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THE RESEARCH OF PARAMETERS OF DYNAMIC STRESS-STRAINED STATE OF ASYMMETRIC BIMORPH PIEZOCERAMIC ELEMENTS

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Abstract. *The work is devoted to actual problems of the study of dynamic stress-strained state of asymmetric bimorph piezoceramic elements in radiation mode. The results of the calculation of resonant frequencies and deflections of asymmetric bimorph piezoceramic elements in radiation mode of acoustic waves by different methods of fixing of outer contour of metal disk (rigid fixing, pinning and free fixing of outer contour of metal disk) are discussed in the article. The data presented in the work can be used for designing of bimorph acoustic waves radiators.*

Keywords: *piezoceramics, transfer characteristic, bimorph element, asymmetric piezoelements, acoustic waves, fluctuations equation.*

ИССЛЕДОВАНИЕ ПАРАМЕТРОВ ДИНАМИЧЕСКОГО НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ АСИММЕТРИЧНЫХ БИМОРФНЫХ ПЬЕЗОКЕРАМИЧЕСКИХ ЭЛЕМЕНТОВ

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Аннотация. *Работа посвящена актуальным вопросам изучения динамического напряженно-деформированного состояния асимметричных биморфных пьезокерамических элементов в режиме излучения. В статье обсуждаются результаты расчета резонансных частот и прогибов асимметричных биморфных пьезокерамических элементов в режиме излучения акустических волн при различных способах закрепления внешнего контура металлического диска (жестком защемлении, шарнирном закреплении и свободном закреплении внешнего контура металлического диска). Представленные в работе данные могут быть использованы при проектировании биморфных излучателей акустических волн.*

Ключевые слова: *пьезокерамика, передаточная характеристика, биморфный элемент, ассимметричные пьезоэлементы, акустические волны, уравнение колебаний.*

Asymmetric bimorph piezoelectric elements are widely used in electroacoustic and hydroacoustic transducers [1–4].

In [5] the basic relations for the calculation of the parameters and characteristics of dynamic stress-strained state of asymmetric bimorph

piezoelectric element are obtained. The calculation scheme of asymmetric bimorph piezoelectric element is shown in Fig. 1.

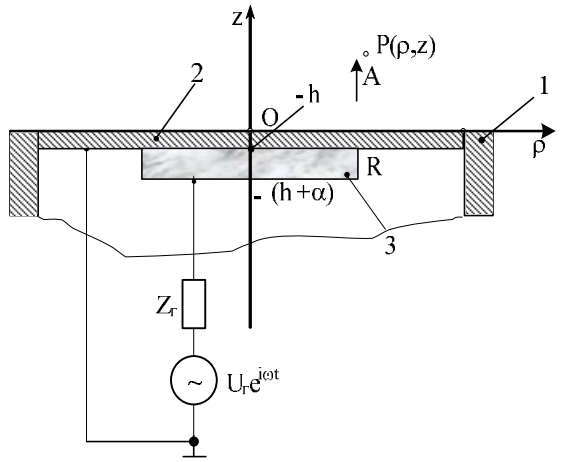


Fig. 1. The calculation scheme of asymmetric bimorph piezoelectric element in radiation mode of acoustic waves

Reference numeral 1 in Fig. 1 designates a housing element of bimorph radiator of acoustic waves. Reference numeral 2 designates a metal disc, which is glued with the disk through the thickness of polarized piezoelectric element by conductive high-molecular adhesive (position 3). When applying to the surfaces with electrodes $z = -h$ and $z = -(h + a)$ (h and a - thickness of metal and piezoceramic disks respectively) potential differences $U_{\tilde{a}} e^{iwt}$, where $U_{\tilde{a}}$ – amplitude; $i = \sqrt{-1}$; w – angular frequency of the change in the sign of potential difference; t – time, harmonic oscillations of transverse bending arise in the construction. As follows from (43) in [5], this type of stress-strained state arises under the condition that

$$\frac{h}{a} > \sqrt{\frac{c_{11}^D}{E}\left(1 + k^*\right)\left(1 - n\right)},$$

where c_{11}^D - elastic modulus of piezoceramics, qualified by concerted action of Coulomb forces and elastic forces; $k^* = c_{12}^D / c_{11}^D$; c_{12}^D - elastic modulus of piezoceramics; E and n - Young's modulus and Poisson's ratio of metal disk. Fig. 2 shows a plot of the dependence of the ratio h/a of elastic modulus ratios c_{11}^D / E. The calculation was performed at typical values $K^* = 0,585$ and $n = 0,3$.

In [5] it is shown that at the recorded above inequality the amplitude of harmonically time-varying deflections $w(r)$ of active zone $0 \le r \le R$ of bimorph radiator is given by

$$w(r) = C_1 J_0(l_0 r) + C_2 I_0(l_0 r), \quad (1)$$

where C_1 and C_2 - constants to be determined; $J_0(l_0 r)$ and $I_0(l_0 r)$ - Bessel function and modified Bessel function, respectively; l_0 - wave number of active zone bending vibrations of an asymmetric bimorph element (see comment to (30) in [5]).

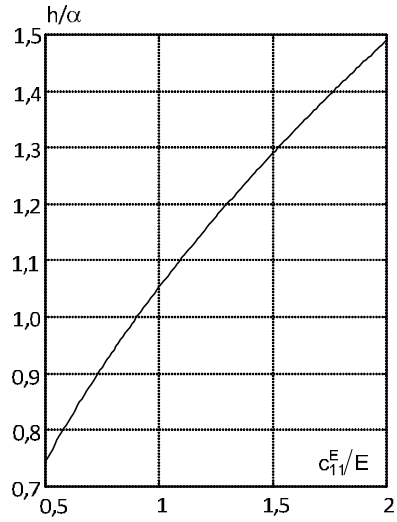


Fig. 2. The minimum values of relative thickness of metal disk for different Young's modulus

In passive zone of bimorph transducer, i.e. in the region $R \le r \le R_1$, amplitudes of metal disk deflections are determined as follows:

$$\begin{aligned} w_1(r) = {} & C_3 J_0(l_1 r) + C_4 N_0(l_1 r) + \\ & + C_5 I_0(l_1 r) + C_6 K_0(l_1 r) \end{aligned}, \quad (2)$$

where C_3, …, C_6 – unknown constants to be determined; $J_0(x)$, $N_0(x)$, $I_0(x)$ and $K_0(x)$ – Bessel function, Neumann, modified Bessel function and Macdonald function of zero order; $x = l_1 r$; $l_1 = \sqrt[4]{h r_j w^2 / D_j (h/2)}$ - wave number of bending vibrations of metal disk (r_M – density of metal; w – angular frequency; $D_j (h/2) = Eh^3 / \left[12(1 - n^2)\right]$ – flexural rigidity of metal plate, calculated on the assumption that neutral plane divides in half the thickness of the plate).

Deflections $w(r)$ and $w_1(r)$ must satisfy kinematic and dynamic coupling on the contour $r = R$, that separates active and passive zones of bimorph piezoelectric element. These conditions can be written as follows:

$$w(r)-w_1(r)=0\forall r=R,$$
$$\frac{\partial w(r)}{\partial r}-\frac{\partial w_1(r)}{\partial r}=0\forall r=R, \qquad (3)$$
$$M_r-M_r^{(1)}=0\forall r=R,$$
$$Q-Q^{(1)}=0\forall r=R,$$

where M_r and $M_r^{(1)}$ – linear density of bending moments in active and passive zones of bimorph element, respectively, determined relative to the same curvilinear axis; Q and $Q^{(1)}$ – linear density of transverse forces in active and passive areas.

To uniquely identify six constants $C_1, \dots, C_4$, four (3) conjugation conditions of solutions are not enough. Missing two conditions are written in the contour $r=R_1$ and identified by the way of outer contour of metal disk fixing.

The purpose of this paper is to calculate resonant frequencies and deflections of asymmetric bimorph elements with rigidly fixed, pinned and free fixed outer contour $r=R_1$ of metal disk.

1. The parameters of dynamic stress-strained state of asymmetric bimorph element with rigidly fixed outer contour of metal disk.

When outer contour of metal disc is rigidly fixed, a cylindrical cross section $r=R_1$ is no capable of moving up and down and cannot rotate relatively a neutral plane. Formally, these physically obvious conditions are written as follows:

$$w_1(R_1)=0,\ \left.\frac{\partial w_1(r)}{\partial r}\right|_{r=R_1}=0. \qquad (4)$$

Coupling conditions (3) and boundary conditions (4) form inhomogeneous system of algebraic equations where six equations contain six unknown constants $C_1, \dots, C_6$. This system of equations is solved for the constants in a unique way.

Writing explicitly the relations (3) and (4), we obtain the following notation for the system of equations, mentioned above:

$$m_{ab}C_b=n_a,\ a,b=1,2,...,6, \qquad (5)$$

where $m_{11}=J_0(lR)$; $m_{12}=I_0(lR)$;

$$m_{13}=-J_0(l_1R);\ m_{14}=-N_0(l_1R);$$
$$m_{15}=-I_0(l_1R);\ m_{16}=-K_0(l_1R);$$
$$m_{21}=-J_1(lR);\ m_{22}=I_1(lR);\ m_{23}=\frac{l_1}{l}J_1(l_1R);$$
$$m_{24}=\frac{l_1}{l}N_1(l_1R);\ m_{25}=-\frac{l_1}{l}I_1(l_1R);$$
$$m_{26}=\frac{l_1}{l}K_1(l_1R);\ m_{31}=-J_0(lR)+\frac{1-x}{lR}J_1(lR);$$
$$m_{32}=I_0(lR)-\frac{1-x}{lR}I_1(lR);$$
$$x=\left(nD_i+k^*D_{i\dot{y}}+D_0\right)/D_1;$$
$$m_{33}=-z_0\left[-J_0(l_1R)+\frac{1-n}{l_1R}J_1(l_1R)\right];$$

$z_0=\dfrac{l_1^2D_i(z_0)}{l^2D_1}$; $D_i(z_0)$ – flexural rigidity of metal disk converted relatively the coordinate z_0 of active zone neutral layer of bimorph element;

$$m_{34}=-z_0\left[-N_0(l_1R)+\frac{1-n}{l_1R}N_1(l_1R)\right];$$
$$m_{35}=-z_0\left[I_0(l_1R)-\frac{1-n}{l_1R}I_1(l_1R)\right];$$
$$m_{36}=-z_0\left[K_0(l_1R)+\frac{1-n}{l_1R}K_1(l_1R)\right];$$
$$m_{41}=J_1(lR);\ m_{42}=I_1(lR);\ m_{43}=-z\,J_1(l_1R);$$
$$z=\frac{l_1^3D_i(z_0)}{l^3D_1};\ m_{44}=-z\,N_1(l_1R);$$
$$m_{45}=-z\,I_1(l_1R);\ m_{46}=z\,K_1(l_1R);\ m_{51}=0;$$
$$m_{52}=0;\ m_{53}=J_0(l_1R_1);\ m_{54}=N_0(l_1R_1);$$
$$m_{55}=I_0(l_1R_1);\ m_{56}=K_0(l_1R_1);\ m_{61}=m_{62}=0;$$
$$m_{63}=-J_1(l_1R_1);\ m_{64}=-N_1(l_1R_1);$$
$$m_{65}=I_1(l_1R_1);\ m_{66}=-K_1(l_1R_1);$$
$$n_1=n_2=n_4=n_5=n_6=0;\ n_3=-U_0\frac{\hat{M}_0^*}{\lambda^2D_1};$$
$$\hat{M}_0^*=\frac{e_{31}\alpha}{2(1+\xi)}\left[\xi^2+\xi+1-\frac{c_{11}^D}{E}(1-\nu)(1+k^*)\right];$$
$$x=h/a.$$

Table 1 shows numerical values of dimensionless frequencies of the first two resonances of piezoelectric bimorph element. Values $(wt_0)_1$ and $(wt_0)_2$ ($t_0=R/v^D$ – time scale; $v^D=\sqrt{c_{11}^D/r_0}$; r_0 – density of piezoceramics) are determined by the first two local minimums of the main determinant of the system of equations (5).

Table 1

Dimensionless frequencies of the first two resonances of asymmetric bimorph element with rigidly fixed contour of passive layer

R_1/R	$h/a = 0,8$			R_1/R	$h/a = 1,2$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,601711	2,342514	3,893089	1,00	0,739316	2,878225	3,898361
1,05	0,508830	2,012930	3,955998	1,05	0,648763	2,541812	3,917936
1,10	0,445701	1,808284	4,057168	1,10	0,579418	2,294497	3,960003
1,15	0,399004	1,645419	4,123815	1,15	0,523626	2,090080	3,991551
1,20	0,361710	1,497917	4,141211	1,20	0,477109	1,908173	3,999448
1,25	0,330536	1,356968	4,105356	1,25	0,437298	1,740066	3,979131
1,30	0,303653	1,222204	4,025001	1,30	0,402561	1,583352	3,933197
1,35	0,279968	1,097080	3,918591	1,35	0,371817	1,438777	3,869584
1,40	0,258795	0,985911	3,806143	1,40	0,344322	1,307838	3,798300
1,45	0,239685	0,887414	3,702416	1,45	0,319546	1,191333	3,728205
1,50	0,222329	0,803795	3,615341	1,50	0,297099	1,088978	3,665372
1,55	0,206506	0,732610	3,547644	1,55	0,276683	0,999672	3,613057
1,60	0,192051	0,671982	3,498977	1,60	0,258064	0,921903	3,572383
1,65	0,178830	0,620106	3,467570	1,65	0,241050	0,854077	3,543151
1,70	0,166730	0,575401	3,451096	1,70	0,225479	0,794688	3,524444
1,75	0,155653	0,536554	3,447119	1,75	0,211210	0,742409	3,515029
1,80	0,145509	0,502497	3,453374	1,80	0,198122	0,696106	3,513524
1,85	0,136216	0,472371	3,467810	1,85	0,186104	0,654831	3,518631
1,90	0,127698	0,445492	3,488635	1,90	0,175057	0,617799	3,529130
1,95	0,119886	0,421312	3,513273	1,95	0,164892	0,584364	3,543917
2,00	0,112715	0,399396	3,543412	2,00	0,155528	0,553995	3,562027

In the calculation of dimensionless frequencies material constants of bimorph element components were recorded. Thus, the parameters of a piezoelectric correspond to the basic parameters of piezoceramics PZT–19 ($c_{11}^E = 112,2\,GPa$; $c_{12}^E = 62,2\,GPa$; $e_{31} = -8,4\,C/m^2$; $c_3^e = 8,274\cdot10^{-9}\,F/m$; $r_0 = 7,4\cdot10^3\,kg/m^3$; $v^D = 3918\,m/s$; quality factor of a piezoelectric $Q = 80$). For metal layer following constants have been recorded: $E = 210GPa$, $n = 0,31$, that approximately corresponds to average index of steel alloyed with manganese or chromium or both elements together; density of the metal $r_i = 7,9\cdot10^3\,kg/m^3$. Such types of steel are used in instrument making for various extensions and suspensions, including such that work on the bend. While doing calculations of dimensionless frequency of the resonances, dimensionless wave number $l\,R$ was calculated using the following formula:

$$l\,R = \sqrt[4]{\frac{3(wt_0c)^2 d^2\left(1+\frac{r_i h}{r_0 a}\right) - i(wt_0c)d^3\left(x_c^{(+)}+x_c^{(-)}\right)}{\left[\left(1+\frac{z_0}{a}\right)^3-\left(\frac{z_0}{a}\right)^3\right]\left(1+d_{i\,ïý}-d_{0ïý}\right)}}, \quad (6)$$

where $c = 1 - i/(2Q)$; Q – quality factor of piezoceramics; $d = R/a$ – geometric parameter of piezoceramic disk; $x_c^{(\pm)} = Z_c^{(\pm)}/(r_0 v^D)$ – relative acoustic impedance of front and rear environment or acoustic load; $d_{i\,ïý} = D_i/D_{ïý}$; $d_{0ïý} = D_0/D_{ïý}$ – relative rigidities.

As follows from the data in Table 1, which are determined for $x_c^{(+)} = x_c^{(-)} = 0$, $Q \to \infty$ and $d = 10$, i.e. the absence of loss in the materials of bimorph element and the absence of radiation losses to the environment, changes in relative thickness (parameter h/a in Table 1) and relative radius (parameter R_1/R) quite significantly affect the position of the first electro-mechanical resonance of bimorph piezoelectric element on frequency axis (parameter $(wt_0)_1$). At the same time the value of

dimensionless frequency of the second electromechanical resonance $(wt_0)_2$ changes even more. The numerical values of the first and second electro-mechanical resonances go smoothly to lower frequencies with the growth of relative radius. This can be explained by the fact that the increasing of metal layer size is a reduction of integral bending rigidity of bimorph element. Thus, by varying the thickness and radius of passive metal layer of bimorph piezoelectric element, its frequency dependent characteristics can be controlled in a wide range, i.e. transfer coefficient from an electric side to mechanical output and electrical impedance value, that is very important quantity in designing real devices, can also be controlled. Electrical impedance of the radiator almost completely determines the direction of designing of output stage amplifier, which generates an electrical impulse that excites transverse vibrations of a piezoelectric bimorph element.

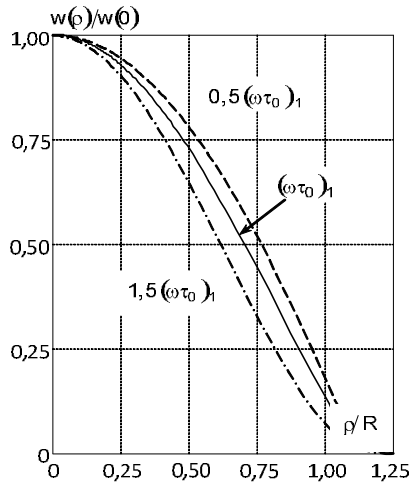
Table 1 (continued)

Dimensionless frequencies of the first two resonances of asymmetric bimorph element with rigidly fixed contour of passive layer

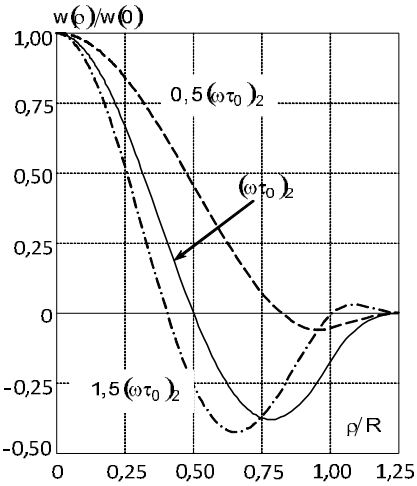
R_1/R	$h/a = 1,6$			R_1/R	$h/a = 2,0$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,880453	3,427696	3,893091	1,00	1,024496	3,988456	3,893091
1,05	0,785053	3,065262	3,904483	1,05	0,921042	3,590754	3,898578
1,10	0,707724	2,777459	3,924494	1,10	0,834663	3,261749	3,907864
1,15	0,643196	2,533176	3,938420	1,15	0,761125	2,977802	3,912369
1,20	0,588133	2,315931	3,937768	1,20	0,697528	2,724896	3,906504
1,25	0,540323	2,117634	3,919238	1,25	0,641827	2,495442	3,888029
1,30	0,498248	1,935270	3,884150	1,30	0,592540	2,285979	3,857282
1,35	0,460830	1,768324	3,837259	1,35	0,548572	2,095296	3,819547
1,40	0,427298	1,617178	3,784662	1,40	0,509096	1,922964	3,777214
1,45	0,397057	1,481906	3,732224	1,45	0,473470	1,768449	3,735082
1,50	0,369661	1,361913	3,684222	1,50	0,441189	1,630776	3,696321
1,55	0,344756	1,256030	3,643243	1,55	0,411844	1,508565	3,662954
1,60	0,322052	1,162765	3,610487	1,60	0,385098	1,400207	3,635976
1,65	0,301311	1,080537	3,586120	1,65	0,360666	1,304041	3,615646
1,70	0,282328	1,007834	3,569729	1,70	0,338306	1,218480	3,601711
1,75	0,264929	0,943287	3,560649	1,75	0,317807	1,142085	3,593642
1,80	0,248958	0,885701	3,557833	1,80	0,298985	1,073584	3,590763
1,85	0,234280	0,834057	3,560084	1,85	0,281676	1,011882	3,592363
1,90	0,220772	0,787492	3,566993	1,90	0,265738	0,956045	3,597696
1,95	0,208324	0,745287	3,577537	1,95	0,251039	0,905279	3,606130
2,00	0,196840	0,706839	3,590020	2,00	0,237466	0,858918	3,617016

Table 1 shows the ratio of dimensionless frequencies of electromechanical resonances. This parameter allows to determine relative sensitivity of these frequencies to the changes of bimorph radiator design parameters.

Fig. 3 shows the change of deflections along the radius of bimorph piezoelectric element composed of a piezoceramics PZT-19 and the passive layer of manganese steel. Geometric parameters of the element are: $h/a = 1,2$, $R_1/R = 1,25$, $d = R/a = 10$. Piezoelectric quality factor is $Q = 100$. Energy losses in metal disk are not taken into account. Potential difference between the surfaces with electrodes is $U_0 = 1V$. Contour $r = R_1$ is rigidly fixed. All the curves in Fig. 3 are normalized by the value of deflections in the center of bimorph piezoelectric element active zone. Frequency-dependent change of the absolute value of deflections at $r = 0$ for different values of R_1/R and for fixed parameter $h/a = 1,2$ is shown in Fig. 4.



a)



b)

Fig. 3. Deflections diagrams of asymmetric bimorph piezoceramic element with rigidly fixed contour of metal disk

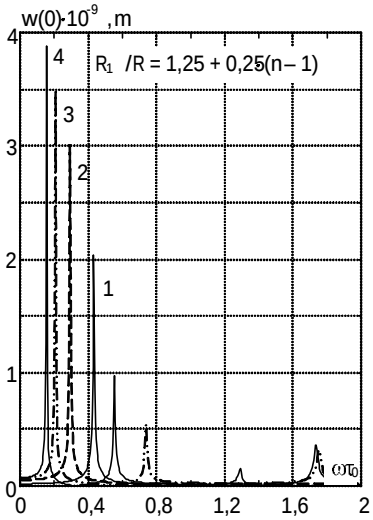


Fig. 4. Frequency-dependent change of the deflection at the center of bimorph element active zone with rigidly fixed contour of metal disk

Fig. 3, *a* shows deflections distribution in the first electromechanical resonance frequency and frequencies smaller and larger than the frequency of the first resonance. The values of these frequencies are determined by the numbers affixed near corresponding curves in the figure. It is clearly seen that single nodal line where deflection is zero, is a circle $r = R_1$, i.e. rigidly fixed contour of metal disk. At frequencies less than or equal to the second resonance frequency (Fig. 3, *b*), there is the second nodal line, that displaces to the center of bimorph element with the frequency increasing. At the frequency of $1{,}5(wt_0)_2$ there is the third nodal line. Direct calculations show that the number of nodal lines is determined by the number of electromechanical resonance.

2. The parameters of dynamic stress-strained state of asymmetric bimorph element with pinned outer contour of metal disk.

With pinned contour $r = R_1$ of metal disk this circuit must satisfy the following boundary conditions:

$$w_1(R_1) = 0\,,\ M_r^{(1)}\Big|_{r=R_1} = 0\,. \qquad (7)$$

This leads to the change of the coefficients $m_{63},...,m_{66}$ in the sixth equation of equations system (5). The values of these coefficients are of the form:

$$m_{63} = -J_0(l_1R_1) + \frac{1-n}{l_1R_1}J_1(l_1R_1)\,;$$

$$m_{64} = -N_0(l_1R_1) + \frac{1-n}{l_1R_1}N_1(l_1R_1)\,;$$

$$m_{65} = I_0(l_1R_1) - \frac{1-n}{l_1R_1}I_1(l_1R_1)\,;$$

$$m_{66} = K_0(l_1R_1) + \frac{1-n}{l_1R_1}K_1(l_1R_1)\,.$$

Table 2 shows dimensionless frequencies calculation results of the first two electromechanical resonances of asymmetric bimorph element with pinned outer contour of metal disk.

The calculations were performed with the same values of material parameters of piezoceramic and metal disks, which were used in the calculations of frequencies and deflections of bimorph piezoceramic element with rigidly fixed contour of metal disk.

Comparing data between Table 1 and Table 2, it is easy to see that numerical values of electro-mechanical resonances dimensionless frequencies with pinned contour of metal disk are almost twice less than similar frequencies with rigidly fixed one. This is because an integral rigidity of bimorph element with pinned contour is less than an integral rigidity of oscillatory system with rigidly fixed one. The character of the change of numeric values of electromechanical resonances dimensionless frequencies of bimorph element with pinned contour is the same as in the case with rigidly fixed contour of metal disk. The increase of relative thickness of metal disk leads to significant differences in resonance frequencies for small values of R_1/R. These differences gradually decrease with increase of the radius of metal disk.

Table 2

Dimensionless frequencies of the first two resonances of asymmetric bimorph element with pinned contour of passive layer

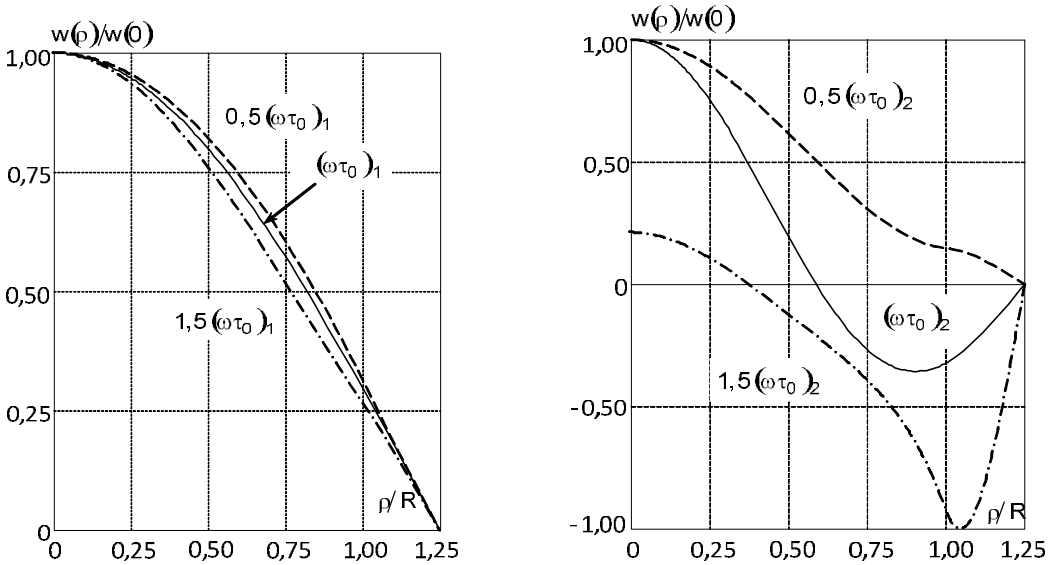
R_1/R	$h/a = 0,8$			R_1/R	$h/a = 1,2$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,301937	1,759537	5,872750	1,00	0,369326	2,160560	5,852008
1,05	0,272422	1,591354	5,841503	1,05	0,333984	1,956094	5,856851
1,10	0,246386	1,431164	5,808626	1,10	0,302951	1,767731	5,835040
1,15	0,223132	1,273853	5,708967	1,15	0,275417	1,590357	5,774359
1,20	0,202245	1,123855	5,556900	1,20	0,250847	1,425429	5,682463
1,25	0,183459	0,988835	5,389950	1,25	0,228859	1,276546	5,577870
1,30	0,166581	0,873381	5,242983	1,30	0,209162	1,146072	5,479349
1,35	0,151452	0,777557	5,134013	1,35	0,191512	1,034039	5,399345
1,40	0,137923	0,698824	5,066766	1,40	0,175696	0,938765	5,343118
1,45	0,125848	0,633966	5,037551	1,45	0,161522	0,857834	5,310943
1,50	0,115084	0,580030	5,040600	1,50	0,148814	0,788789	5,300503
1,55	0,105494	0,534608	5,067661	1,55	0,137412	0,729442	5,308431
1,60	0,096945	0,495828	5,114527	1,60	0,127170	0,677971	5,331219
1,65	0,089319	0,462267	5,175456	1,65	0,117958	0,632906	5,365523
1,70	0,082505	0,432847	5,246314	1,70	0,109657	0,593080	5,408503
1,75	0,076405	0,406754	5,323654	1,75	0,102163	0,557571	5,457661
1,80	0,070931	0,383366	5,404770	1,80	0,095384	0,525651	5,510894
1,85	0,066009	0,362208	5,487245	1,85	0,089238	0,496745	5,566515
1,90	0,061571	0,342913	5,569386	1,90	0,083654	0,470394	5,623087
1,95	0,057559	0,325196	5,649788	1,95	0,078569	0,446232	5,679496
2,00	0,053923	0,308835	5,727332	2,00	0,073928	0,423966	5,734849

Fig. 5 shows deflections diagrams $w(r)$ of bimorph element neutral layer with pinned outer contour of metal disk. Geometric parameters of the element are: $h/a = 1,2$, $R_1/R = 1,25$, $d = R/a = 10$. Piezoelectric quality factor is $Q = 100$. All the curves in Fig. 5, except one line in Fig. 5, *b*, that corresponds to the deflections of dimensionless frequency $1,5(wt_0)_2$, are normalized to deflection value $w(0)$ of bimorph element center. Fig. 6 shows the plots of frequency dependence of absolute deflections values $w(0)$ for bimorph element composed of a piezoelectric PZT-19 and manganese steel. Potential difference between the surfaces with electrodes of piezoceramic disk is $U_0 = 1V$. Specific differences that could be interpreted as a fundamental between the curves, which are shown in Fig. 3 – Fig. 6, in this series of calculations have not been established.

Table 2 (continued)

Dimensionless frequencies of the first two resonances of asymmetric bimorph element with pinned contour of passive layer

R_1/R	$h/a = 1,6$			R_1/R	$h/a = 2,0$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,438516	2,571946	5,865114	1,00	0,509056	2,991732	5,877077
1,05	0,397113	2,329825	5,866907	1,05	0,461404	2,710999	5,875542
1,10	0,360836	2,110311	5,848339	1,10	0,419705	2,458883	5,858598
1,15	0,328747	1,907837	5,803360	1,15	0,382891	2,229173	5,821951
1,20	0,300200	1,722346	5,737329	1,20	0,350201	2,020677	5,770050
1,25	0,274716	1,555500	5,662213	1,25	0,321062	1,833773	5,711585
1,30	0,251918	1,408270	5,590193	1,30	0,295019	1,668430	5,655329
1,35	0,231496	1,280140	5,529859	1,35	0,271697	1,523580	5,607643
1,40	0,213182	1,169418	5,485536	1,40	0,250774	1,397310	5,571988
1,45	0,196740	1,073854	5,458237	1,45	0,231973	1,287307	5,549382
1,50	0,181962	0,991130	5,446909	1,50	0,215051	1,191230	5,539290
1,55	0,168660	0,919128	5,449591	1,55	0,199793	1,106926	5,540366
1,60	0,156669	0,856024	5,463901	1,60	0,186009	1,032523	5,550929
1,65	0,145840	0,800304	5,487549	1,65	0,173534	0,966438	5,569157
1,70	0,136043	0,750734	5,518356	1,70	0,162220	0,907363	5,593409
1,75	0,127161	0,706313	5,554476	1,75	0,151938	0,854221	5,622169
1,80	0,119093	0,666235	5,594240	1,80	0,142574	0,806133	5,654137
1,85	0,111748	0,629849	5,636334	1,85	0,134028	0,762378	5,688197
1,90	0,105047	0,596627	5,679621	1,90	0,126213	0,722365	5,723379
1,95	0,098921	0,566141	5,723163	1,95	0,119050	0,685608	5,758993
2,00	0,093309	0,538039	5,766209	2,00	0,112473	0,651705	5,794325



a) *b)*

Fig. 5. Deflections diagrams of asymmetric bimorph piezoceramic element with pinned contour of metal disk

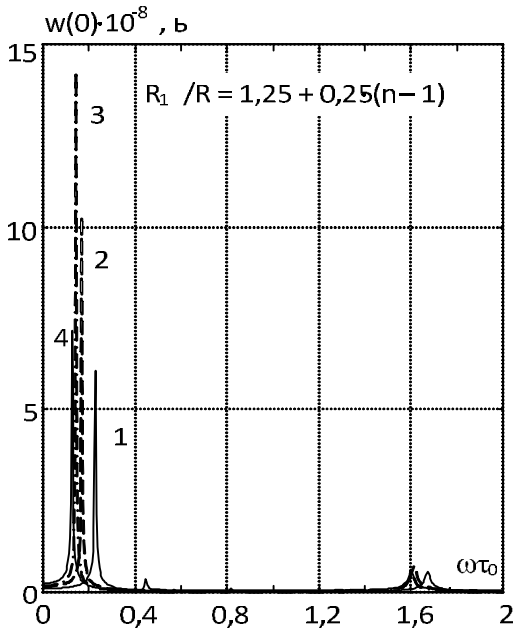


Fig. 6. Frequency-dependent change of the deflection at the center of bimorph element active zone with pinned contour of metal disk

3. The parameters of dynamic stress-strained state of asymmetric bimorph element with free fixed outer contour of metal disk.

With absolutely soft free fixed outer contour of metal disk the following conditions apply on the contour $r = R_1$:

$$M_r^{(1)}\Big|_{r=R_1} = 0,\ Q^{(1)}\Big|_{r=R_1} = 0. \qquad (8)$$

In this case the coefficients in the fifth and sixth equations of the system (5) change. These coefficients are determined as follows:

$$m_{51} = m_{52} = 0;$$

$$m_{53} = -J_0(l_1R_1) + \frac{1-n}{l_1R_1}J_1(l_1R_1);$$

$$m_{54} = -N_0(l_1R_1) + \frac{1-n}{l_1R_1}N_1(l_1R_1);$$

$$m_{55} = I_0(l_1R_1) - \frac{1-n}{l_1R_1}I_1(l_1R_1);$$

$$m_{56} = K_0(l_1R_1) + \frac{1-n}{l_1R_1}K_1(l_1R_1);$$

$$m_{61} = m_{62} = 0;$$

$$m_{63} = J_1(l_1R_1);\ m_{64} = N_1(l_1R_1);$$

$$m_{65} = I_1(l_1R_1);\ m_{66} = -K_1(l_1R_1).$$

Table 3 shows dimensionless frequencies calculation results of the first two electromechanical resonances of asymmetric bimorph element with the same set of material constants that are used in two previous series of calculations.

As a particular feature, you can specify relatively high values of dimensionless frequencies of the first electromechanical resonance.

The reason for this is clarified by diagrams of the deflections of bimorph element neutral layer, as shown in Fig. 7. Geometric parameters of the element are: $h/a = 1,2$, $R_1/R = 1,25$, $d = R/a = 10$. Piezoelectric quality factor is $Q = 100$. As before, mechanical vibrations energy losses in metal disk are not taken into account.

All the curves in Fig. 7 are normalized to deflection value $w(0)$ of bimorph element center. Fig. 8 shows the plots of frequency dependence of absolute deflections values $w(0)$ for bimorph element composed of a piezoelectric PZT-19 and manganese steel. Potential difference between the surfaces with electrodes of piezoceramic disk is $U_0 = 1V$. As shown in Fig. 7, the first nodal circle, neutral plane deflections of which are zero, is located at low frequencies $wt_0 < (wt_0)_1$ near the boundary of piezoceramic disk. By increasing dimensionless frequency this nodal circle moves to bimorph element center and after passing the second electromechanical resonance frequency the second nodal circle appears. Such behavior of oscillating system is explained by the law of conservation of momentum. From Fig. 6 and Fig. 8 it follows that bimorph elements with pinned and free fixed outer contour of metal disk have good mobility – displacements amplitudes of bimorph element center reach the fractions of a micrometer feeding on the surface with electrodes of piezoelectric disk just one volt of electrical potential difference. The latter allows recommending these types of metal disk fixing in the manufacture of high-performance acoustic wave radiators.

Table 3

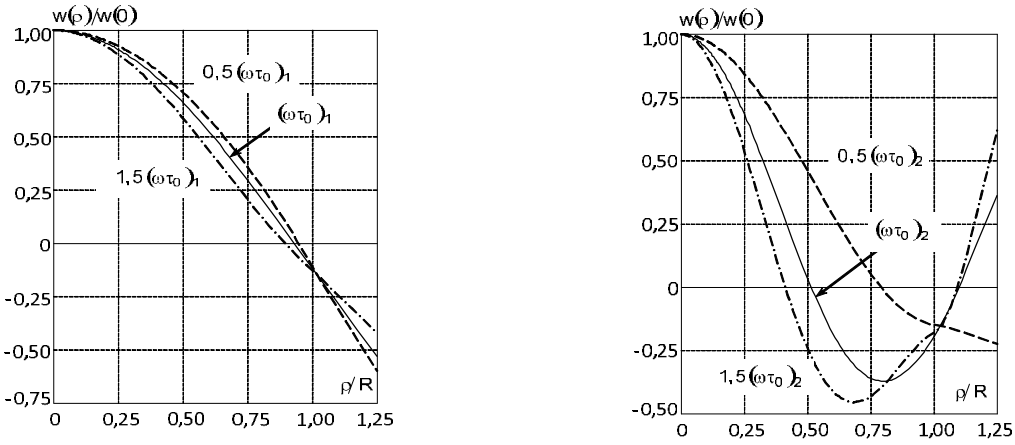
Dimensionless frequencies of the first two resonances of asymmetric bimorph element with free fixed contour of passive layer

R_1/R	$h/a = 0,8$			R_1/R	$h/a = 1,2$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,547021	2,280901	4,169677	1,00	0,669674	2,800033	4,181189
1,05	0,463597	1,973839	4,257661	1,05	0,573634	2,438603	4,251148
1,10	0,407627	1,776826	4,358951	1,10	0,507072	2,198578	4,335831
1,15	0,365552	1,616007	4,420730	1,15	0,456250	2,004569	4,393576
1,20	0,331625	1,468070	4,426899	1,20	0,4155008	1,831654	4,413539
1,25	0,303002	1,325936	4,375999	1,25	0,380178	1,671100	4,395572
1,30	0,278126	1,190507	4,280460	1,30	0,349968	1,521145	4,346525
1,35	0,256081	1,065862	4,162207	1,35	0,323287	1,382940	4,277748
1,40	0,236300	0,955336	4,042896	1,40	0,299433	1,258041	4,201410
1,45	0,218411	0,859927	3,937195	1,45	0,277930	1,147128	4,127398
1,50	0,202159	0,7778731	3,852074	1,50	0,258438	1,049800	4,062094
1,55	0,187359	0,709933	3,789161	1,55	0,240703	0,964902	4,008683
1,60	0,173864	0,651515	3,747268	1,60	0,224525	0,890937	3,968095
1,65	0,161555	0,601611	3,723877	1,65	0,209741	0,826356	3,939889
1,70	0,150324	0,558632	3,716185	1,70	0,196212	0,769725	3,922923
1,75	0,140076	0,521276	3,721382	1,75	0,183817	0,719783	3,915756
1,80	0,130722	0,488500	3,736935	1,80	0,172451	0,675462	3,916839
1,85	0,122181	0,459471	3,760578	1,85	0,162016	0,635872	3,924748
1,90	0,114377	0,433532	3,790376	1,90	0,152427	0,600278	3,938134
1,95	0,107241	0,410160	3,824652	1,95	0,143606	0,568076	3,955794
2,00	0,100708	0,388939	3,862045	2,00	0,135481	0,538770	3,976719

Table 3 (continued)

Dimensionless frequencies of the first two resonances of asymmetric bimorph element with free fixed contour of passive layer

R_1/R	$h/a = 1,6$			R_1/R	$h/a = 2,0$		
	$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$		$(\omega\tau_0)_1$	$(\omega\tau_0)_2$	$(\omega\tau_0)_2/(\omega\tau_0)_1$
1,00	0,795575	3,332609	4,188911	1,00	0,923956	3,876028	4,195035
1,05	0,687645	2,918864	4,244725	1,05	0,804120	3,409984	4,240640
1,10	0,610554	2,634558	4,315029	1,10	0,716432	3,080673	4,300022
1,15	0,550755	2,404403	4,365650	1,15	0,647507	2,812967	4,344303
1,20	0,501843	2,201932	4,387692	1,20	0,590728	2,578961	4,365733
1,25	0,460397	2,017086	4,381189	1,25	0,542448	2,367393	4,364276
1,30	0,424422	1,846691	4,351073	1,30	0,500485	2,174007	4,343799
1,35	0,392672	1,690459	4,305016	1,35	0,463448	1,993517	4,310120
1,40	0,364325	1,548836	4,251249	1,40	0,430397	1,837566	4,299467
1,45	0,338808	1,421919	4,196830	1,45	0,400670	1,693724	4,227289
1,50	0,315704	1,309151	4,146767	1,50	0,373777	1,565155	4,187404
1,55	0,294703	1,209439	4,103925	1,55	0,349341	1,450649	4,152531
1,60	0,275549	1,121403	4,069703	1,60	0,327066	1,348781	4,123880
1,65	0,258041	1,043587	4,044368	1,65	0,306705	1,258072	4,101894
1,70	0,242006	0,974602	4,027180	1,70	0,288054	1,177103	4,086397
1,75	0,227296	0,913191	4,017630	1,75	0,270937	1,104581	4,076892
1,80	0,213784	0,858260	4,014612	1,80	0,255201	1,039361	4,072713
1,85	0,201355	0,808870	4,017134	1,85	0,240712	0,980450	4,073124
1,90	0,189907	0,764231	4,024237	1,90	0,227353	0,927000	4,077360
1,95	0,179349	0,723679	4,035029	1,95	0,215016	0,878290	4,084764
2,00	0,169599	0,686658	4,048715	2,00	0,203609	0,833709	4,094657



a) *b)*

Fig. 7. Deflections diagrams of asymmetric bimorph piezoceramic element with free fixed contour of metal disk

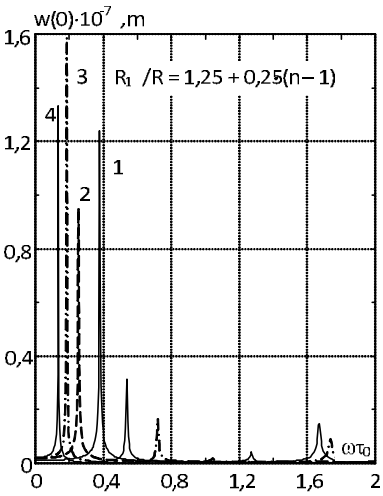


Fig. 8. Frequency-dependent change of the deflection at the center of bimorph element active zone with free fixed contour of metal disk

Conclusions: Presented in the work data can be used for designing of bimorph acoustic waves radiators.

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