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THE STABILITY ANALISIS FOR DYNAMIC MARKET MODELS

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АНАЛІЗ СТІЙКОСТІ ДИНАМІЧНИХ МОДЕЛЕЙ РИНКУ

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The work considers the problem of modeling and control of the stability of the market of goods taking into account the different types of factors associated with deterministic characteristics of market participants and the stochastic nature of economic processes. Significant for the stability of the market factors identified and the directions of development models with a view to its integration in a system-dynamic model of regional economy with regard to industrial and social factors are proposed .
Keywords: *mathematical modeling, decision making, systems' control, analysis, market equilibrium, market stability, computer modeling, system dynamic approach .*

State of the problem. Market, market relations of producers and consumers are an integral part of the socio-economic structure of society. The interaction of agents in the industrial sector, finance and the social sector are determined by market relations. Therefore, the study of the conditions of equilibrium and stability of the market has always attracted the attention of researchers. Laid Walras ideas of market equilibrium, based on the interaction of supply and demand, clothed in mathematical form [1,2,3,4,8,10,11], find application in different sectors of the economy [5,6,7,9]. In this article market will be interested as a subsystem of the regional economy as a whole with the inclusion of further model of production, social and environmental components.

The aim of this work is the development of a model of Walras in the direction of incorporating elements of the dynamics of production, elements of interaction (coordination, competition) of the market participants. Furthermore , the use of advanced modeling allows us to estimate the influence of various different factors on the process of achieving equilibrium.

The main part. Consider a market model of goods for Walras in general, believing that in the market of m goods present n producers of these goods. Regarding market participants we will require the following: each of them is a manufacturer of at least one item (or a fixed number) and consumer of other commodities (or their subsets). That is, if K={1,2,...,m} is the set of indices of goods in the market, and for each participant $K_j^-, K_j^+, j=\overline{1,n}$ sets of indices of sold and purchased , respectively, of goods, then:

$$K_j^- \neq \emptyset, K_j^+ \neq \emptyset, K_j^- \subset K, K_j^+ \subset K, K_j^- \cap K_j^+ = \emptyset, \quad \bigcup_{j=1}^n K_j^- = \bigcup_{j=1}^n K_j^+ = K, \quad (1)$$

i.e. every merchandise someone produces and each product has a consumer.

We introduce the following notation:

$$x = \|x_{ij}\|, x_{ij} = x_{ij}(t) \geq 0, i = \overline{1,m}, j = \overline{1,n}, \quad (2)$$
$$y = \|y_{ij}\|, y_{ij} = y_{ij}(t) \geq 0, i = \overline{1,m}, j = \overline{1,n}, \quad (3)$$

where $x_{ij}(t), y_{ij}(t)$ - are demand and supply respectively of the i-th product for the j-th market participant in each moment of time t .

Let each moment of time on the market set prices for all goods:

$$p(t) = (p_1, p_2, \dots, p_m)^T.$$

Then participants of the market cost of a basket of demand are determined by the vector-line:

$$M(t) = p^T(t)x(t), \quad (4)$$

and the budget of sales:

$$m(t) = p^T(t)y(t). \quad (5)$$

Under the terms of the equilibrium models of Walras [4] for each market participant must satisfied the condition

$$M(t) - m(t) = 0, \quad (6)$$

and the driving force for the formation of the equilibrium price in the market is the difference between the amounts of supply and demand for goods. A column vector of excess demand will build as the difference between those for each item:

$$Z(t) = \left(Z_1(t), Z_2(t), \dots, Z_m(t)\right)^T,$$
$$Z_i(t) = X_i(t) - Y_i(t), i = \overline{1,m}, \quad (7)$$

where
$X_i(t) = \sum_{j=1}^{n} x_{ij}$, $Y_i(t) = \sum_{j=1}^{n} y_{ij}$, $i = \overline{1,m}$.
and for the formation of equilibrium prices we write the equation Walras [1] in the form:

$$\dot{p}(t) = W \cdot Z(t),$$
$$p(0) = \left(p_1(0), p_2(0), \dots, p_m(0)\right)^T, \quad (8)$$

where $W = \|w_{ij}\|, i,j = \overline{1,m}$ - matrix of weight coefficients that determine the nature and the dynamics of the process of agreeing prices.

Unlike the standard model [4], where W is assumed a scalar, and even W=1 , we do not believe the identity or a diagonal matrix with equal elements. In the general case, we believe that $w_{ij} > 0, i = \overline{1,m}$ and will not require $w_{ij} = 0$ for $i \neq j$.Thus we will provide the opportunity for the "regulators" of prices in (8) to use a different management strategy given the behaviour of other regulators.

This is due primarily to the fact that each of participants of the market the consumer has a utility function $f_i\left(X^j(t)\right), j = \overline{1,n}$, were $X^j(t)$- is the demand function , which is j-th column in the matrix (2) and it is solves the problem of reaching the maximum in each moment of time:

$$f_j\left(X^j(t)\right) \xrightarrow[X^j(t)]{} max, \quad (9)$$

$$M_j(t) - m_j(t) = 0 .$$

Depending on the structure f_j goods can be complementary, neutral or interchangeable. Therefore, the "regulator" may impose a corrective communication to the driver of prices (8) using the nondiagonal elements in the matrix W. The absolute values of the elements in the matrix W reflects the rate of the reaction "regulators" to the imbalance in (7), but,as noted in [4] , not the nature of the behavior of the overall market and relative values specify exactly what the nature of market behavior. However, we note that this can be true only if W is a scalar and there is no time delay in the system.

For computer simulation, consider the market of 3 goods, because the market of 2 goods is always stable [4] and there is no possibility simultaneously to observe different types of interactions manufacturers.

As utility functions we will use multiplicative function of the form:

$$f_j = \prod_{i=1}^{m} x_{ij}{}^{a_{ij}} , \sum_{i=1}^{m} a_{ij} = 1, a_{ij} \geq 0, j = \overline{1,n}, \quad (10)$$

moreover, without reducing the generality , we simplify the situation by putting that if the j -th participant produces the k - th product, then $a_{kj} = 0$, i.e. it does not consume him, rather, it is accounted for in internal flows. In these conditions after solving problem (9) the demand vector for the j-th participant is:

$$x_{ij}(t) = \frac{a_{ij}}{p_i(t)} \cdot \frac{\sum_{i=1}^{m} p_i(t) y_{ij}(t)}{\sum_{i=1}^{m} a_{ij}}, i = \overline{1,m}, j = \overline{1,n}. \quad (11)$$

As the base model use the following: m=3, n=3,

$$p(0) = \begin{pmatrix} 60 \\ 20 \\ 30 \end{pmatrix}, y = \begin{pmatrix} 30 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{pmatrix},$$
$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \|a_{ij}\| = \begin{pmatrix} 0 & 0.6 & 0.4 \\ 0.4 & 0 & 0.6 \\ 0.4 & 0.6 & 0 \end{pmatrix} \quad (12)$$

i.e. we register the offer and installed the same reaction "regulators" of prices in the market., Producers are regulators, because they are monopolists. The results are presented in Fig.1, where the parameter R(t) given the value

$$R(t) = \sqrt{p_1^2(t) + p_2^2(t) + p_3^2(t)},$$

which due to the equality of diagonal elements in W retains its value throughout its trajectory $(p_1(t), p_2(t), p_3(t))$, i.e. the motion occurs in a sphere, the radius of which determined by the initial price vector.

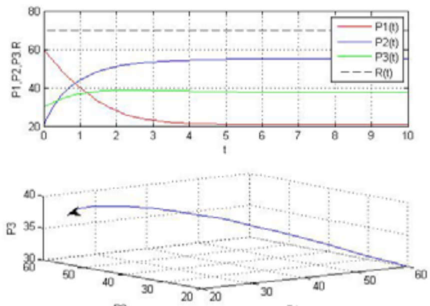

Fig. 1. Trajectories for case of (12)

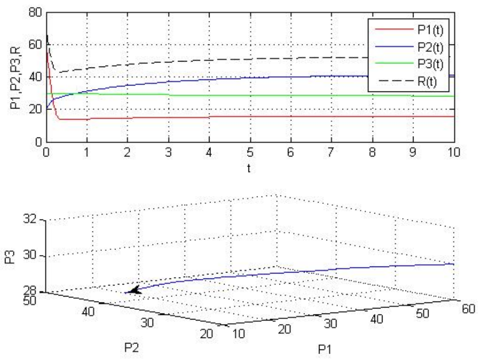

Fig. 2. Trajectories for different strength reactions (13)

To assess the influence of the coefficients of the matrix W , leaving it diagonal, substantially change the relative values of the elements

$$W = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{pmatrix}, \qquad (13)$$

in the order of increasing reaction $p_1(t)$ and the decreasing for $p_3(t)$. As a result, the equilibrium price is achieved at lower prices. The largest decline of prices was observed for the item with the largest response $(p_1(t))$and for item with "slow" reaction $(p_3(t))$ the price was virtually unchanged. It is also worth noting the change in the behavior of R(t), since now the trajectory $((p_1(t), p_2(t), p_3(t)))$ lies in this case on a very "deformed" ellipsoid (see Fig.2).

Next, we assume that the vectors of the supply of goods by manufacturers

$$Y^j(t) = \left(y_{1j}(t), y_{2j}(t), \dots, y_{mj}(t)\right)^T, j = \overline{1,n} \qquad (14)$$

are the result of industrial activity, which displays the production function of each of the participants

$$y_{ij}(t) = F_{ij}\left(K_{ij}^a(t), L_{ij}^a(t)\right), i = \overline{1,m}, j = \overline{1,n} \qquad (15)$$

and the manufacturer can regulate the amount of products offered, varying the volumes involved (active) capital (K^a) and labour (L^a).

As a simple control strategy we use natural response: if demand exceeds supply, it is necessary to increase the offer , i.e.

$$\dot{Y}^j(t) = G_j \cdot Z(t), G_j = \|g_{ik}^j\|, g_{ii}^j \geq 0,$$
$$i, k = \overline{1,m}, j = \overline{1,n}. \qquad (16)$$

At constant conditions (12), where we assume that the matrix $y = \|y_{ij}\|$ is only an initial condition for (16), we use the matrix

$$G_1 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, G_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$G_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.1 \end{pmatrix} \qquad (17)$$

which set of manufacturer dependence in (16) only the value of the balance of the manufactured product.

The results of the calculations are presented in Fig. 3-4. It is obvious that the process of achieving equilibrium is decreasing in time at the expense of control offer.

In the next stage of modeling will change G_j and make different rates of reaction of producers to the imbalance as shown in (18) and we get the results shown in Fig. 5-6 . The diagrams show that the more intense reaction to the imbalance allows producers 2 and 3 to reach the equilibrium state large volume of sales, however with some reduction in the price level.

$$G_1 = \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, G_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$G_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 10 \end{pmatrix}. \qquad (18)$$

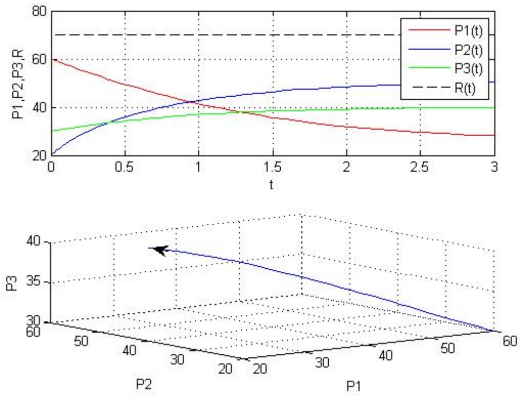

Fig. 3. Trajectories for equal reactions by supplying

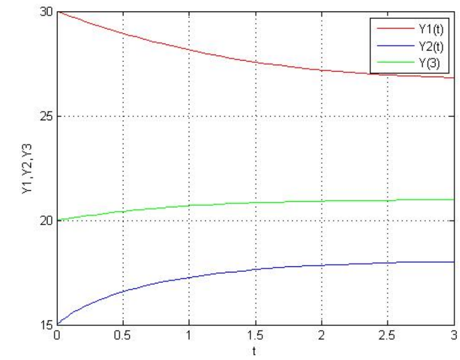

Fig. 4. Trajectories for suppliers' reactions

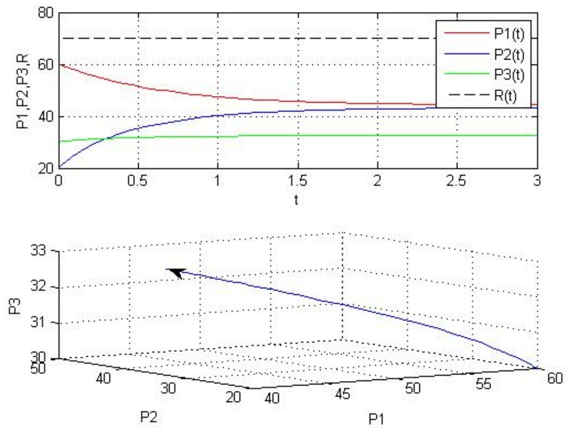

Fig. 5. Trajectories for different reactions by supplying

Additionally, using the matrix W from (13) and performing calculations, obtain the results presented in Fig.7 -8. In this case, the more intense the reaction of the members 1 and 2 in the formation of prices does not allow to leave the member 3 at the previous level in terms of volume. The trajectory of prices is already happening under the ellipsoid.

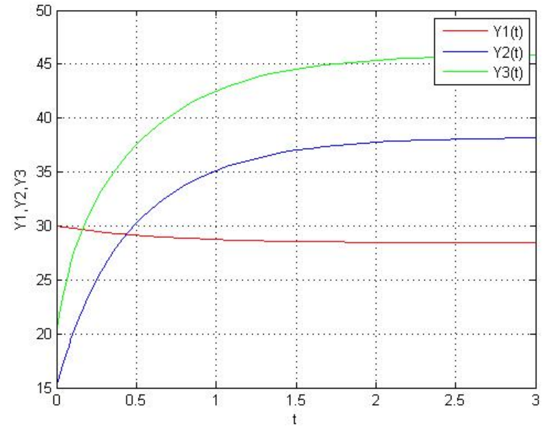

Fig. 6. Trajectories for supplying reactions in case (18)

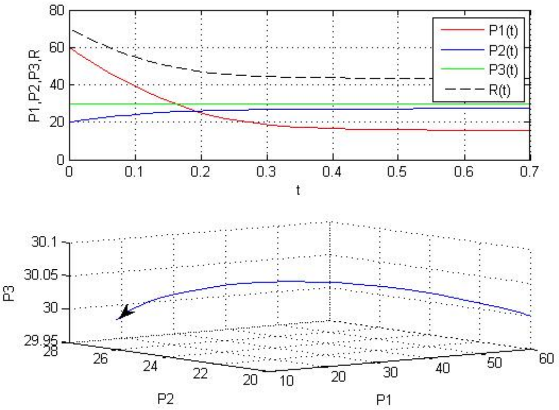

Fig. 7. Trajectories for combined case of matrixes W and G

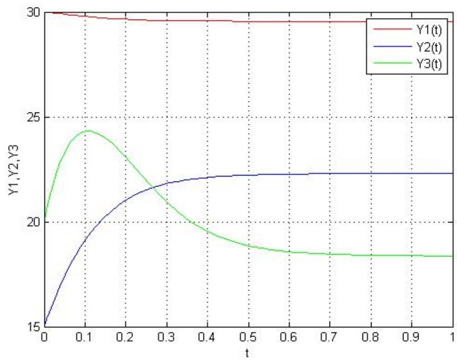

Fig .8. Trajectories for supplying reactions for W and G from (13) and (18)

However, it should be noted that equation (16) does not provide growth, since price stabilization their right side becomes zero, although it is possible, when we have obtained the equilibrium prices are really proportional development of all industries, i.e. the growth of the economy.

In this situation, consider the real limitations for the manufacturer:

-issue manufacturer's limited available capital ($\overline{K_j}$) and human resources ($\overline{L_j}$), i.e. in (15) $L_{ij}^a \leq \overline{L_J}$, and consequently, for the matrix $y(t) = \|y_{ij}(t)\|$ in (3) there exists a matrix of limiting values of releases $y = \|\overline{y_{\iota J}}\|$. Then at any point in time must satisfy the condition $y_{ij}(t) \leq \overline{y_{\iota J}}$;

- each manufacturer there is a rate of return determined by production technologies and subjective factors, and accordingly there and the lower limit price $p^l = \|p_i^l\|, i = \overline{1,m}$ below which production is unprofitable . Hence the condition for prices :$p_i(t) \geq p_i^l$.

In this situation it is proposed that the controller (16) use equation:

$$\dot{Y}^J(t) = G_j \cdot \Delta P(t) \cdot \left(\overline{Y_J} - Y^j(t)\right), G_j = \|\vartheta_{ik}^j\|, \vartheta_{ik}^j \geq 0,$$
$$i, k = \overline{1,m}, j = \overline{1,n},$$

were

$$\Delta P(t) = \begin{pmatrix} p_1(t) - p_1^l & 0 & 0 \\ 0 & p_2(t) - p_2^l & 0 \\ 0 & 0 & p_3(t) - p_3^3 \end{pmatrix} (19)$$

that take into account both the above factors.

In Fig. 9-10 presents calculations with matrices W and G from (12) and (17), i.e. with the equivalent reactions of the participants on the dynamics of market processes. It is obvious that the process of achieving equilibrium, is more complicated and only participant 2 reached a download of all their"capacity". Next was introduced the matrix W from (13), with a large change participants ' reactions to price dynamics (Fig. 11-12). This led to a "jump" at the beginning of the process and long reaching the stationary regime.

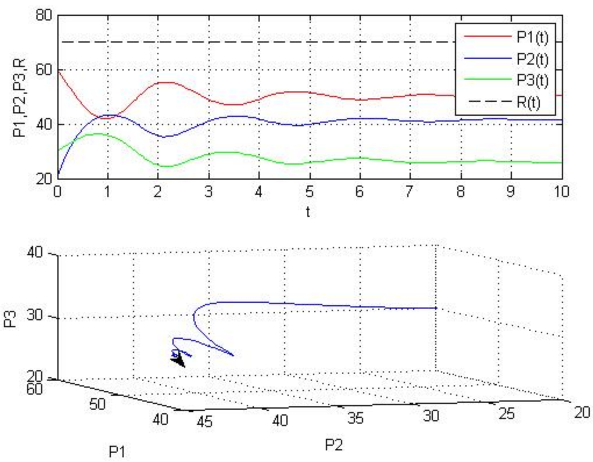

Fig. 9. The case for equivalent reactions and including price reaction (19)

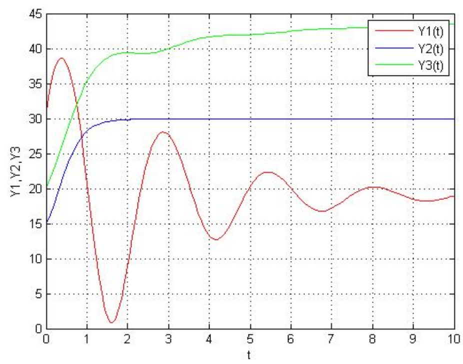

Fig. 10. Reactions of suppliers on regulator (19)

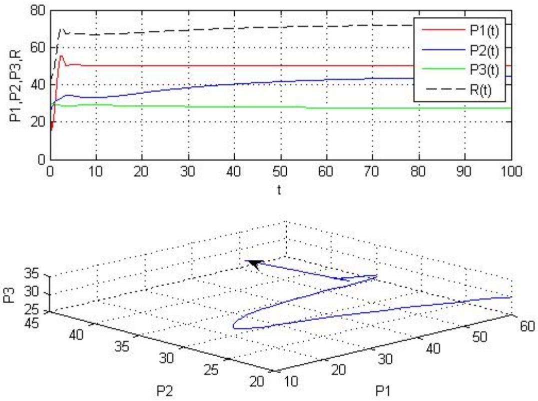

Fig. 11. The case for different reactions (13), (18)

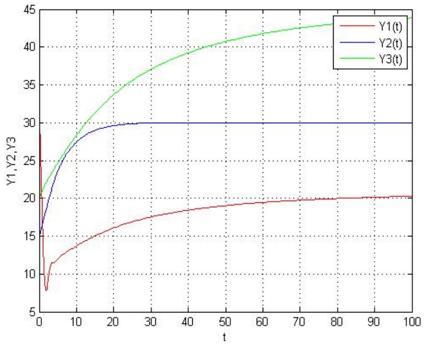

Fig. 12. Reactions of suppliers in case of (13), (18)

Using matrix G from (17) that provide additional compensating response of producers with a lesser reaction to price changes is increased by the reaction of the price level and the volume of production taking into account the profitability and the available production capacity. In the specific case, we can say that the 1st party is "responsible" changes in the price of the product, and the 3rd is the change in volume of production (Fig.13-14).

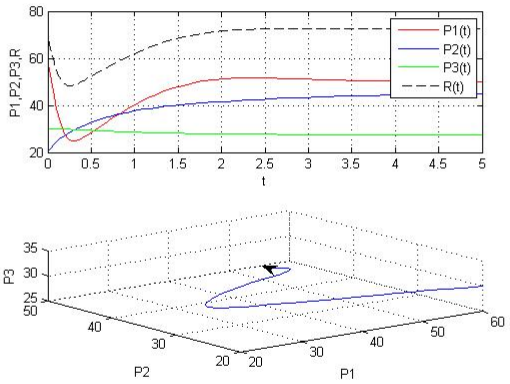

Fig. 13. Trajectories for added conditions of (17)

It should be noted that in reality the market situation is complicated by such processes as the delay in the decision-making processes and applying stochastic processes to the volumes. The study of such systems is possible only by simulation methods of system dynamics.

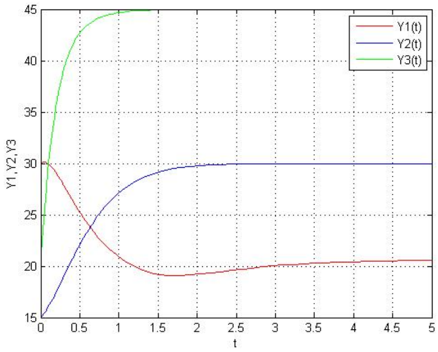

Fig. 14. Reactions of suppliers for added conditions of (17)

Conclusions. The proposed development model in the exchange market with the inclusion in the modeling process:

- the controls supply of goods on the market;

-elements lag "controllers" to make decisions on the price of the goods;

- control the intensity of the reaction participants to change the situation on the market;

-allowed us to estimate the degree of influence of various factors on the stability of the market.

The proposed model and its computer implementation has allowed to overestimate the value of some of the parameters of the model, which previously not focused the attention of researchers.

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Істомін Л.Ф.Аналіз стійкості динамічної моделі ринку

В роботі розглянуто проблеми моделювання та управління стійкістю ринку товарів з урахуванням різних видів факторів , пов'язаних як з детермінованими характеристиками суб'єктів ринку, так і стохастичною природою самих економічних процесів. Виявлено істотні для стійкості ринку чинники та запропоновано напрямки розвитку моделі з метою її інтеграції в системно-динамічну модель регіональної економіки з урахуванням виробничих і соціальних факторів.

Ключові слова: *математичне моделювання, прийняття рішень,управління, аналіз, ринок,рівновагу та стійкість,комп'ютерне моделювання,системно-динамічний підхід.*

Истомин Л.ф. Анализ устойчивости динамической модели рынка

В работе рассмотрены проблемы моделирования и управления устойчивостью рынка товаров с учетом различных видов факторов, связанных как с детерминированными характеристиками субъектов рынка, так и стохастической природой самих экономических процессов. Выявлены существенные для устойчивости рынка факторы и предложены направления развития модели с целью ее интеграции в системно-динамическую модель региональной экономики с учетом производственных и социальных факторов.

Ключевые слова: *математическое моделирование, принятие решений,управление, анализ, рынок,равновесие и устойчивость,компьютерное моделирование,системно динамический подход.*

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