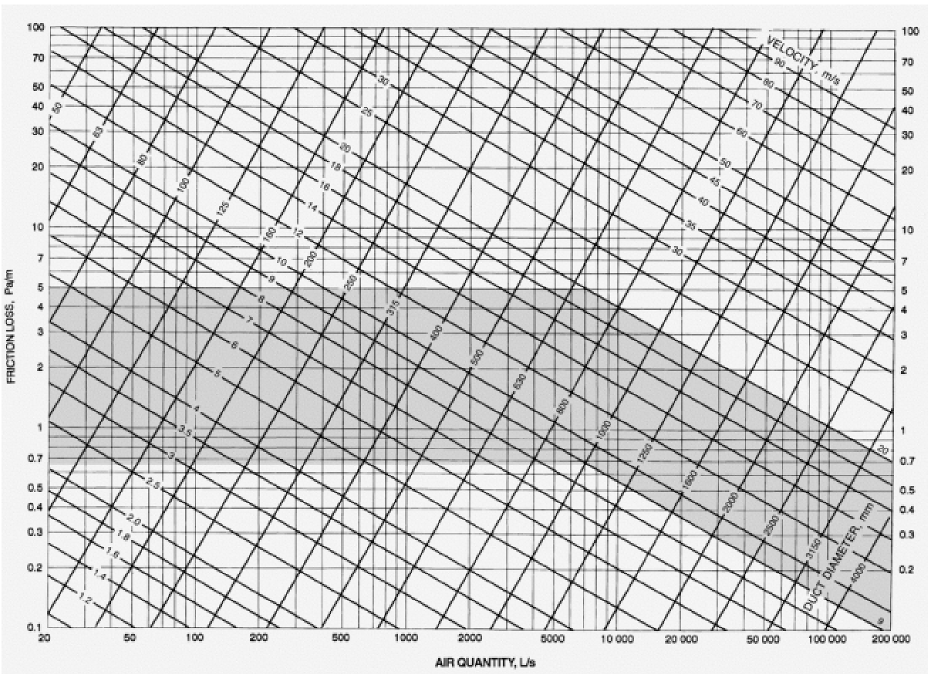

Fig. 2. ASHRAE chart for determining ventilation ducts friction losses

Conclusions. The traditional hydraulic design tool offered to engineers in handbooks was a chart (nomograph), giving directly the pressure drop per unit length (Pa/m), thus hiding the complexity of finding the friction factor (Fig. 2). Sometimes, instead of a chart, a big table full of numbers was given, inviting the engineer to repeteadly interpolate in order to obtain his design solution. Obviously, that was a slow and tedious work.

Another traditional instrument found in firms was the "Air Duct Calculator", in the form of a wheel-chart or sliding-scale calculator (Fig. 3). No maths, just fitting/adjusting the device and reading.

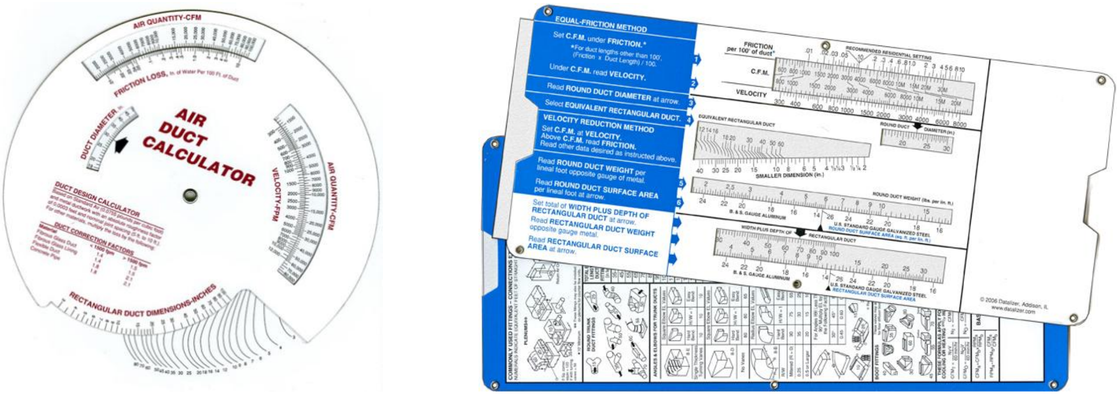

Fig. 3. Traditional wheel-chart and sliding-scale calculator

Recently, the digital revolution provided modern tools for duct sizing. This mobile software is often called "Ductulator" and is found on Windows, Android and IOS platforms. As an example, we used "HVAC Calculator" (www.softhvac.com) on Android and the results were comparable to those obtained by reading the charts (Fig. 2) or making calculations via Colebrook-White equation in spreadsheet.

For those engineers who prefer complex professional HVAC software instead of a DIY customized spreadsheet, one answer may be (for example) Lindab CADvent. CADvent is an AutoCAD© application with a complete toolbox for drafting, dimensioning, calculation, quantification and presentation of complete HVAC installations.

And for those engineers wanting to see what's running "behind the scenes", the present paper tries to make an overview of the most used alternatives to Colebrook-White equation, analyzing their complexity and mathematical accuracy along a wide range of Reynolds numbers and relative roughnesses. While the discussed scenarios are by no means exhaustive, these results may be used by building services engineers as guidance if they want to avoid iterative calculations.

Acknowledgments. This article was elaborated in the framework of the project VEGA 1/0202/15.

1. ASHRAE Handbook – Fundamentals, 2009, CD Edition. 2. Cengel Y, Cimbala J. Fluid Mechanics: Fundamentals and Applications. New York: McGraw-Hill, 2013, 1024 p. 3. Chen N. H. An explicit equation for friction factor in pipe. Industrial and Engineering Chemistry Fundamentals, Vol. 18(3), pp. 296–297, 1979. 4. Clamond D. Efficient resolution of the Colebrook equation. Industrial & Engineering Chemistry Research, Vol. 48(7), pp. 3665–3671, 2009. 5. Colebrook C.F. Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws, Journal of the Institution of Civil Engineers, London, Vol. 11, pp. 133–156, 1939. 6. Haaland S.E. Simple and explicit formulas for the friction factor in turbulent flow. Transactions of ASME, Journal of Fluids Engineering, Vol. 103, pp. 89–90, 1983. 7. Lester T. Solving for friction factor. ASHRAE Journal, Vol. 45(7), pp. 41–44, July 2003. 8. Swamee P.K., Jain A.K. Explicit equations for pipe-flow problems. Journal of the Hydraulics Division (ASCE), Vol. 102(5), pp. 657–664, 1976