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THE SIMULATION MODEL FOR ANALYSIS OF DYNAMIC MARKET

Istomin L.F.

ІМІТАЦІЙНА МОДЕЛЬ АНАЛІЗУ ДИНАМІКИ РИНКУ

Істомін Л.Ф.

The problem of modeling and control of the stability of the goods' market is considered.The study is taking into account the different types of factors associated with deterministic characteristics of market participants and the stochastic nature of economic processes. Base model of Walras got new elements to wide opportunities for taking in account as competitive interactions of market's participants and their reaction by changing their production activity too. The results of investigation was got by using developed by author program in SIMULINK of MATLAB. Structure of proposed computer model have great variability to change technical characteristics and type of behaviour for market's participants. Significant for the stability of the market factors identified and the directions of development models with a view to its integration in a system-dynamic model of regional economy with regard to industrial and social factors are proposed .

Keywords: *mathematical modeling, simulation model, SIMULINK, decision making system, systems' control, analysis, market equilibrium,market stability,computer modeling,system dynamic approach .*

State of the problem. The interaction of producers and consumers through the market is an integral part of the socio-economic structure of society. Similarly, the interaction of agents in the field of finance and social sphere are determined by market relations. The condition's problem of equilibrium and stability of the market and analysis of the principles and methods of regulation of the factors determining the situation on the market has attracted the attention of researchers both from the theoretical positions and problems of applied nature. Laid Walras' and Marshall's ideas of market equilibrium, based on the interaction of supply and demand find its development in theoretical aspect [1,2,3,4,7] and applied research and even experimental [5,6,8,11]. Clothed in mathematical form [1,2,,8,11,12], they find application in various sectors of the economy [5,6,7,11]. However, given the complexity of modeling the behavior of the market with many agents, this area has not received sufficient attention, especially the models that include elements of competition, regulation and factors associated with the stochasticity of economic processes and time lags in decision-making.

The aim of this work is the further exploration and development of the Walras model proposed in [10] in the direction of incorporating elements of the dynamics of production, elements of interaction (competition) of the market participants. Furthermore, the use of advanced modeling allows to investigate the influence factors of stochasticity and time delays in decisions by market agents.

The main part. As in [10] we consider a market model of goods by Walras in general, believing that the market of m goods present n producers of these goods (n≥m). Regarding market participants we will require the following: each of them is a manufacturer of at least one item (or a fixed range) and consumer of other commodities (or their subsets).

Supply and demand will enter as follows:

$$x = \left\| x_{ij} \right\|, x_{ij} = x_{ij}(t) \ge 0, i = \overline{1,m} \;, j = \overline{1,n},$$
$$y = \left\| y_{ij} \right\|, y_{ij} = y_{ij}(t) \ge 0, i = \overline{1,m}, j = \overline{1,n}, \quad (1)$$

where $x_{ij}(t), y_{ij}(t)$ - demand and supply are respectively of the i-th product by the j-th participant of market at time t . At time t baskets of supply and demand are determined by the vectors:

$$M(t) = p^T(t)x(t), m(t) = p^T(t)y(t), \quad (2)$$

where $p(t) = (p_1, p_2, ..., p_m)^T$ - is the current vector of prices.

The formation of equilibrium market prices is based on the equation:

$$\dot{p}(t) = W \cdot Z(t), \quad (3)$$

$$p(0) = (p_1(0), p_2(0), ..., p_m(0))^T,$$

where: $Z(t) = (Z_1(t), Z_2(t), ..., Z_m(t))^T$,

$$Z_i(t) = X_i(t) - Y_i(t), i = \overline{1,m}$$

is the excess demand for goods;

$$X_i(t) = \sum_{j=1}^{n} x_{ij}, Y_i(t) = \sum_{j=1}^{n} y_{ij};$$

$W = \|w_{ij}\|, i, j = \overline{1,m}$ - matrix of weight coefficients is determining the nature and pace of the agreeing process for prices.

In the general case, we believe that $w_{ii} \geq 0, i = \overline{1,m}$, but will not require $w_{ij} = 0$ for $i \neq j$. Thereby it provide the opportunity for the "regulators" of prices in (3) to use a different management strategy given the behaviour of other regulators.

Note that each of the market participants in the role of the consumer has a utility function $f_j(X^j(t))$, where $X^j(t)$ is a vector- function of demand , which is j-th column in the matrix $X(t)$ and it is the solution in each moment of time of the problem about reaching the maximum:

$$f_j(X^j(t)) \xrightarrow[X^j(t)]{} \max, \quad (4)$$

$$M_j(t) - m_j(t) = 0, \quad (5)$$

where and are the j –th columns of the respective matrices.

Goods can be complementary, neutral or mutually substitute depending on the structure of f_j . Therefore, the "regulator" may impose a corrective communication to the driver of prices (3) using the nondiagonal elements in the matrix W. The absolute values of the elements in the matrix W reflects the rate of the reaction "regulators" to the imbalance in (5), but, as noted in [12] , not the nature of the behavior of the overall market and relative values specify exactly what the nature of market behavior. However, it must be noted that this can be true only if W is a scalar and there is no lag in the system.

As utility functions we will use multiplicative function of the form:

$$f_i = \prod_{i=1}^{m} x_{ij}^{a_{ij}}, \sum_{i=1}^{m} a_{ij} = 1, a_{ij} \geq 0, j = \overline{1,n},$$

moreover, without reducing the generality we simplify the situation by putting that if the j -th participant produces the k - th product, then $a_{ik} = 0$, i.e. he does not consume this product, rather, it is accounted for in internal flows. In these conditions after solving the problem (3) the demand vector for the j-th participant is:

$$x_{ij}(t) = \frac{a_{ij}}{p_i(t)} \cdot \sum_{i=1}^{m} p_i(t) y_{ij}(t), i = \overline{1,m}, j = \overline{1,n}.$$

As the base model we use the following:

$$m = 3, n = 3,$$

$$p(0) = \begin{pmatrix} 60 \\ 20 \\ 30 \end{pmatrix}, y = \begin{pmatrix} 30 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 20 \end{pmatrix}, \quad (6)$$

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \|a_{ij}\| = \begin{pmatrix} 0 & 0.6 & 0.4 \\ 0.4 & 0 & 0.6 \\ 0.4 & 0.6 & 0 \end{pmatrix},$$

i.e. there are fixed offer and equal reaction of prices' "regulators" in the market. As regulators we have producers, because they are monopolists. The research results of the basic model presented in [10].

Put now that the vectors in the supply of goods by manufacturers:

$$Y^j(t) = (y_{1j}(t), y_{2j}(t), ..., y_{mj}(t)), j = \overline{1,n},$$

are the result of industrial activity, which displays the production function of each of the participants

$$y_{ij}(t) = F_{ij}(K_{ij}^a(t), L_{ij}^a(t)), i = \overline{1,m}, j = \overline{1,n} \quad (7)$$

and the manufacturer can regulate the amount of products offered, varying the volumes involved (active) capital (K^a) and labour (L^a).

In this situation, consider the real limitations for the manufacturer:

- every manufacturer is limited by available capital $(\overline{K_j})$ and labor resources $(\overline{L_j})$, i.e. $K_{ij}^a(t) \leq \overline{K_j}$ and $L_{ij}^a(t) \leq \overline{L}_j$, consequently, for the matrix $y(t) = \|y_{ij}(t)\|$ in (6) a matrix of limiting values of releases $\overline{y = \|\overline{y_{ij}}\|}$ exists. Then at any time must satisfied the condition $y_{ij}(t) \leq \overline{y_{ij}}$;
- each manufacturer has level of profitability determined by production technologies and subjective factors, and accordingly there is the lower limit price $p^l = \|p_i^l\|, i = \overline{1,m}$, below which production activity is unprofitable . Hence the condition for prices : $p_i(t) \geq p_i^l$.

Thus as a regulator of production volumes, it is possible to offer a system of equations:

$$\dot{Y}(t) = G_j \cdot \Delta P(t) \cdot (\overline{Y_j} - Y^j(t)), G_j = \left\| g_{ik}^j \right\|,$$
$$g_{jj}^j \geq 0, i,k = \overline{1,m}, j = \overline{1,n}, \quad (8)$$

where $\Delta P(t) =$

$$= \begin{pmatrix} p_1(t) - p_1^l & 0 & 0 \\ 0 & p_2 - p_2^l & 0 \\ 0 & 0 & p_3 - p_3^l \end{pmatrix}, \quad (9)$$

that take into account both the above factors.

It should be noted that in reality the market situation is complicated by such processes as the delay in the decision-making processes and applying stochastic processes to the supply volumes. The study of such systems is possible only by simulation methods of system dynamics. Below the developed in the SIMULINK of MATLAB [9] (Fig. 1) simulation model is presented that take into account as a time delaying , elements of control, stochastic processes and technological restriction too.

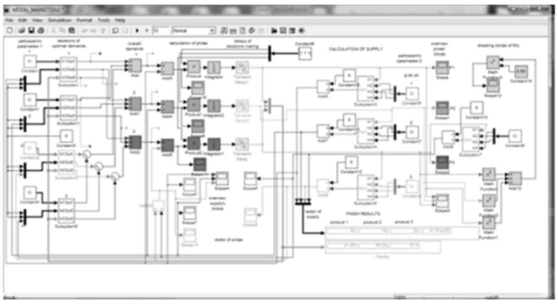

Fig. 1. Structure of the system's model in SIMULINK

In the system two types of subsystems are used(Fig.2-3). The first of these ones solves the problem of a market participant as a consumer, and whose output is the demand vector that optimizes an individual utility function. The second type of subsystem solves the task of decision-making on changes in the volume of output taking in account prices in the market and the achieved level of production volume.

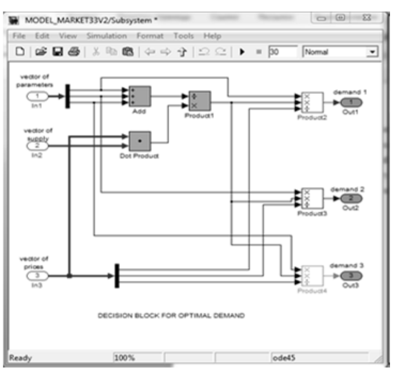

Fig. 2. Subsystem for consumer's problem decision

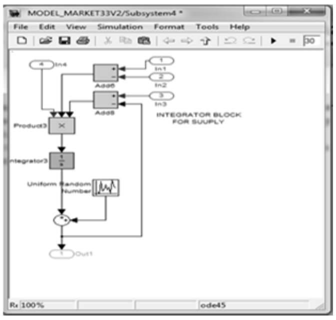

Fig. 3. Subsystem for manufacturer's calculation

The model incorporates of elements for regulation of time lag for each "controller" and the generator of the stochastic component for the supply of each producer too. There is also the possibility of modification of control matrix for the reaction of the "regulators" of prices W, and manufacturers of G in the model.

Thus, in this model, at the time t price of the-th item is equal to $p_i(t) = \tilde{p}_i(t - \tau_i)$ where $\tilde{p}_i(t - \tau_i)$ is the solution of the differential equation (3) for time $t - \tau_i$, i.e. accepted decision for price is delayed by the time τ_i .

With regard to the formation of volume of deliveries of goods by each manufacturer, it is determined as the solution of the differential equation (8) with additive random component $\varepsilon_j(t)$ in right side , i.e.

$$\dot{Y}^j(t) = G_j \cdot \Delta P(t) \cdot (\overline{Y}_j - (Y^j(t) + \varepsilon_j(t)),$$
$$j = \overline{1,n} , \quad (10)$$

here $\varepsilon_j(t)$ is a random process with a uniform distribution on the interval $[-b_j , b_j]$, that is set on base of the characteristics of volumes of deliveries of each producer.

Additionally, the model includes the 4-th market participant that can be included in the system to study the effects of competitive relations of producers with different production technologies.

The first series of calculations produced for the initial data (6) with input delays "regulators" τ=[0,0,2] (Fig.4) and τ=[0,0,10] (Fig.5). Graphs $p_1(t), p_2(t), p_3(t), R(t)$ are presented consistently vertical.

It is obvious that the delay $\tau_3 = 2$ (Fig.4) leads to oscillatory processes, which quickly fade and balance is achieved when the price level is equal to unperturbed problem [10]. However , for $\tau_3 = 10$ (Fig.5) we get high-amplitude long-term transitional process of agreeing prices, which ends at the level of prices in 3.46 times higher than in the previous case. It should be noted that the condition of equilibrium of the prices $p(t) = \lambda \cdot p^*(t)$, where $\lambda \geq 0, p^*(t)$ - is the equilibrium price, persists.

At the next step we include in the study the regulator for the supply of goods (9) with the matrix G:

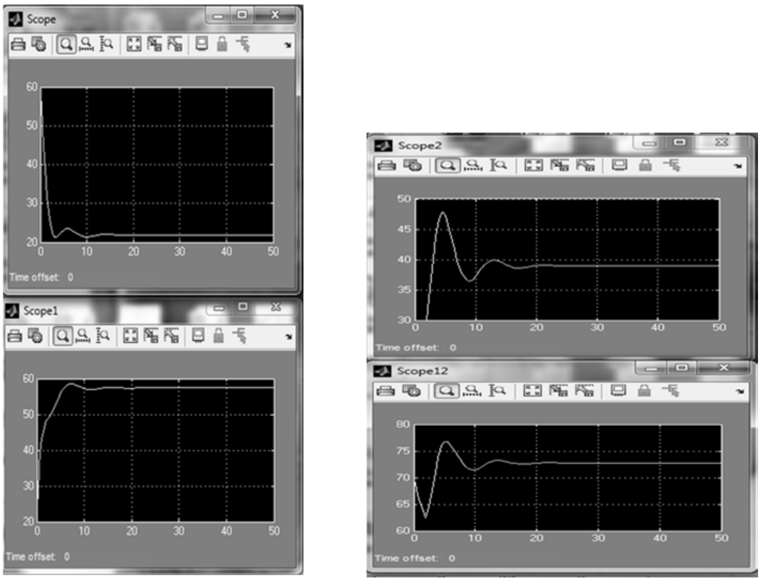

Fig. 4. Results for $\tau_3 = 2$

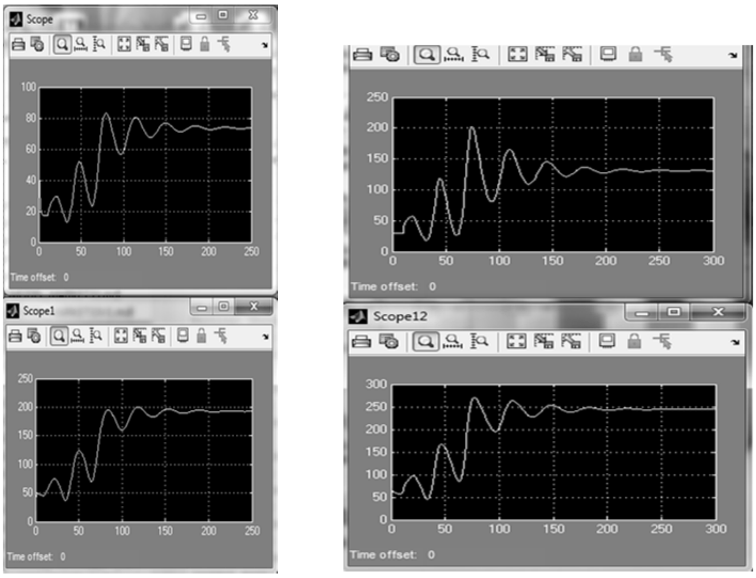

Fig. 5. Results for $\tau_3 = 10$

$$g_{jj}^{j} = 0.1, g_{ik}^{j} = 0.1, i,k = \overline{1,m}, i,k \neq j, j = \overline{1,n}.$$

without the use of time delay with the following information:

$$\overline{Y} = (80\ 30\ 45), P^{l} = (30\ 15\ 20).$$

The results demonstrate a quick convergence to the equilibrium prices and volume $y(t)$ (Fig.6). The structure of the vector of equilibrium prices has changed compared to the original task due to changes in volumes.

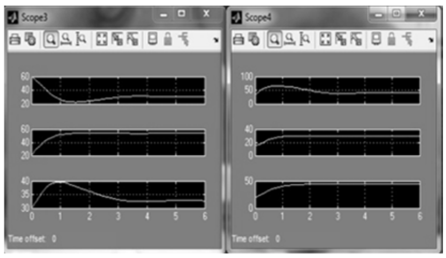

Fig. 6. Results for prices $p(t)$ (left) and volumes $y(t)$ (right)

The inclusion of the time lag $\tau_3 = 10$ leads only to a shift in prices, but the equilibrium state is achieved at the same pace as in the previous case (with the time shift by 10 units of course)(Fig.7).

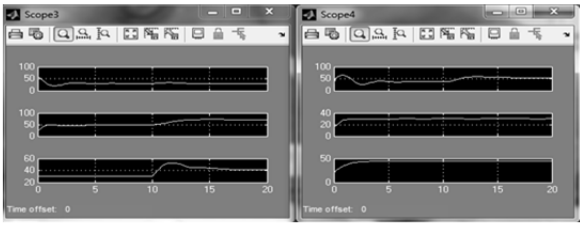

Fig. 7. Results of $p(t)$(left) and $y(t)$ (right) for $\tau_3 = 10$

The superposition of random components of different durations on the supply in the amount of $\le 10\%$ from initial values does not change the essence of the process as is shown in Fig.6. We have stable and equilibrium prices and volumes of supply have not changed their values (Fig.8).

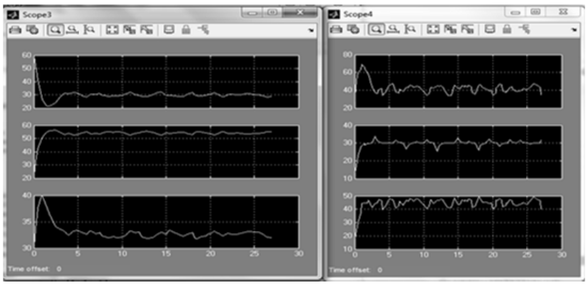

Fig. 8. Results with inclusion of random components

However, the input of time lag $\tau_3 = 10$ ceteris same conditions leads to the imbalance of the market's prices and supply too(Fig.9).

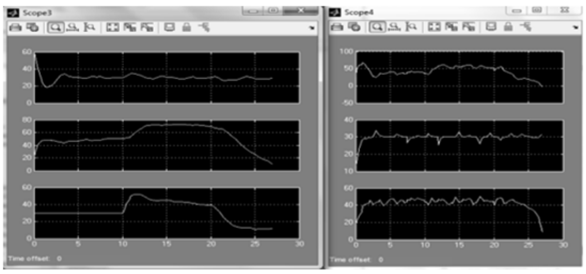

Fig. 9. Imbalanced market for time lag $\tau_3 = 10$

At the same time it should be noted that, if instead of using the matrix W from (6) we'll use matrix with a lower degree of response to price changes , i.e. $w_{ii} = 0.1, i = \overline{1,n}$, we obtain a picture(Fig.10), indicating that the market reaches its equilibrium state. Thus , it can be argued that in the presence of delays in decision-making and the presence of random omponents in the supply, the role of the matrix W is changed fundamentally. Now important for the sustainability of the market become not only the relative values of the elements of the matrix W, but also their absolute values.

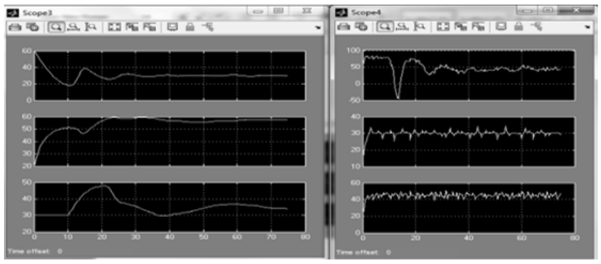

Fig. 10. The equilibrium of market for decreasing reaction in W

The next step in the study is the input of the 4-th market participant. Enter it as a competitor of party 3, i.e. it produces a 3-rd product, consuming the 1-st and 2-nd, but with a different utility function, which in a certain way displays using alternative technologies . The upper limit of the volume of deliveries will fixed for the new party on higher level and set higher the lower limit of the acceptable price. When competitors is entered on the market, volumes of initial offerings of competitors are equal one another and equal to half of the proposals in the previous calculations:

$$m = 3, n = 4,\ p(0) = (60\ 20\ 30)^T ,$$

$$y = \begin{pmatrix} 30 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 \\ 0 & 0 & 10 & 10 \end{pmatrix}, \qquad (11)$$

$$W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \left\| a_{ij} \right\| = \begin{pmatrix} 0 & 0.4 & 0.4 & 0.8 \\ 0.6 & 0 & 0.6 & 0.2 \\ 0.4 & 0.6 & 0 & 0 \end{pmatrix},$$

$$\overline{Y} = (80\ 30\ 45\ 50)^T , P^l = (30\ 15\ 20\ 28)^T .$$

The results of calculations (Fig.11) show that the market equilibrium is reached with stable prices that is closed to those in the original problem (changes in the structure of demand has not affected the result for the equilibrium prices) ,and in competition the participant's 3 position is a lot more preferable because its production capacity is fully loaded, and the 4-th one only 20% (Fig.12). Thus, we can say that elements of competition have been reflected in the proposed model.

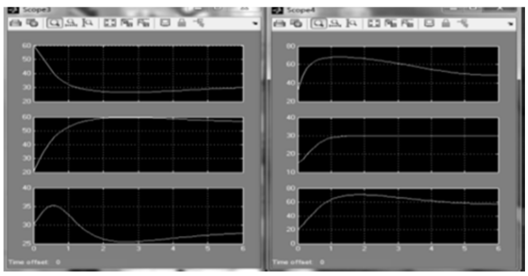

Fig. 11. The market's equilibrium of (11): $p(t)$ (left) and $y(t)$ (right)

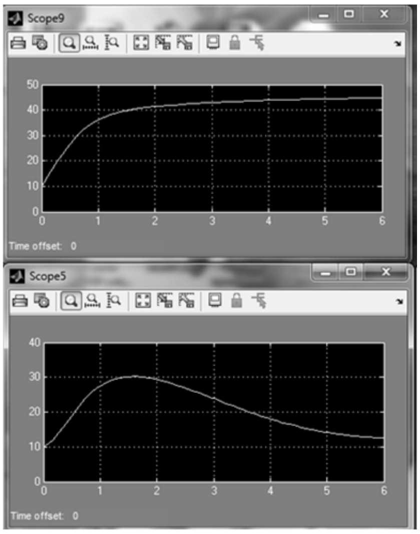

Fig. 12. Data for $y_3(t)$ (above) and $y_4(t)$ (below)

As a result we can do conclusion that matrix W in (3) is playing a regulator's role for participants reaction's intensity and simultaneously ask the level of competitive interaction of manufacturers. As for matrices G and ΔP in (10) these are regulators with restrictions for outputs and prices too.

Conclusion. The proposed development of Walras model in the exchange market with the inclusion to the modeling process:

- controls supply of goods on the market;

- elements time lag "controllers" to make decisions on the price of the goods;

- control the intensity of the participant's reaction to change the situation on the market;

- inclusion of stochastic components in the supply market ;

- enable us to estimate the degree of influence of various factors on the stability of the market, its handling too.

The proposed model and the computer implementation has allowed to overestimate the value of the entity some of model's parameters, which previously not focused the attention of researchers.

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Істомін Л.Ф. **Імітаційна модель аналізу динаміки ринку**

Розглянуто проблему моделювання та контролю стабільності ринку товарів. Дослідження виконані з урахуванням різних типів факторів, пов'язаних з детермінованими характеристиками учасників ринку і стохастичним характером економічних процесів. Базова модель Вальраса отримала нові елементи і широкі можливості для врахування таких факторів, як конкурентна взаємодія учасників ринку та їх реакції змінами в їх виробничої діяльності. Результати дослідження виконані за допомогою розробленої автором програми в середовищі СИМУЛИНК пакету МАТЛАБ. Структура пропонованої комп'ютерної моделі мають велику варіабельність щодо зміни технічних характеристик і типу поведінки для учасників ринку. В результаті роботи виявлено істотні для стабільності ринку фактори та напрямки розвитку моделі з метою інтеграції в системно-динамічну модель регіональної економіки з урахуванням виробничих і соціальних факторів.

Ключові слова*: математичне моделювання, імітаційне моделювання, прийняття рішень, управління, аналіз, ринок,рівновага та стійкість, комп'ютерне моделювання, МАТЛАБ, СИМУЛИНК, системно - динамічний підхід.*

Истомин Л.ф. Имитационная модель анализа динамики рынка

Рассмотрена проблема моделирования и контроля стабильности рынка товаров. Исследования выполнены с учетом различных типов факторов, связанных с детерминированными характеристиками участников рынка и стохастическим характером экономических процессов. Базовая модель Вальраса получила новые элементы и широкие возможности для принятия во внимание таких факторов как конкурентное взаимодействие участников рынка и их реакции изменениями в их производственной деятельности. Результаты исследования выполнены с помощью разработанной автором программы в среде СИМУЛИНК пакета МАТЛАБ. Структура предлагаемой компьютерной модели имеют большую вариабельность по изменению технических характеристик и типа поведения для участников рынка. В результате работы выявлены существенные для стабильности рынка факторы и направления развития модели с целью интеграции в системно-динамическую модель региональной экономики с учетом производственных и социальных факторов.

Ключевые слова: *математическое моделирование, имитационное моделирование, принятие решений, управление, анализ, рынок,равновесие и устойчивость, компьютерное моделирование, МАТЛАБ, СИМУЛИНК, системно- динамический подход.*

Істомін Л.Ф. – к.т.н., доцент кафедри економіки та підприємництва, Інститут економіки і управління Східноукраїнського національного університету імені Володимира Даля. istominvl@mail.ru

Рецензент: д.е.н,професор ***Чернявська Є.І***.

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