

## STUDY OF ACCURACY OF MEASUREMENT RANGE OF MULTIFREQUENCY PHASE METHOD

The article is devoted to the researches the accuracy of the measurement of range by the multifrequency phase method. The sources of error of probing signals are identified in the paper. These include: instability of frequency probing signal, error of measurement amplitude and phase shift of the total reflected signals. There; was shown influence on total reflected signal. Also, when calculating the distances to objects, the condition number of matrix system of linear equations has a significant impact on the accuracy of measuring distances. The resulting graphs after mathematical modeling showed that the least impact on the accuracy occurs when frequency range of the probing signal is conformed with the amplitude-frequency response of the total reflected signal.

Keywords: range, multy frequency phase method, summarize signals, the condition number.

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### ДОСЛІДЖЕННЯ ТОЧНОСТІ ВИМІРЮВАННЯ ДАЛЬНОСТІ БАГАТОЧАСТОТНИМ ФАЗОВИМ МЕТОДОМ

Стаття присвячена дослідженням точності вимірювання дальності багаточастотним фазовим методом. В статті визначені джерела похибки зондуючих сигналів. До них відносяться: нестабільність частоти зондуючого сигналу, помилка вимірювання амплітуди і фазового зсуву зсумарного відбитого сигналів. Показано їх вплив на сумарний відбитий сигнал. Крім того, при розрахунку відстані до об'єктів, число обумовленості матриці системи лінійних рівнянь робить значний впливає на точність вимірювання відстаней. Отримані графіки після математичного моделювання показали, що найменший вплив на точність відбувається, коли частотний діапазон зондуючого сигналу відповідає амплітудно-частотній характеристиці загального відбитого сигналу.

Ключові слова: дальність, багаточастотний фазовий метод, сумарний сигнали, число обумовленості.

#### 1. Introduction

Previous works by the authors are devoted to the study of possibility of measuring distances to many objects [1] and to the development of methods of measuring range by multifrequency phase method [2, 3].

Ranging of the objects by the multifrequency phase method consists of several steps. On the first step the probing harmonic signals formed at some frequencies. On the second step, the objects are probed by the probe signals which were formed. The third step is to measure the phase shifts and the amplitude of the total reflected signals. On the final fourth stage, distances are calculated of by solving a system of linear equations, and solving of polynomial equation and finding finally arguments of the solutions [3].

On each stage of measurement and mathematical transformations the methodological errors is introduced into the measurement result. Let us consider each source of an error. On the first stage, frequency of probing signal is formed inaccurately because of is the instability of generator and frequency is changing step by step or continuously [4]. Therefore, the wavelength also will not match a given value. This leads to the fact that the total phase shift and amplitude of the reflected signal at this frequency will differ from the true values.

#### 2. Main part

For one object which is located at distance  $l_x$  for probe signal which has frequency value  $f_o$ , the phase shift will be as follows [1]:

$$\varphi_x = \frac{4\pi l_x}{c} f_o \quad (1)$$

If you make an error in the value of the probe frequency  $\Delta f$  the phase shift is as follows:

$$\varphi_{x\Delta} = \frac{4\pi l_x}{c} (f_o + \Delta f) \quad (2)$$

Then the error of phase shift is as follows:

$$\Delta\varphi = |\varphi_{x\Delta} - \varphi_x| = \frac{4\pi l_x}{c} \Delta f \quad (3)$$

In this case, the amplitude of the signal reflected from one object is not changed. In this case an error vector of the signal which reflected from one object will be forming:

$$\begin{aligned} \overline{\Delta a} &= a \cdot k_x \cdot e^{j\varphi_{x\Delta}} - a \cdot k_x \cdot e^{j\varphi_x} = a \cdot k_x \cdot (e^{j\varphi_{x\Delta}} - e^{j\varphi_x}) = \\ &= 2 \cdot a \cdot k_x \cdot \sin\left(\frac{\varphi_{x\Delta} - \varphi_x}{2}\right) \cdot e^{j\frac{\varphi_{x\Delta} + \varphi_x + \pi}{2}} \end{aligned} \quad (4)$$

From the resulting expression we can see that the amplitude of the error vector is determined by the value

of the double sine from the phase shift error. The phase of error vector depends on the frequency of the signal, frequency error and is shifted by 90 ° degrees relative to the vector signal:

$$\frac{\varphi_{x\Delta} + \varphi_x + \pi}{2} = \frac{\frac{4\pi l_x}{c} f_o + \frac{4\pi l_x}{c} (f_o + \Delta f) + \pi}{2} = \frac{4\pi l_x}{c} f_o + \frac{4\pi l_x \Delta f}{2c} + \frac{\pi}{2} \quad (5)$$

The process of origination of the error is shown in Fig. 1.

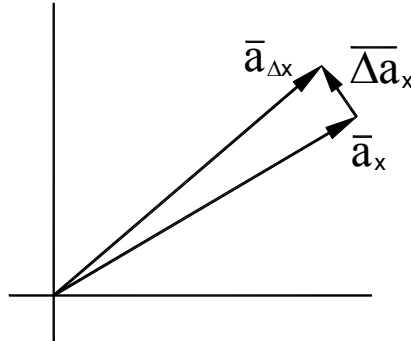


Fig. 1. The formation of the error signal reflected from one object due to instability of the oscillator frequency.

The total signal is the geometrical sum of signals reflected by each object:

$$\overline{a_\Sigma} = \sum_{i=1}^N \overline{a_i} \quad (6)$$

If there are multiple objects, error vectors are formed for each object. In this case, the error signal will be equal to the sum of vectors error of the signals reflected from each object:

$$\overline{\Delta a_\Sigma} = \sum_{i=1}^N \overline{\Delta a_i} \quad (7)$$

Due to the random nature of the frequency instability the total error will also be random. Distribution of errors from frequency instability has normal law.

When objects are probed and signals are passed through tracts of transmitter and receiver, noises are superimposed to signals. They are reason of errors, which become apparent in the process of measuring the phase shift [5, 6] and amplitude [6]. The law of error of measurement of digital measurement methods is uniform. A mathematical model of the vector error of total harmonic signal reflected from any number of objects can be described by the following expression:

$$\Delta a_\Sigma e^{j\varphi_\Sigma} = 2 \cdot a \cdot \sum_{i=1}^N \sin\left(\frac{4\pi l_x}{c} \Delta f\right) \cdot e^{j\left(\frac{4\pi l_x}{c} (f_o + \frac{\Delta f}{2}) + \frac{\pi}{2}\right)} + \Delta a e^{j\Delta\varphi} \quad (8)$$

Due to the emergence of errors of measurement and the amplitude and phase shift of the total signal error vector will have a random direction and modulus of the vector. In general it will be within the circle (Fig. 2).

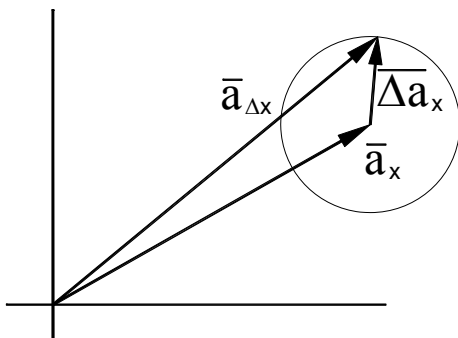


Fig. 2. The formation of the error signal reflected from one object due to errors in measurement of phase shift and amplitude

The last step is to compute distances of objects. In this case, the errors of measurements of parameters of signals (total value vector signals with errors) will be transferred into the distance errors. Also during the computation there will be presented errors of calculation that occur due to rounding of numbers. However, the latter sources of error can be ignored because it is possible to choose a sufficient accuracy of calculations. In work <sup>3</sup> there was proposed the expression of error of distances when the expressions of multifrequency phase method for measurement of distances are used. It was shown that the result of measurement has a significant impact from condition number of the matrix of the linear equations

on the first step of calculations. It is clear that the condition number of the matrix will depend on the values of the elements of matrix. These values depend on the frequency of the probe signal. If frequency and step of the first probe signal is changing, then the value of condition number of the matrix will vary. On Fig. 3 the results of mathematical modeling of frequency dependence of condition number of the matrix with different initial conditions are shown. There were set the following value of range and reflection coefficients of four objects:  $l_1 = 1000 \text{ M}$ ,  $l_2 = 1500 \text{ M}$ ,  $l_3 = 3000 \text{ M}$ ,  $l_4 = 3500 \text{ M}$ ,  $k_1 = 0.5$ ,  $k_2 = 0.3$ ,  $k_3 = 0.15$ ,  $k_4 = 0.07$ . The value of the initial frequency is changing from 1 kHz to 0.4 MHz with step 1 kHz. There was set the step of changing frequency

15 kHz, 30 kHz and 45 kHz.

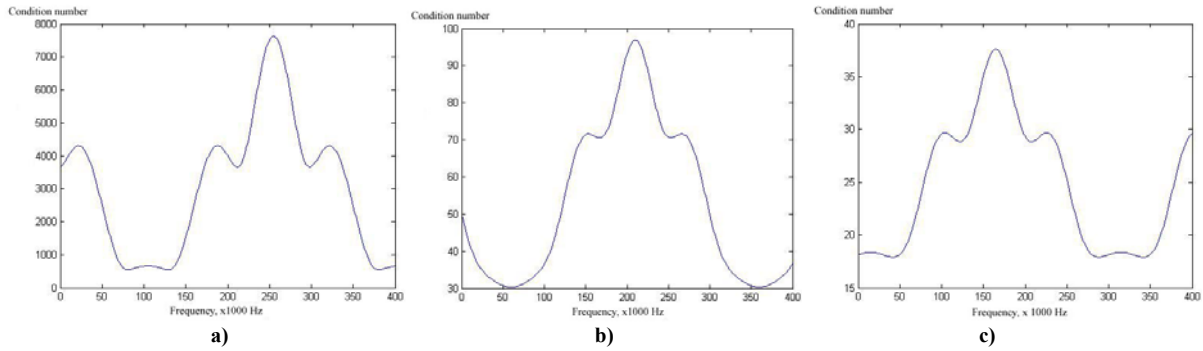


Fig. 3. Frequency dependence of the condition number of the matrix.

(a – step, with which frequency has been changing, is 15 kHz, b - step, with which frequency has been changing, is 30 kHz, c - step, with which frequency has been changing, is 45 kHz)

To identify regularity, we have to compare obtained characteristics with amplitude-frequency characteristic for the total reflected signal which is shown on Fig. 4.

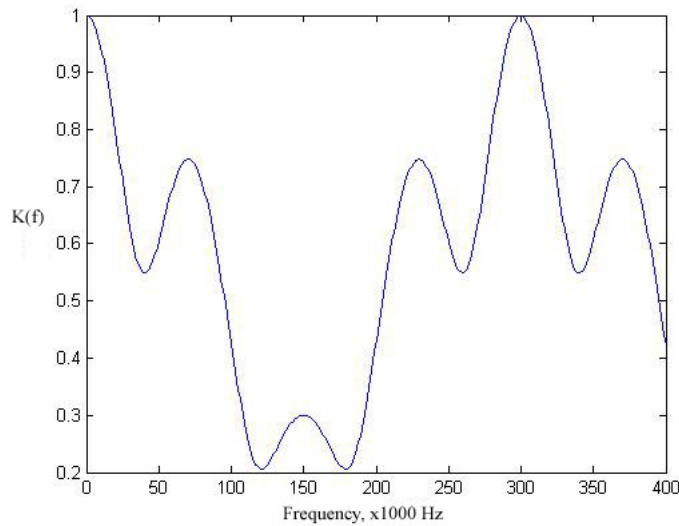


Fig. 4. Amplitude-frequency characteristic of the total reflected signal

Mathematical modeling was held using mathematical model of the error of measurement of the total reflected probe signal. There were set the following parameters for the calculation of the distances with error:

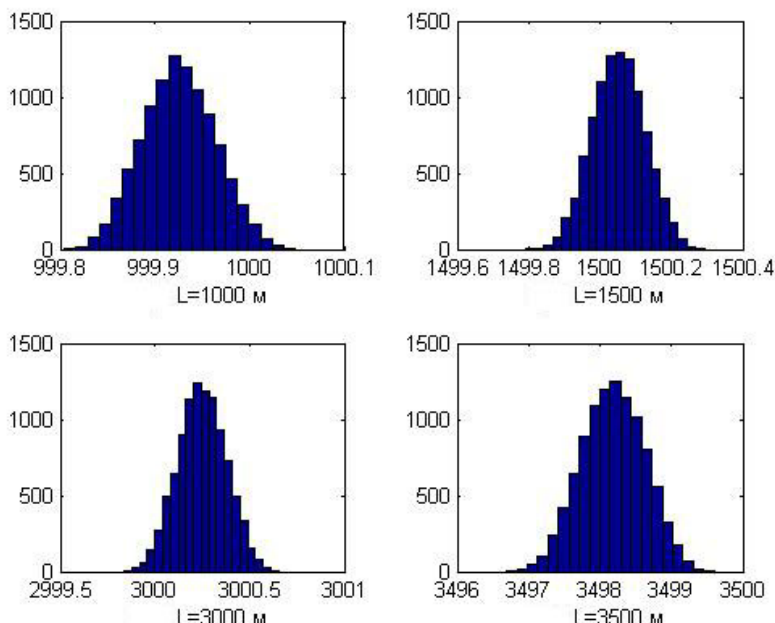


Fig. 5. Histograms of distributions in modeling of multifrequency phase method of measuring distances

- frequency instability  $\Delta f = 10^{-6}$  Hz;
- error of the measurement of amplitude  $\Delta a = 1/4096$ , that is 12 bits of analog-to-digital converter
- error of the measurement of the phase shift  $\Delta \varphi = 0.01^\circ$ .

The frequency of probing signal has been changing ranging from 37.5 kHz to 300 kHz. In this frequency range, as defined above, the condition number of the matrix is the smallest. The number of objects, their distance and the reflection coefficient was chosen as for the first option. 10 thousand simulations have been. The histograms conducted obtained as a result of mathematical modeling is shown in Fig. 5.

The figure shows that the distributions correspond to the normal

law As a result of statistical treatment of the mathematical modeling results there were obtained expectation and standard deviation are presented in Table 1.

Table 1

<b>Results of simulation of ranging by multifrequency phase method.</b>		
Distance, m	Expectation, m	Standard deviation, m
1000	999.92	0.0381
1500	1500.1	0.0730
3000	3000.2	0.1266
3500	3498.2	0.4397

Increasing the first frequency of the probe signal from the point minimum of condition number of matrix at 7.5 kHz upwards, we can see an increase in the standard deviation (Table 2).

Table 2

**Results of simulation of ranging by multifrequency phase method when increasing the first frequency.**

Distance, m	Expectation, m	Standard deviation, m
1000	999.9	0.0616
1500	1500.1	0.0807
3000	3000.2	0.1611
3500	3499,8	0.6247

### 3. Conclusion

The results of the study allow us to determine the scope of multifrequency phase method of measuring distances to develop improved algorithms and for measuring distances with high accuracy. Improved algorithms have to search for points where we can see minimum condition number of matrix of system of linear equations. The point where we can see minimum of condition number are changing, when objects are placed at different distances. The need to find points of minimum on the frequency axis will lead to increase of time sensing and measurement. But it will significantly increase accuracy. In this example, the accuracy was increased twice. In this case, the frequency of probe signal was lower than in the case where the accuracy was lower with increased accuracy. From the perspective of the classical theory of phase measuring distances it is impossible. However, taking into account the theoretical propositions of multifrequency phase method of measurement distances, it is possible.

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