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META-MINIMAX APPROACH FOR OPTIMAL ALTERNATIVES SUBSET REGARDING THE CHANGE OF THE RISK MATRIX IN ENSURING INDUSTRIAL AND MANUFACTURING LABOR SAFETY

Industrial and manufacturing labor safety is considered from the point of view of minimizing risks of getting human harm or injuries. To ensure minimal damage under possibly worst conditions and poor predictability, risks are minimaxed regarding the change of the risk matrix. The change is explained as an aftermath of that the point evaluation of the risk matrix is impossible, and so this matrix is represented as a finite set of matrices implying the risk matrix change through this set. Each version of the risk matrix corresponds to a state which is called the metastate. Thus the change of the risk matrix is formalized. Formally, the finite change of the risk matrix can be substituted with the three-dimensional risk matrix. Optimal alternatives subset regarding the change of the risk matrix is obtained by the suggested criterion. This criterion is a meta-minimax approach considering, however, four cases. These four cases are generated from that there are probabilistic measures relating to ordinary states and a probabilistic measure over metastates. And each of the two types of the probabilistic measure is considered available or unavailable. Finally, a way of sorting metastates is discussed. In this way, either versions of the risk matrix or stochastic matrices with probabilistic measures relating to ordinary states can be sorted using a distance in the space of real-valued matrices of the corresponding size.

Keywords: labor safety, industrial and manufacturing labor risks, risk matrix, minimax, alternative, metastate, meta-minimax, minimaximax.

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МЕТА-МІНІМАКСНИЙ ПІДХІД ДЛЯ ПІДМНОЖИНИ ОПТИМАЛЬНИХ АЛЬТЕРНАТИВ, ЩО ЗВАЖАЄ НА ЗМІНУ МАТРИЦІ РИЗИКІВ У ЗАБЕЗПЕЧЕННІ ОХОРОНИ ПРОМИСЛОВОЇ ТА ВИРОБНИЧОЇ ПРАЦІ

Розглядається безпека промислової та виробничої праці з точки зору мінімізації ризиків отримання людиною уражень або пошкоджень. Заради забезпечення мінімальних збитків за ймовірно найгірших умов та слабкої передбачуваності ризики визначаються на основі мінімаксу, що зважає на зміну матриці ризиків. Ця зміна пояснюється як наслідок того, що точкове оцінювання матриці ризиків неможливе, і тому ця матриця представляється у виді скінченної множини матриць, яка продукує зміну матриці ризиків. Кожен варіант матриці ризиків відповідає деякому стану, котрий називається метастаном. Відповідним чином матриця ризиків і формалізується. Скінченна зміна матриці ризиків може бути формально замінена тривимірною матрицею. Підмножина оптимальних альтернатив, що зважає на зміну матриці ризиків, отримується за запропонованим критерієм. Цим критерієм є мета-мінімаксний підхід, котрий допускає, між іншим, чотири можливості. Ці чотири випадки випливають з того, що розрізняють імовірнісні міри щодо звичайних станів й імовірнісну міру щодо метастанів. І кожен з цих двох типів імовірнісної міри розглядається доступним та недоступним. Зрештою, обговорюється спосіб сортування метастанів. За цим способом можуть сортуватися або варіанти матриці ризиків, або стохастичні матриці з імовірнісними мірами щодо звичайних станів з використанням деякої метрики у просторі дійснозначних матриць відповідного розміру.

Ключові слова: безпека праці, ризики промислової та виробничої праці, матриця ризиків, мінімакс, альтернатива, метастан, мета-мінімакс, мінімаксимакс.

Importance of minimizing the industrial and manufacturing labor risks

Ensuring labor safety and protection is the main task in any industrial branch. While manufacture process is projected, the manufacturing human labor is to be designed safe. This especially concerns heavy industrial branches, i. e. machine-building, metal processing, large-sized constructing, etc. In these branches, risks of getting human harm or injuries are higher than elsewhere. Thus, minimization or reduction of the risks must be early implemented, before the process starts.

Approaches to minimizing risks

Due to poor predictability, industrial risks are minimized under uncertainty. Uncertainty can be reduced as statistics grows. When statistics is unavailable, risks are minimaxed in order to ensure minimal damage under possibly worst conditions [1, 2]. Formally, this is about the criterion of Wald or the criterion of Savage [3, 4]. Growing statistics allows to estimate some situations statistically in the sense of probabilistic measure [4, 5]. Then, probabilistic criterions can be applied to calculate risks and make an appropriate decision. Basically, they are criterions of Bayes — Laplace (BL), Germeyer, Hodges — Lehmann (HL) and other hybridized rules [4, 6, 7]. These criterions, however, do not admit that the decision matrix or risk matrix (RM) may change. And if RM changes, what reflects real influences of factors in industrial processes, even the best-assurance minimax criterion (or BL criterion related to minimax) comes inconsistent.

Goal of the article and tasks to be accomplished

Based on that RM may change, generating uncertainties in its elements, the goal is to state a criterion which could regard the change. Both unavailability of probabilistic measures and their availability must be considered. For reaching the goal, the following tasks are to be accomplished:

1. Formalize the change of RM.
2. Suggest rules for obtaining the optimal alternatives subset (OAS) regarding the change of RM. For doing

that appropriately, consider four cases:

2.1. Any probabilistic measures are unavailable or uncertain.

2.2. Probabilistic measures are available. Note that these measures relate to ordinary states whose evaluations are in columns of RM.

2.3. Probabilistic measures are unavailable, but a probabilistic measure over states of the changing RM is available. Henceforward, these states are called metastates.

2.4. Both probabilistic measures relating to ordinary states and the probabilistic measure over states of the changing RM (over metastates) are available.

Formalization of RM change

Let $\mathbf{R}_k = \left(r_{ijk}\right)_{M \times N}$ be RM at the k-th metastate of an environment, influencing on elements of RM, by $M \in \mathbb{N} \setminus \{1\}$ and $N \in \mathbb{N} \setminus \{1\}$. May there be K metastates by $K \in \mathbb{N} \setminus \{1\}$. Denote the set of alternatives by $X = \{x_i\}_{i=1}^{M}$. Then r_{ijk} is the risk at the j-th state in the k-th metastate when the alternative x_i is selected. Formally, the finite change of RM can be substituted with the risk $M \times N \times K$ matrix.

Probabilistic measures are unavailable

This is the most pessimistic case relating to the classical minimax approach [3, 4, 8, 9]. It does not rely, however, on any statistics, because

$$X^* = \arg \min_{x_i,\, i=\overline{1, M}} \left\{ \max_{j=\overline{1, N}} \max_{k=\overline{1, K}} r_{ijk} \right\} \subset X . \tag{1}$$

OAS by (1) is effective when conditions under which one has to make a decision occur rarely or just a few times [3, 5, 9]. Seemingly, minimaximax by (1) comes severer than the classical minimax approach owing to risks are maximized twice.

Probabilistic measures relating to ordinary states are available

Let $\mathbf{P}_k = \left(p_{ijk}\right)_{M \times N}$ be the stochastic matrix whose value p_{ijk} is the probability of the j-th state at the k-th metastate when the i-th alternative is selected. Obviously,

$$\sum_{j=1}^{N} p_{ijk} = 1 \quad \forall\, i = \overline{1, M} \quad \text{and} \quad \forall\, k = \overline{1, K} . \tag{2}$$

Then

$$X^* = \arg \min_{x_i,\, i=\overline{1, M}} \left\{ \max_{k=\overline{1, K}} \sum_{j=1}^{N} p_{ijk} r_{ijk} \right\} \subset X . \tag{3}$$

OAS by (3) is effective when conditions under which one has to make a decision occur frequently [4].

Only probabilistic measure over metastates is available

Let $\{\mu_k\}_{k=1}^{K}$ be a probabilistic measure over K metastates, so

$$\sum_{k=1}^{K} \mu_k = 1 . \tag{4}$$

Although matrices $\{\mathbf{P}_k\}_{k=1}^{K}$ are unknown, the probabilities $\{\mu_k\}_{k=1}^{K}$ allow to form

$$X^* = \arg \min_{x_i,\, i=\overline{1, M}} \left\{ \max_{j=\overline{1, N}} \sum_{k=1}^{K} \mu_k r_{ijk} \right\} \subset X . \tag{5}$$

OAS by (5) is effective when RM changes frequently [4, 5]. When industrial and manufacturing labor risks are evaluated, this case is much more probable than availability of probabilistic measures in matrices $\{\mathbf{P}_k\}_{k=1}^{K}$ by (2). Nonetheless, evaluation of $\{\mu_k\}_{k=1}^{K}$ by (4) is a separate problem.

Probabilistic measures relating to ordinary states and the probabilistic measure over metastates are available

In this case, OAS

$$X^* = \arg \min_{x_i,\, i=\overline{1,\, M}} \left\{ \sum_{k=1}^{K} \mu_k \sum_{j=1}^{N} p_{ijk} r_{ijk} \right\} \subset X \quad (6)$$

is effective when conditions under which one has to make a decision occur frequently, and RM changes frequently as well. Obviously, this case relies on huge statistics, what is unfeasible for industrial and manufacturing labor risks. OAS by (6), as a consequence, bears a little practical use.

Discussion

In real practice, point evaluation of RM is impossible, so RM is represented as the set of K matrices $\{\mathbf{R}_k\}_{k=1}^{K}$ implying the RM change through this set. Clearly, the neighboring numbered matrices $\mathbf{R}_k$ and $\mathbf{R}_{k+1}$ by $k=\overline{1,\, K-1}$ may have non-changed elements. Matrices $\{\mathbf{R}_k\}_{k=1}^{K}$ can be sorted variously. In a partial case, if $\mathbf{R}_1$ is given,

$$\mathbf{R}_k \in \arg \min_{\mathbf{R}_m,\, m=\overline{k,\, K}} \rho_{M\times N}\left(\mathbf{R}_{k-1},\, \mathbf{R}_m\right) \quad \text{for} \quad k=\overline{2,\, K-1} \quad (7)$$

by the distance $\rho_{M\times N}\left(\mathbf{A},\, \mathbf{B}\right)$ in the space of real-valued $M \times N$ matrices $\mathbf{A}$ and $\mathbf{B}$, where matrix $\mathbf{R}_K$ remains itself after matrices $\{\mathbf{R}_k\}_{k=1}^{K-1}$ are known (by their numbers). Otherwise, the metastates' numbers $\{k\}_{k=1}^{K}$ may be obtained after sorted matrices $\{\mathbf{P}_k\}_{k=1}^{K}$ in the same way, i. e. when $\mathbf{P}_1$ is given,

$$\mathbf{P}_k \in \arg \min_{\mathbf{P}_m,\, m=\overline{k,\, K}} \rho_{M\times N}\left(\mathbf{P}_{k-1},\, \mathbf{P}_m\right) \quad \text{for} \quad k=\overline{2,\, K-1}. \quad (8)$$

The initial matrix $\mathbf{R}_1$ or $\mathbf{P}_1$ is chosen free. At least, any reasons to take some $\mathbf{R}_1$ or $\mathbf{P}_1$ are going to be specific rather than general.

Conclusion

Here, in (1), (3), (5), (6) a meta-minimax approach is suggested. This is caused with that RM, whose elements are evaluated by experts of the industrial labor safety assurance, cannot be expressed as an ordinary $M \times N$ matrix. An OAS by (1), (3), (5), (6) regards the RM change. Note that this change is finite, not infinite or continuous. Availability of statistics is not excluded, so the rigorous minimaximax (1) can be softened with (3), (5), or (6). An OAS by (3), (5), or (6) corresponds to a meta-BL criterion. Based on minimaximax (1) and meta-BL criterion (3), (5), or (6), a meta-HL criterion may be formalized also. However, this formalization should contain substantiation of how to select two parameters of relying on probabilistic measures in $\{\mathbf{P}_k\}_{k=1}^{K}$ and $\{\mu_k\}_{k=1}^{K}$.

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