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T.I. KULIK, B.M. ZLOTENKO
Kyiv National University of Technology and Design**STRESS AND DEFORMATION IN ROD AND TUBE
POLYMERIC SHOE PARTS AT TORSION AND BENDING**

The article deals with the mathematical modelling of torsion and bending of rod and tubular bottom parts of athletic shoes during operation. Modern athletic shoe soles contain rod and tubular parts which increase sportsperson's performance by playing active role in transferring energy to the direction of locomotion. As the rod or tubular elements are parts of the sole and serve to absorb dynamic loads their design should be based on the relationships between stresses deformations which has not been established yet. For successful design of sport shoe soles it is needed to determine the dependencies of stresses and deformation that occur in the rod and tubular elements on the values of torsional and bending loads. It has been established that during operation upon the tubular elements act two opposing torques applied in parallel to its axis planes, which leads to their torsion. Rod elements whose axes are perpendicular to the surface of the sole bend under the action of the force which occurs as a result of interaction with the ground when the athlete pushes back to accelerate. The difficulty of mathematical modelling lies in the fact that in the shoe industry different polymer materials are widely used: rubber, polyurethane, polyethylene and polystyrene. And all of these materials are characterized by non-linear relationship between the stress and strain arising under the applied load, which complicates the calculations. So the study was undertaken to determine the dependencies of stresses and deformation that occur in the rod and tubular elements on the values of torsional and bending loads. As a result of the study the analytical expressions are obtained that determine dependencies of the stress and strain on the values of operating loads. The research results can be used to design the soles of sports shoes and other plastic parts, working at torsion and bending.

Keywords: Sports shoes, sole, torsion, bending, polymer beam.

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Київський національний університет технологій та дизайну**НАПРУЖЕННЯ ТА ДЕФОРМАЦІЇ У СТРИЖНЕВИХ ТА ТРУБЧАСТИХ
ПОЛІМЕРНИХ ДЕТАЛЯХ ВЗУТТЯ ПІД ЧАС КРУЧЕННЯ ТА ЗГИНУ**

У статті представлені результати математичного моделювання кручення та згину стрижневих і трубчастих деталей низу спортивного взуття в процесі експлуатації. Отримані аналітичні вирази, які встановлюють залежності напружень та деформацій від величини експлуатаційних навантажень. Результати досліджень можуть бути використані для проектування підшов спортивного взуття та інших полімерних деталей, які працюють на кручення та згин.

Ключові слова: спортивне взуття, підшова, кручення, згин, полімерна балка.

Introduction

Sports footwear has constantly changed, with new designs and materials being developed by sporting goods manufacturers to meet players' demands. It has been showed that in sprinting performance improvements were possible by changing the mechanical properties of the shoe [1]. Bending stiffness has been demonstrated as an important performance parameter in running and football footwear [2]. In football players' movements tend to be low intensity actions, such as walking, with short bursts of high intensity movements types such as sprinting [3]. The effect of the different designs and the materials used in ski-boot construction on the final performances and the prevention of injuries is emphasized in [4].

Analysis of published data and problem definition

The modern day boots consist of two major components; the outsole and the upper, designed to protect the foot and enhance performance [5]. In sports footwear outsoles production the principle of directional energy transfer is used where energy applied to the sole by the athlete is returned as force acting in a direction other than the direction in which energy was put into the system [6]. The energy transfer was realized by reciprocal rotation of the tubes in the process of running [7]. Directional energy transfer in running shoes increases athletic performance as running speed by transferring energy to the direction of locomotion. The concept was applied to the Adidas Bounce style shoe. The shoe uses tubes incorporated into the shoe sole. Directional energy transfer was optimized by rotation of the tubes of Adidas shoes and the overall stiffness changed by altering the tube length [8]. But any theoretical prediction of the tubes' strength and deformation has not been made.

The purpose and objectives of the research

As the rod or tubular elements are parts of the sole and serve to absorb dynamic loads their design should be based on the relationships between stresses deformations which are not established yet.

The purpose of this research is to develop a mathematical model to predict the behaviour of the footwear sole elements under loads that occur during walking and running.

When upon the element act two opposing torques applied in parallel to its axis planes, it is subjected to torsion. Rod elements whose axes are perpendicular to the surface of the foot, they bend under the action of the force which occurs as a result of interaction with the ground.

Tasks of the study are to determine the dependencies of stresses and deformation that occur in the rod and tubular elements on the values of torsional and bending loads.

Materials for soles and methods of describing their mechanical properties

In the shoe industry different polymer materials are widely used: rubber, polyurethane, polyethylene and polystyrene. All of these materials are characterized by non-linear relationship between the stress and strain arising under the applied load that can be represented as a power law.

In the case of torsion, this dependence has the form:

$$\tau^m = \gamma G, \quad (1)$$

where τ – shear stress;

m – exponent which varies from 0.6 to 1 (when $m=1$ the body has the elastic properties);

γ – relative shear deformation;

G – shear modulus.

The analogous relationship between the bending normal stresses and strains is expressed as:

$$\sigma^m = E\varepsilon, \quad (2)$$

where σ – normal tension in cross-section;

E – modulus of elasticity of the material sample;

ε – relative elongation of the sample.

Results of analytical research of torsion and bending of rod and tubular elements of polymeric soles

Consider torsion of a polymer beam one end of which is fixed and the other loaded with torque.

Relative shear deformation of the beam is given by [9]:

$$\gamma = \rho \frac{d\varphi}{dx}, \quad (3)$$

where ρ – the radius of an elementary cross-sectional area;

φ – the angle of beam's twist;

x – axial coordinate.

The greatest shear stress is acting on the surface of the beam was obtained as:

$$\tau = \frac{T}{I_{0m}} \rho^{\frac{1}{m}}. \quad (4)$$

The value I_{0m} for solid beams will be:

$$I_{0m} = \int_A \rho^{\frac{1}{m}+1} dA = \int_0^R \rho^{\frac{1}{m}+1} \rho d\rho = \int_0^R \rho^{\frac{1}{m}+2} d\rho = \frac{1}{\frac{1}{m}+3} \rho^{\frac{1}{m}+3} \Big|_0^R = \frac{1}{\frac{1}{m}+3} R^{\frac{1}{m}+3}. \quad (5)$$

In the case of hollow beams we get:

$$I_{0m} = I_{0m}(R) - I_{0m}(r) = \frac{1}{\frac{1}{m}+3} \left(R^{\frac{1}{m}+3} - r^{\frac{1}{m}+3} \right), \quad (6)$$

where $I_{0m}(R)$ and $I_{0m}(r)$ – the values defined for solid beams with the radii R and r respectively.

The angle of beam's twist will be:

$$\varphi = \frac{\left(\frac{T}{I_{0m}} \right)^m}{G} l. \quad (7)$$

Let us define stress in rod shoe element at bending:

$$\sigma = \frac{1}{I_m} Mz^m, \quad (8)$$

where

$$I_m = \int_A z^{\frac{1}{m}+1} dA = \int_{-h/2}^{h/2} z^{\frac{1}{m}+1} b dz = \frac{b}{\frac{1}{m}+2} \left[z^{\frac{1}{m}+2} \right]_{-h/2}^{h/2} = \frac{2b}{\frac{1}{m}+2} \left(\frac{h}{2} \right)^{\frac{1}{m}+2}. \quad (9)$$

For hollow beams we get:

$$I_m = I_m(B, H) - I_m(b, h) = \frac{2}{\frac{1}{m}+2} \left[B \left(\frac{H}{2} \right)^{\frac{1}{m}+2} - b \left(\frac{h}{2} \right)^{\frac{1}{m}+2} \right], \quad (10)$$

where $I_m(B, H)$ and $I_m(b, h)$ – the values defined for solid beams of cross-sections dimensions $B \times H$ and $b \times h$ respectively.

For a beam of round cross-section we have:

$$I_m = \int_A z^{\frac{1}{m}+1} dA = 2 \int_0^r z^{\frac{1}{m}+1} y dz = 2 \int_0^r z^{\frac{1}{m}+1} \sqrt{r^2 - z^2} dz. \quad (11)$$

Then we get:

$$I_m = -2 \int_0^r x^{\frac{1}{m}+1} \frac{1}{x^2} \frac{dx}{2x^2} = \frac{2}{\frac{1}{m}+3} r^{\frac{1}{m}+3}. \quad (12)$$

For hollow beams we obtain:

$$I_m = I_m(R) - I_m(r) = \frac{2}{\frac{1}{m}+3} \left(R^{\frac{1}{m}+3} - r^{\frac{1}{m}+3} \right), \quad (13)$$

where $I_m(R)$ and $I_m(r)$ – the values defined for solid beams with radii R and r respectively.

Figures 1 and 2 show graphs calculated by the formula (26) taking into account (28).

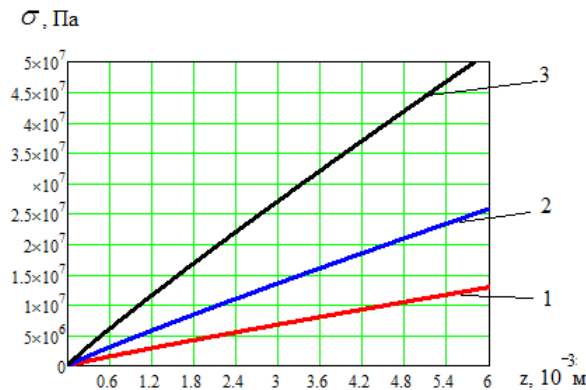


Fig. 1. The dependence of stress σ on the vertical coordinate z for the beam of square cross-section:
1 – $T = 0,5 \text{ N}\cdot\text{m}$; 2 – $T = 1 \text{ N}\cdot\text{m}$; 3 – $M = 1,5 \text{ N}\cdot\text{m}$

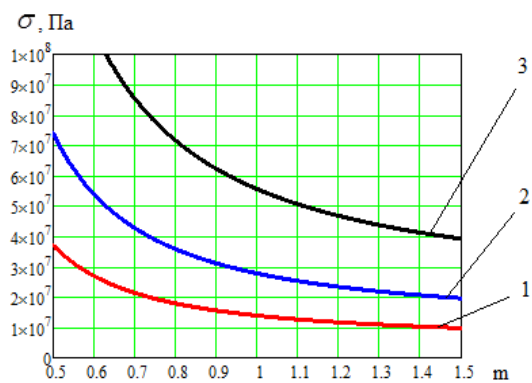


Fig. 2. The dependence of the stresses σ on the exponent m for beams of square cross-section:
1 – $T = 0,5 \text{ N}\cdot\text{m}$; 2 – $T = 1 \text{ N}\cdot\text{m}$; 3 – $M = 1,5 \text{ N}\cdot\text{m}$

Let us define the parameters of deformation of the cantilever beam under the influence of concentrated efforts applied to its free end.

For the axis of the beam we can get [10]:

$$EI_m \frac{d^2 y}{dx^2} = M^m. \quad (14)$$

Taking into account that positive growth of dx corresponds to negative growth of dy , the bending moment can be expressed by the dependence:

$$M = -P(l-x). \quad (15)$$

For the free end of the cantilever beam with concentrated force we have:

$$\theta_{\max} = -\frac{P^m l^{m+1}}{(m+1)EI_m};$$

$$f = y_{\max} = -\frac{P^m l^{2m+1}}{(m+2)EI_m}. \quad (16)$$

Figures 3 and 4 show graphs calculated by the formulae (49) taking into account (28).

Let us consider the deformation of the cantilever beam under a uniformly distributed load. For this case the bending moment can be expressed by the dependence:

$$M = -q \frac{(l-x)^2}{2}. \quad (17)$$

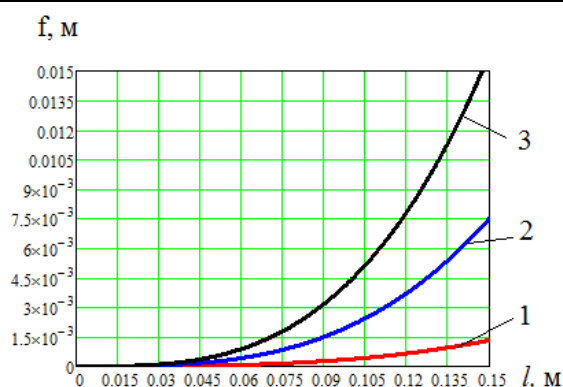


Fig. 3. The dependence of the maximum deflection of the cantilever beam on its length:
1 – $P=1$ N; 2 – $P=5$ N; 3 – $P=10$ N

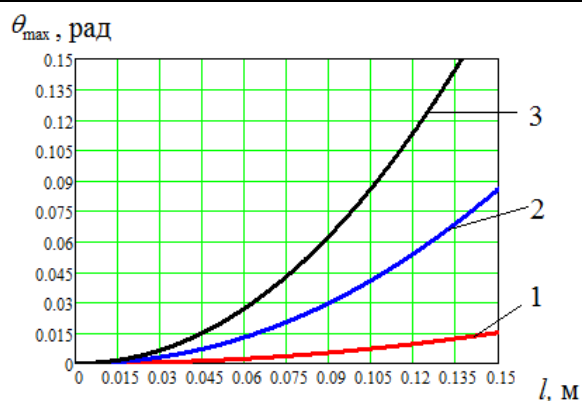


Fig. 4. The dependence of the maximum angle of rotation of the cantilever beam's axis on its length:
1 – $P=1$ N; 2 – $P=5$ N; 3 – $P=10$ N

For the free end of the cantilever beams with distributed load we obtain:

$$\theta_{\max} = -\frac{q^m l^{2m+1}}{2^m (2m+1) EI_m};$$

$$f = y_{\max} = -\frac{q^m l^{2m+2}}{2^m (2m+2) EI_m}.$$
(18)

Discussion of research results

As shown in the graph in Figure 2 increase of the exponent m corresponds to decrease of the stress in the surface fibers of the beam. In general, when values of the exponent m accede 1, the value of the shear stress is less than when it is less than 1. The rest of the graphs show that the character of the dependencies of stresses and deformations on the value of loads in a plastic beams is analogous to that in elastic beams.

Conclusions

The mathematical model of the work of rod and tubular elements polymeric the shoe bottom is evolved that establish dependencies of the stresses and deformations on the value of operating loads. Obtained analytical expressions can be used to design the soles of sports shoes and other plastic parts that work at twisting and bending.

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