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EQUIVALENT CIRCUIT OF PIEZOELECTRIC TRANSDUCER WITH HELMHOLTZ RESONATOR

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Abstract. *The work is devoted to actual problems of the improvement of piezoelectric electroacoustic transducers. The change of piezoelectric transducers characteristics is possible due to external circuits for piezoelectric element – electric, mechanical or acoustic ones. This article describes a piezoelectric transducer with Helmholtz resonator. The equivalent circuit of electroacoustic transducer with Helmholtz resonator is built. The use of the proposed equivalent circuit allows by means of application programs to assess characteristics, to predict the parameters and operation mode of piezoelectric electroacoustic transducers.*

Keywords: *piezoelectric transducer, Helmholtz resonator, amplitude-frequency response, equivalent circuit.*

ЭКВИВАЛЕНТНАЯ СХЕМА ПЬЕЗОЭЛЕКТРИЧЕСКОГО ПРЕОБРАЗОВАТЕЛЯ С РЕЗОНАТОРОМ ГЕЛЬМГОЛЬЦА

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Аннотация. *Работа посвящена актуальным вопросам совершенствования пьезоэлектрических электроакустических преобразователей. Изменение характеристик преобразователя возможно за счет внешних для пьезоэлемента цепей – электрических, механических или акустических. В статье рассмотрен пьезоэлектрический преобразователь с резонатором Гельмгольца. Построена эквивалентная схема электроакустического преобразователя с резонатором Гельмгольца. Использование предложенной эквивалентной схемы позволяет при помощи прикладных программ производить оценку характеристик, прогнозировать параметры и режим пьезоэлектрических преобразователей.*

Ключевые слова: *пьезоэлектрический преобразователь, резонатор Гельмгольца, амплитудно-частотная характеристика, эквивалентная схема.*

As is known, devices which convert an electric signal into an acoustic one are called as electroacoustic transducers (EAT). EAT are widely used for the work in air environment (security systems, measuring equipment), in water (radars, sonar, underwater communication), to create sound waves in solids (nondestructive control), etc. [1-8].

Piezoelectric transducers are widely used in electroacoustics.

In [9, 10] the synthesis technologies of piezoelectric transducers are described. These technologies allow you to create transducers with necessary characteristics.

Among the described technologies of special interest is the technology of additional elements, since in this case the change in characteristics of the transducer is due to external circuits of piezoelectric element. The essence of this technology is that additional oscillatory systems (electric, mechanical, electromechanical or acoustic systems) are attached to piezoelectric element [10]. Helmholtz resonator is an example of simple acoustic oscillating system.

Analytical expressions for amplitude-frequency characteristics (AFC) of such oscillatory systems are not available, so the definition of AFC is often carried out experimentally; it is not always convenient and increases the time of piezoelectric transducers designing.

So the **purpose** of this work is to develop an equivalent circuit of piezoelectric transducer with Helmholtz resonator. The use of such circuits allows with application programs to assess characteristics and to predict the parameters and modes of piezoelectric transducers.

Helmholtz resonator is a spherical cavity with an open neck. The air in the neck is oscillating mass and the volume of air in the cavity is an elastic element. Of course, such separation is only approximately true, because some of air in the cavity has an inertial resistance. However, for a sufficiently large value of ratio of the orifice area and the sectional area of the cavity accuracy of this approximation is quite satisfactory. Most of kinetic energy of oscillations is concentrated in the neck of resonator, where vibration particle velocity of air is the greatest [11].

Helmholtz resonator with volume V and neck length l and cross-section $S=\pi r^2$ is shown in Fig. 1 and its modified version – in Fig. 2.

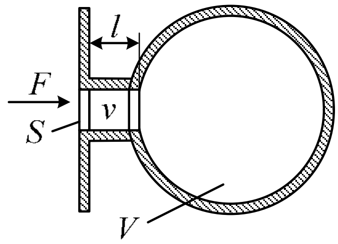


Fig. 1. Helmholtz resonator

Strictly speaking, the resonator is a system with distributed parameters. However, if the dimensions of the resonator are small compared with the wavelength of oscillations acting on the resonator, such system can practically be considered as a system with lumped parameters.

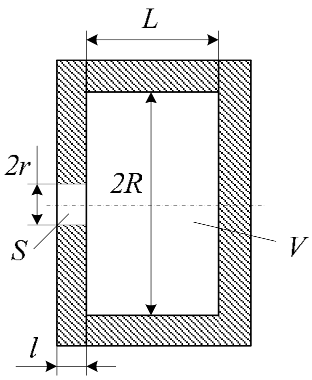


Fig. 2. Modified Helmholtz resonator

For using a well-designed apparatus of electrical quadripoles theory in analysis processes occurring in complex mechanical systems, the method of electromechanical analogies is used. It allows to transform mechanical systems to electric ones. By this method the pressure P is considered as an analog of the voltage, vibration velocity v – as an analog of current density and volume vibration velocity $V_a=vS$ (where S is acoustic line cross-section) – as an analog of the current [12].

In Fig. 1 the force $F=PS$, where S is orifice area, and the P is sound pressure. The force F is applied to the mass M_a, corresponding to the mass of air in the orifice. In the same orifice is also the resistance R_a (orifice wall friction, the viscosity of air, radiation, etc.) and therefore the force is applied also to this resistance. The air mass in the orifice is almost incompressible, so the force F fully influences the volume behind it, that is, the flexibility C_a of air volume V. So we have a contour consisting of M_a, R_a and C_a.

The equivalent circuit of Helmholtz resonator is shown in Fig. 3 [11].

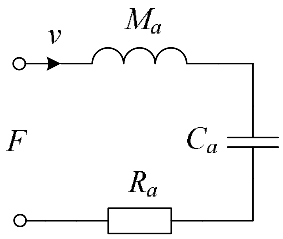


Fig. 3. Equivalent circuit of Helmholtz resonator

In Fig. 3 acoustic mass M_a is given by [13]:

$$M_a = \frac{\rho_0 (l+\Delta l)}{\pi r^2}, \tag{1}$$

where l and r are the length and the radius of the orifice; ρ_0 is the environment density; Δl – the correction, that takes into account entrained air near the neck of the resonator;

$$\Delta l = 0,85r \cdot \left(1 - 0,7\frac{r}{R}\right) + 0,85r, \tag{2}$$

where R is the radius of the cavity V.

C_a is the cavity acoustic compliance:

$$C_a = \frac{V}{\rho_0 c_0^2}, \tag{3}$$

where $V = \pi R^2 L$; L is the depth of the cavity; c_0 is the velocity of sound in environment.

Acoustic impedance of the cavity V is therefore:

$$Z_V = \frac{1}{j\omega C_a} = \frac{1}{j\omega\left(V/\rho_0 c_0^2\right)} = \frac{\rho_0 c_0^2}{j\omega\left(\pi R^2 L\right)}. \tag{4}$$

R_a is the neck resistance:

$$R_a = \frac{l}{r}\cdot\frac{\sqrt{2\mu\rho_0\omega}}{\pi r^2} + 2\frac{\sqrt{2\mu\rho_0\omega}}{\pi r^2} + \frac{\rho_0 c_0}{\pi r^2}\left[1 - \frac{2J_1(2kr)}{2kr}\right], \tag{5}$$

where μ is dynamic viscosity of air; $J_1(x)$ is Bessel function of the first kind. The neck resistance R_a includes several contributions. The first term represents viscous loss in the neck wall boundary layer, which is derived assuming the fluctuating flow through the neck due to acoustic excitation is hydro-dynamically incompressible viscous flow. The second term represents viscous loss at the neck ends (also refers to resistance end correction). The third term represents radiation loss at outer neck end. At low frequencies

$$R_a \approx \frac{\rho_0 c_0 k^2}{2\pi}. \tag{6}$$

The expression for acoustic impedance of Helmholtz resonator is thus:

$$Z_{RH} = R_{RH} + jX_{RH} \approx$$
$$\approx \frac{\rho_0 c_0 k^2}{2\pi} + j\omega\frac{\rho_0 (l+\Delta l)}{\pi r^2} + \frac{\rho_0 c_0^2}{j\omega V}, \tag{7}$$

where R_{RH} and X_{RH} are acoustic resistance and reactance of the resonator, respectively.

Resonant frequency of the resonator is the frequency at which X_{RH} approaches zero:

$$f_0 = \frac{c_0}{2\pi}\sqrt{\frac{\pi r^2}{(l+\Delta l)V}}. \tag{8}$$

At resonant frequency the pressure amplification, defined as the ratio of amplitude between the cavity pressure and harmonic incident pressure, reaches a maximum.

Acoustic compliance C_a increases with the volume V (3), meaning that the resonance frequency f_0 falls with increasing of cavity volume V. Acoustic mass M_a (inertance) in the duct increases with the length of the duct t and decreases with increasing of duct area $S=\pi r^2$ (1). Expression (8) shows that the resonance frequency f_0, is proportional to square root of the duct area πr^2 and inversely proportional to square root of the duct length l, so the resonance frequency f_0 falls with increasing of duct length l and increases with the duct area πr^2.

Equivalent circuit of piezoelectric transducer is shown in Fig. 4 [1-4, 13].

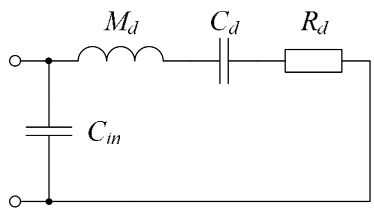


Fig. 4. Simplified equivalent circuit of piezoelectric transducer

In Fig. 4 C_{in} is piezoelectric transducer input capacitance [13]; M_d is acoustic mass of piezoelectric bimorph diaphragm:

$$M_d = 2\pi\int_0^{R_2} \rho_A \left(\left.\frac{w(r)}{\Delta V}\right|_{U_\sim=0}\right)^2 r\,dr + \frac{8\rho_0}{3\pi^2 R_2}, \tag{9}$$

where ρ_A is the surface density of piezoelectric bimorph plate;

C_d is piezoelectric bimorph plate acoustic compliance:

$$C_d = \frac{\int_0^{R_2} \left.w(r)\right|_{U_\sim=0} 2\pi r\,dr}{P}. \tag{10}$$

R_d is piezoelectric bimorph diaphragm resistance:

$$R_d = 2z\sqrt{\frac{M_{ä}}{C_{ä}}}\,, \qquad (11)$$

where ζ is damping factor.

Piezoelectric transducer with Helmholtz resonator is shown in Fig. 5.

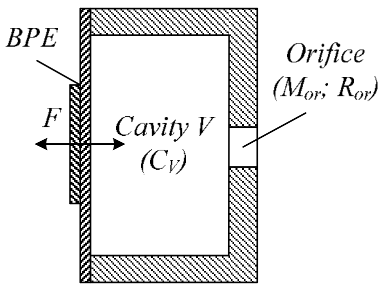


Fig. 5. Piezoelectric transducer with Helmholtz resonator

Sound pressure which is created in the volume above the diaphragm in the form of bimorph piezoelectric element (BPE), which is the circuit of M_d, C_d and R_d, must overcome the resistance of air volume above the diaphragm C_V. So its elasticity $s_V=1/C_V$ is added to the elasticity of the diaphragm $s_d=1/C_d$. Sound pressure which is created in this volume, acts the mass, located in the orifice M_{or}. Additionally, this orifice represents the resistance R_{or}.

The synthesis of piezoelectric transducer with Helmholtz resonator equivalent circuit can be made by Belov's method [11]. According to this method it is necessary to build contour for each unit combining them by similar elements (Fig. 6).

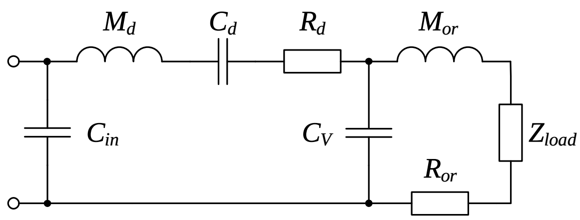
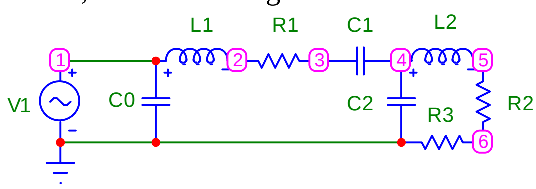
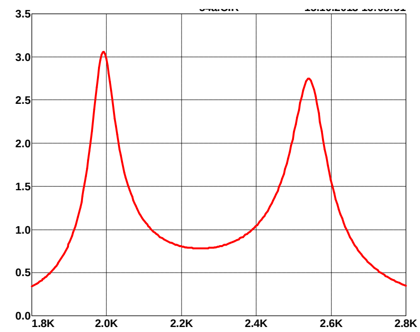


Fig. 6. Equivalent circuit of piezoelectric transducer with Helmholtz resonator

The investigation of equivalent circuit of piezoelectric transducer with Helmholtz resonator is conducted by application of Micro-Cap software, as shown in Fig. 7.



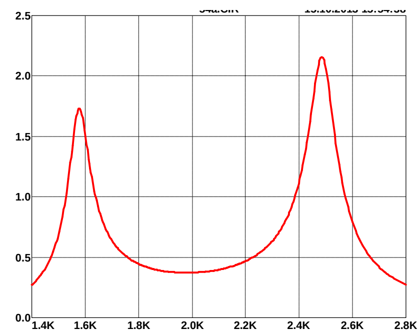
a)



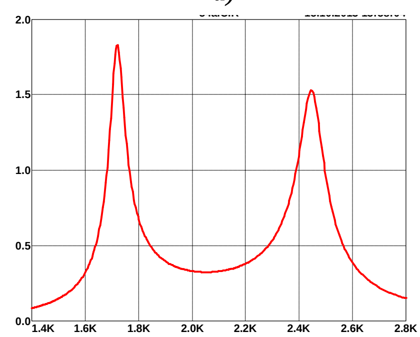
b)

Fig. 7. Schematic model of piezoelectric transducer with Helmholtz resonator (*a*) and simulation results in Micro-Cap program (*b*)

Fig. 8 shows equivalent circuit AFC with increasing of acoustic mass M_a, acoustic compliance C_a and orifice resistance R_a.



a)



b)

Fig. 8. AFC of piezoelectric transducer with Helmholtz resonator with increasing of acoustic mass M_a (*a*) and acoustic compliance C_a (*b*) in Micro-Cap program

As can be seen from Fig. 7, 8, the resonance frequency of piezoelectric transducer with Helmholtz resonator falls with the increasing of acoustic mass M_a (inertance). It also falls with the increasing of acoustic compliance C_a. The orifice resistance R_a affects the transducer quality factor.

Fig. 9 shows the AFC of piezoelectric transducer with Helmholtz resonator based on bimorph element from ZP-19.

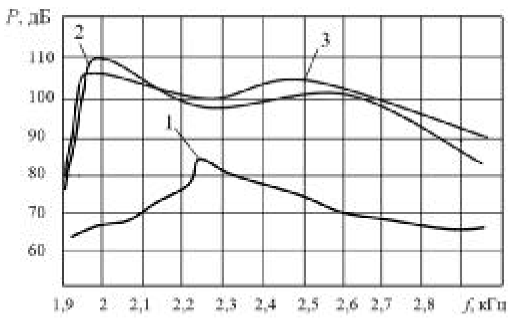


Fig. 9. AFC of piezoelectric transducer:
1 – without Helmholtz resonator,
2 – with Helmholtz resonator (V = 8 cm^3);
3 – with Helmholtz resonator (V = 16 cm^3)

Fig. 9 shows that the resonance frequency of piezoelectric transducer with Helmholtz resonator falls with the increasing of the volume V, and hence with the increasing of acoustic compliance C_a.

As can be seen from Fig. 7, 9, the behavior of amplitude-frequency characteristics of experimental sample and equivalent circuit is practically coinciding.

Conclusions:

1. Equivalent circuit of piezoelectric transducer with Helmholtz resonator is built.

2. Proposed equivalent circuit allows with application programs to assess characteristics, to predict the parameters and operation mode of piezoelectric transducers.

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