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MATHEMATICAL MODELS AND INFORMATION TECHNOLOGY OF TRANSFORMATION STOCHASTIC AND FUZZY INFORMATION TO QUANTUM INFORMATION

Abstract – In the article mathematical model have developed. Mathematical models is basic of information technology transformation stochastic and fuzzy information to quantum information. Mathematical models of transformation are unitary operators. Attribute unitary of operators are condition for physical realization. Stochastic information are stochastic data or random variables. Fuzzy information are fuzzy data or fuzzy variables. Quantum information are quantum stochastic data or quantum stochastic variables. Unitary operators transformation probability density function of random variables and membership function of fuzzy variables to wave function of quantum stochastic variables. The mathematical model of transformation of stochastic and fuzzy information to quantum information is developed, which represented as multiplication of unitary operators from noncommutative von Neumann *group. This group is theoretical basic of information technologies of quantum processor' quantum-logic schemes for stochastic and fuzzy data coding to quantum fuzzy data.*

Keywords: classical information, stochastic information, fuzzy information, quantum information.

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МАТЕМАТИЧНІ МОДЕЛІ ТА ІНФОРМАЦІЙНА ТЕХНОЛОГІЯ ПЕРЕТВОРЕННЯ СТОХАСТИЧНОЇ ТА НЕЧІТКОЇ ІНФОРМАЦІЇ У КВАНТОВУ ІНФОРМАЦІЮ

У статті розроблено математичну модель, яка є основою інформаційної технології перетворення стохастичної та нечіткої інформацій у квантову інформацію.

Ця модель представлена у вигляді множення унітарних операторів з некомутатичної групи фон Неймана, що формує теоретичну основу інформаційних технологій квантово-логічного програмування квантового процесора для кодування стохастичних та нечітких даних у квантові нечіткі дані.

Ключові слова: Класична інформація, стохастична інформація, нечітка інформація, квантова інформація.

The development of quantum computing happens rapidly in recent years: quantum computations, quantum information theory and quantum cryptography. The quantum information theory occupies an important place among these directions of quantum computing. Using methods and tools of quantum information theory we can transmit classical information through a quantum communication channel efficiently and reliably. The efficiency of transmission of classical information through a quantum communication channel is ensured thanks to the quantum parallelism, and reliability is ensured thanks to confusing ensembles of quantum transmitters. However, in order to transmit the classical information through a quantum communication channel, we must at first convert it in quantum information. This also applies to private kinds of classical information – stochastic and fuzzy information. The importance of stochastic and fuzzy information transmission through a quantum communication channel is conditioned by its wide use in various areas. But to effectively and reliably transmit the stochastic and fuzzy information through a quantum communication channel, you need to have a mathematical model and overall information technology to transform stochastic and fuzzy information in quantum information.

The Grover`s article [1] is the first work, which concerns the transformation of fuzzy information in quantum information. However, more fully and deeply this question has been considered in the work [2]. Also the article [2] analyses the attempts of stochastic information convertation in quantum information. But none of these works does not solve a question about the transformation of stochastic and fuzzy information in quantum information through unitary operators, which are necessary and sufficient conditions of the physical implementation.

The goal of this article is to propose mathematical models and information technology for the stochastic and fuzzy information conversion in quantum information.

The task of this article is to develop mathematical models and on their basis to build information technology of the stochastic and fuzzy information transformation in quantum information using the unitary operators.

Depending on the physical nature of uncertainty, which is contained in classical information, it is divided into stochastic information and unclear information.

The essence of stochastic information is stochastic data that is random variables ξ or random functions. Because of random functions are the reflection of the set of the argument values in many random variables, stochastic information can always be considered in the format of random variables.

Random variables fully and clearly can be defined by function of probability distribution:

$$0 \le P_\xi(x) \le 1, \quad P_\xi(\varnothing) = 0, \quad P_\xi(X = \mathbf{\Omega}) = 1,$$

where $x \in X = \mathbf{\Omega}$ – space of elementary random events or density of probability distribution:

$$0 \le P'_\xi(x) = p_\xi(x) \le 1,$$

where $x \in X = \mathbf{\Omega}$ – space of elementary random events, if it exists.

In the case of discrete random variables the density of probability distribution $p_\xi(x)$ is a discrete function $p_\xi(x_i)$, $i = \overline{1, N}$.

Thus, the stochastic information, which consists of stochastic data which is a random variable, fully and unequivocally specifies by the functions of the probability distribution or density of probability distribution of the random variables. It follows that the mathematical models of stochastic data are distribution functions or density of probability functions.

The essence of fuzzy information is inexact data fa. Fuzzy data means fuzzy values, which are defined by appliance functions. If the appliance functions were normalized, their scope of values is a segment $[0, 1]$ of numerical axis R.

Thus, fuzzy information, which which consists of inexact data that means fuzzy values fa completely and unambiguously can be specified by the appliance functions:

$$0 \le \mu_{fa}(x) \le 1$$

where $x \in X \subset R$.

So the mathematical models of fuzzy data are appliance functions.

As we know [3], one of the data types of quantum information is quantum inexact data qfa. Quantum fuzzy data qfa is set by indicator functions which are the wave functions ψ_{qfa}. Thus, the mathematical models of quantum fuzzy data, which fully and clearly represent them, are the wave functions.

The process of a stochastic information conversion in quantum information is a process of stochastic data ξ coding in quantum fuzzy data qfa, that is the function p_ξ conversion to the appropriate functions ψ_{qfa}.

The process of fuzzy information transformation in quantum information is a process of fuzzy data fa encoding in quantum fuzzy data qfa, namely, the conversion functions to the appropriate function.

Necessary and sufficient conditions for the practical implementation of these transformations is that they must be depicted by unitary operators. Thus, von Neumann algebra (the group) of unitary operators is the theoretical foundation for the information transformation technology of stochastic and fuzzy information in quantum information.

Let examine, for example, the following cases:

- stochastic data ξ, dimensioned by density type of probability distribution

$$p_\xi(x) = \begin{cases} p_\xi(x_1), x = x_1, \\ p_\xi(x_2), x = x_2; \end{cases}$$

- fuzzy data fa, dimensioned by appliance function

$$\mu_{fa}(x) = \begin{cases} \mu_{fa}(x_1), x = x_1, \\ \mu_{fa}(x_2), x = x_2. \end{cases}$$

Formula (1) sets a matrix of a unitary operator, which is the basis of the information coding technology in accordance to stochastic ξ and fuzzy fa data in quantum fuzzy data qfa:

$$U_{\xi \to qfa} = \begin{pmatrix} p_\xi(x_1) & p_\xi(x_2) \\ p_\xi(x_2) & -p_\xi(x_1) \end{pmatrix},$$

$$U_{fa \to qfa} = \begin{pmatrix} \mu_{fa}(x_1) & \mu_{fa}(x_2) \\ \mu_{fa}(x_2) & -\mu_{fa}(x_1) \end{pmatrix}. \quad (1)$$

As we can see from (1), the operator matrixes are unitary, and this means that such transformations are practically implemented.

In general, the basis of information coding technology of stochastic ξ and fuzzy *fa* data in quantum fuzzy data *qfa* is quantum-logic program:

$$U_{\xi \to qfa} \cdot H, \quad U_{fa \to qfa} \cdot H,$$

where H – Hadamard operator.

Thus, a decent multiplication of unitary operators from not commutative von Neumann group is the mathematical model of transformation of stochastic and fuzzy information in quantum information.

Conclusions. The author of this article has been developed the mathematical model of transformation of stochastic and fuzzy information in quantum information in the form of multiplication of unitary operators from not commutative von Neumann group, that forms the theoretical basis of information technologies of quantum-logic programming of quantum processor for coding of stochastic and fuzzy data in quantum inexact data.

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