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V. M. Sharapov[1], *D.Sc.(Engineering), professor,*
O. N. Petrishchev[2], *D.Sc.(Engineering), professor,*
Zh. V. Sotula[1], *Ph.D.,*
K. V. Bazilo[1], *Ph.D.*
[1]Cherkasy State Technological University
Shevchenko blvd., 460, Cherkasy, 18006, Ukraine
v_sharapov@rambler.ru
[2]National Technical University of Ukraine «Kyiv Polytechnic Institute»
Peremogy ave., 37, Kyiv, 03056, Ukraine

STIMULATION OF BENDING VIBRATIONS IN CYLINDRICAL PIEZOELEMENTS

The work is devoted to perfection of piezoelectric transducers. A special place is occupied by piezoelectric transducers in electroacoustics and hydroacoustics, where they are intended for radiation and reception of acoustic vibrations in air or aquatic environment. The general objective in improving of piezoelectric transducers is to increase the range of action. For transducers manufacture monomorphic and bimorph elements are used. The method of stimulation of bending vibrations in monomorphic cylindrical piezoelectric elements is described. Schemes of radially polarized cylindrical piezoelectric element to stimulate bending vibrations at different mutual arrangement of electric field vector of stimulating voltage and polarization vector are given. The creation of two oscillatory circuits in piezoelectric element circuit has allowed to increase the power of sound pressure level.

Keywords: *piezoelectric transducer, projector, bending vibrations, oscillatory circuit.*

Introduction. Piezoelectric transducers are widely used in electroacoustics, hydroacoustics, measuring technology, nondestructive control, piezomotors, scanners of nanomicroscopes, other fields of science and technics [1–4].

A special place is occupied by piezoelectric transducers in electroacoustics and hydroacoustics, where they are intended for radiation and reception of acoustic vibrations in air or aquatic environment [3; 4].

Piezoelectric transducers that are used in hydroacoustics are divided into two major classes:

- transducers-receivers of acoustic signal (sensors);
- transducers-projectors of acoustic signal.

A common task in improving the projectors is in increase of the range of action that can be achieved by:

- reducing of operating (resonant) frequency and (or)
- increasing of radiation power (increasing of sound pressure level).

It is known that low-frequency sound travels in water practically without attenuation at distances up to several thousand kilometers by forming in top layer of the ocean an underwater sound channel – acoustic waveguide of refractive type. Due to this low-frequency acoustics has obvious advantages in a wide range of problems [5; 6].

For making electroacoustic transducers monomorphic piezoelements, i.e. composed of a single piezoelectric element, and bimorph elements, composed of two piezoelectric elements or piezoelectric element and a metal plate connected by gluing or soldering, are used [2; 4].

Most frequently in electroacoustics and hydroacoustics asymmetric bimorph piezoelements (BPE), which have a relatively low resonant frequency and created high sound pressure level, are used, but they are more complex than monomorphic and also include an adhesive compound that reduces the mechanical strength of BPE [1–4].

Monomorphic piezoelements (MPE) have a relatively high resonant frequency, which in some cases (particularly in hydroacoustics) is a disadvantage. To reduce operating (resonant) frequency in MBE it is necessary to create bending vibrations.

Thus, **the purpose** of this work is the stimulation of low-frequency bending vibrations in cylindrical piezoelectric elements.

Let's consider the vibrations of cylinder generated by external harmonic loads [7]. As such next loads can act: an electric field induced by the action of potential difference applied to electrodes cylindrical surfaces $r_0 \pm h$, and (or) mechanical forces such as pressure of external environment on cylindrical surfaces.

According to the character of the loading and, consequently, the type of boundary conditions it is convenient to represent a resolving system of equations in a mixed form. As independent functions we choose such variables: u_z, σ_{rr}, σ_{rz}, u_r, φ, D_r. After simple transformations we can present the resolving system in such form [7]:

$$\frac{\partial u_z}{\partial r} = -\frac{\partial u_r}{\partial z} + \frac{1}{c_{55}^E}\left(\sigma_{rz} - e_{15}\frac{\partial \varphi}{\partial z}\right),$$

$$\frac{\partial \sigma_{rr}}{\partial r} = \frac{1}{r}\left(c_{12}^E + \frac{\Delta_2}{\Delta_1}\right)\frac{\partial u_z}{\partial z} - \frac{1}{r}\left(1 - \frac{\Delta_4}{\Delta_1}\right)\sigma_{rr} - $$

$$-\frac{\partial \sigma_{rz}}{\partial z} - \left[\rho\omega^2 - \frac{1}{r^2}\left(c_{11}^E + \frac{\Delta_2}{\Delta_1}\right)\right]u_r + \frac{1}{r}\frac{\Delta_3}{\Delta_1}D_r,$$

$$\frac{\partial \sigma_{rz}}{\partial r} = -\rho\omega^2 u_z - \left(c_{11}^E + \frac{\Delta_2}{\Delta_1}\right)\frac{\partial^2 u_r}{\partial z^2} - \frac{\Delta_4}{\Delta_1}\frac{\partial \sigma_{rr}}{\partial z} - $$

$$-\frac{1}{r}\sigma_{rz} - \frac{1}{r}\left(c_{12}^E + \frac{\Delta_2}{\Delta_1}\right)\frac{\partial u_r}{\partial z} - \frac{\Delta_3}{\Delta_1}\frac{\partial D_r}{\partial z},$$

$$\frac{\partial u_r}{\partial r} = \frac{1}{\Delta_1}\left(\Delta_4\frac{\partial u_z}{\partial z} + \varepsilon_{33}^S\sigma_{rr} - \frac{1}{r}\Delta_4 u_r + e_{33}D_r\right),$$

$$\frac{\partial \varphi}{\partial r} = \frac{1}{\Delta_1}\left(-\Delta_3\frac{\partial u_z}{\partial z} + e_{33}\sigma_{rr} - \frac{1}{r}\Delta_3 u_r - c_{33}^E D_r\right),$$

$$\frac{\partial D_r}{\partial r} = \frac{1}{c_{55}^E}\left(-e_{31}\frac{\partial \sigma_{rz}}{\partial z} + \Delta_5\frac{\partial^2 \varphi}{\partial z^2}\right) - \frac{1}{r}D_r. \quad (1)$$

Here we use the notation

$$\Delta_1 = e_{33}^2 + c_{33}^E\varepsilon_{33}^S;\ \Delta_2 = c_{33}^E e_{31}^2 - 2c_{13}^E e_{31}e_{33} - c_{13}^2\varepsilon_{33}^S;$$

$$\Delta_3 = c_{13}^E e_{33} - c_{33}^E e_{31};\ \Delta_4 = c_{13}^E\varepsilon_{33}^S + e_{31}e_{33};$$

$$\Delta_5 = e_{15}^2 + c_{55}^E\varepsilon_{11}^S.$$

We introduce a system of basis functions 1, *cos* α, *cos* 2 α, ..., *cos n*α, ... and *sin* α, *sin* 2 α, ..., *sin n*α, ... If we accept

$$\{u_z(r,z);\sigma_{rz}(r,z)\} = \sum_{n=0}^{\infty}\{u_z^{(n)}(r),\sigma_{rz}^{(n)}(r)\}\cos\chi_n z,$$

$$\{\sigma_{rr}(z,z);u_r(r,z);\varphi(r,z);D_r(r,z)\} =$$

$$= \sum_{n=0}^{\infty}\{\sigma_{rr}^{(n)}(r);u_r^{(n)}(r);D_r^{(n)}(r)\}\sin\chi_n z, \quad (2)$$

$$\chi_n = n\pi / l,$$

then boundary conditions

$$\sigma_{zz}(r)\ |_{z=0,l} = u_r(r)\ |_{z=0,l} = 0,\ \varphi(r)\ |_{z=0,l} = 0$$

$$D_z(r)\ |_{z=0,l} = 0$$

are satisfied exactly, and in the system (1) variables separation is possible. In order to formulate boundary conditions on cylindrical surfaces concerning the functions be finding $u_z^{(n)}(r)$, $\sigma_{rr}^{(n)}(r)$, $\sigma_{rz}^{(n)}(r)$, $u_r^{(n)}(r)$, $\varphi^{(n)}(r)$, $D_r^{(n)}(r)$, external force factors also need to be decomposed in basis functions. Thus, electric potential $\varphi(z)|_{r=ro\pm h} = \pm\frac{1}{2}V_0$ on lateral surfaces takes the form

$$\varphi(z)\,|_{r_0\pm h} = \pm 2V_0\sum_n \frac{1}{n\pi}\sin\frac{n\pi}{l}z,\ \text{n=1,3,5. …}(3)$$

Thus, the solution of the problem of forced vibrations of radially polarized cylinder with electric loading and homogeneous conditions at the end faces is reduced to the solution of an infinite sequence of systems of ordinary differential equations

$$\frac{du_z^{(n)}}{dr} = -\chi_n u_r^{(n)} + \frac{1}{c_{55}^E}\left(\sigma_{rz}^{(n)} - \chi_n e_{15}\varphi^{(n)}\right),$$

$$\frac{d\sigma_{rr}^{(n)}}{dr} = -\frac{1}{r}\chi_n\left(c_{12}^E + \frac{\Delta_2}{\Delta_1}\right)u_z^{(n)} - \frac{1}{r}\left(1 - \frac{\Delta_4}{\Delta_1}\right)\sigma_{rr}^{(n)} +$$

$$+\chi_n\sigma_{rz}^{(n)} - \left[\rho\omega^2 - \frac{1}{r^2}\left(c_{11}^E + \frac{\Delta_2}{\Delta_1}\right)\right]u_r^{(n)} + \frac{1}{r}\frac{\Delta_3}{\Delta_1}D_r^{(n)},$$

$$\frac{d\sigma_{rz}^{(n)}}{dr} = \left[-\rho\omega^2 + \chi_n^2\left(c_{11}^E + \frac{\Delta_2}{\Delta_1}\right)\right]u_z^{(n)} - \chi_n\frac{\Delta_4}{\Delta_1}\sigma_{rr}^{(n)} -$$

$$-\frac{1}{r}\sigma_{rz}^{(n)} - \frac{\chi_r}{r}\left(c_{12}^E + \frac{\Delta_2}{\Delta_1}\right)u_r^{(n)} - \chi_n\frac{\Delta_3}{\Delta_1}D_r^{(n)},$$

$$\frac{du_r^{(n)}}{dr} = \frac{1}{\Delta_1}\left(\chi_n\Delta_4 u_z^{(n)} + \varepsilon_{33}^S\sigma_{rr}^{(n)} - \frac{\Delta_4}{r}u_r^{(n)} + e_{33}D_r^{(n)}\right),$$

$$\frac{d\varphi^{(n)}}{dr} = \frac{1}{\Delta_1}\left(\chi_n\Delta_3 u_z^{(n)} + e_{33}\sigma_{rr}^{(n)} - \frac{\Delta_3}{r}u_r^{(n)} - c_{33}^E D_r^{(n)}\right),$$

$$\frac{dD_r^{(n)}}{dr} = \frac{\chi_r}{c_{55}^E}\left(e_{15}\sigma_{rz}^{(n)} - \chi_n^2\Delta_5\varphi^{(n)}\right) - \frac{1}{r}D_r^{(n)} \quad (4)$$

with boundary conditions

$$\varphi^{(n)}\,|_{r_0\pm h} = \pm\frac{2V_0}{n\pi},\ \left[\sigma_{rr}^{(n)}(r) = \sigma_{rz}^{(n)}(r)\right]|_{r_0\pm h} = 0.$$

In expansion (3) the summands with doubles *n* are equal to zero, so doubles harmonics are not excited and the system (4) should be solved for $n = 1, 3, 5, \ldots$.

The conductivity of piezoelectric cylinder is determined as

$$Y = i\omega\frac{2\pi(r_0 + h)}{V_0}\int_0^i D_r(r)\,|_{r=r_0+h}\,dz.$$

Using the expression for radial component of induction vector in terms of basis functions (2), we obtain the equation for determining the conductivity

$$Y = 4i\omega\frac{(r_0 + h)l}{V_0}\sum_n \frac{1}{n}D_r^{(n)},\ n\text{=1,3,5,…}. \quad (5)$$

Series (5) is not harmonic and an alternating, so it converges. Although this ratio includes the potential difference V_0, the conductivity will not depend on it because a particular solution is proportional to V_0, which follows from boundary condition (3). Excluding dissipation the conductivity is purely reactive in nature and has a pole and zero at the frequencies of resonance and antiresonance [7].

Let's consider the next case when the electrodes on cylindrical surfaces are divided into four parts (sections *1–4*, *1'–4'*), as shown in fig. 1.

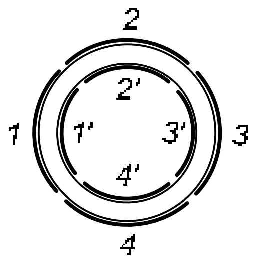


Fig. 1. Piezoelectric element in the form of a hollow cylinder: *1–4*, *1'–4'* – electrodes

Let's stimulate in this piezoelectric element bending vibrations. This problem has been studied experimentally.

For the experiments a radially polarized cylindrical piezoelectric element Ø32×Ø28×20 mm of piezoceramics ЦТС-19 was used.

The main resonant frequency of bending vibrations is 4.15 kHz.

Elements that can create bending oscillations are cylinder sections bounded by electrodes:

a) *1–1'*, *2–2'*, *3–3'*, *4–4'* – in this case, when electric voltage is applied to respective pairs of electrodes (for example, *1–1'*), piezoelectric element in the area of this section increases or decreases the curvature (electric field vector is parallel to polarization vector, α = 0°);

b) when applying electric voltage to the pair of external or internal electrodes (for example, *1–2*, *1'–2'*, *1–2'*, etc.) shear deformations appear which lead to bending of piezoelectric element (electric field vector is at an angle α to polarization vector, α ≈ 90).

The measurement results are shown in table 1. In this table in the second column the electrodes which are connected to signal output of the generator are shown, and in the third column – the electrodes which are connected to "zero" wire.

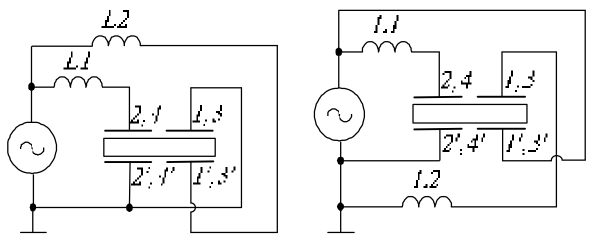
As can be seen from table 1, at an angle α = 0° (variants 1, 2) bending vibrations appear in piezoelectric element and an increasing of the number of electrodes leads to an increase in sound pressure level (variant 2).

Bending vibrations also appear when α → 90° (variants 3, 4).

The highest sound pressure is generated while creating an electric field at α = 0° and α → 90° (variant 5). It should be noted that in this case the phasing of respective voltages, the polarity of electrodes of piezoelectric element and the arising deformations must be taken into account (variants 5, 6).

The electrodes connection by variant 5 provides the maximum sound pressure value for a given piezoelectric element.

A further increase in sound pressure is possible using additional inductance circuits (variants 7, 8 and 9 and also fig. 2).



a) б)

Fig. 2. Connection diagrams of additional inductances to cylindrical piezoelectric element:
a) L_1 = 0,193 H; L_2 = 0,210 H;
b) L_1 = 0,192 H; L_2 = 0,197 H

Both variants shown in fig. 2 create a sound pressure of 112 dB.

Table 1

№	Signal output of generator	"Zero" of generator	*f*, kHz	*C*, nF	*L*, H	*P*, dB
1	2	2'	4,15	3,68		78,7
2	2+4	2'+4'	4,15	7,19		84
3	2+4	1'+3'	4,15	1,12		87,2
4	2+4	1+3	4,15	1,12		88
5	2+4+1'+3'	2'+4'+1+3	4,15	14,5		89,7
6	1+2+3+4	1'+2'+3'+4'	4,15	14,29		<70
7	2+4	2'+4'	4,15	7,19	0,202	108,5
8	2+4	1+3	4,15	1,12	1,28	101
9	2+4+1'+3'	2'+4'+1+3	4,15	14,5	0,1	112

Conclusions:

1. The method of bending vibrations stimulation in cylindrical piezoelectric elements is described.
2. The increase in sound pressure created by cylindrical piezoelectric elements is possible by using schemes with additional inductances.

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В. М. Шарапов[1], *д.т.н., профессор,*
О. Н. Петрищев[2], *д.т.н., профессор,*
Ж. В. Сотула[1], *к.т.н.,*
К. В. Базило[1], *к.т.н.*
[1]Черкасский государственный технологический университет
б-р Шевченко, 460, г. Черкассы, 18006, Украина
v_sharapov@rambler.ru
[2]Национальный технический университет Украины «Киевский политехнический институт»
проспект Победы, 37, г. Киев, 03056, Украина

ВОЗБУЖДЕНИЕ ИЗГИБНЫХ КОЛЕБАНИЙ В ЦИЛИНДРИЧЕСКИХ ПЬЕЗОЭЛЕМЕНТАХ

Работа посвящена совершенствованию пьезоэлектрических преобразователей. Особое место пьезоэлектрические преобразователи занимают в электро- и гидроакустике, где они предназначены для излучения и приема акустических колебаний в воздушной или водной среде. Общей задачей при совершенствовании пьезоэлектрических излучателей является увеличение дальности действия. Для изготовления преобразователей используются мономорфные и биморфные элементы. Описан способ возбуждения изгибных колебаний в цилиндрических мономорфных пьезоэлементах. Приведены схемы подключения радиально поляризованного цилиндрического пьезоэлемента для возбуждения изгибных колебаний при различном взаимном расположении векторов электрического поля возбуждающего напряжения и вектора поляризации. Создание двух возбуждающих контуров в схеме пьезоэлемента позволило повысить мощность уровня звукового давления.

Ключевые слова: *пьезоэлектрический преобразователь, излучатель, изгибные колебания, колебательный контур.*