

V.F. TIMKOV

The Office of National Security and Defence Council of Ukraine

S.V. TIMKOV, V.A. ZHUKOV, K.E. AFANASIEV

Research and Production Enterprise «TZHK»

FRACTAL GENESIS OF THE ANGLES OF THE NEUTRINO MIXING MATRIX

A fractal genesis of the angles of the neutrino mixing matrix hypothesis is being examined. (Pontecorvo – Maki–Nakagawa–Sakata matrix - PMNS matrix). It is assumed that framework for this genesis are gold algebraic fractals mantissas: of the Planck mass, of the Planck time and of the fine-structure constant, namely, sines of angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$ are gold algebraic fractals: of the fine-structure constant, of the Planck mass and of the Planck time respectively. A gold algebraic fractal mantissa of any real number can be represented as additive gold algebraic fractal – an infinite sum of the power series (more exactly – geometrical) with the basis of the gold ratio large number. Assessed the Higgs boson mass based on additive gold algebraic fractal. The value of the angle of the Carioles force is specified in the model of the rotating observable Universe. A new assessment of the contribution of dark matter to the overall energy balance of the observed Universe is given. The universality of Planck's constants and the fine structure constant for all structural and spatial levels of matter has been confirmed.

Key words: fractal genesis, neutrino mixing matrix, gold algebraic fractal, Planck constants, fine-structure constant, Higgs boson, spatial and structural levels of matter.

В.Ф. ТИМКОВ

Аппарат Совета национальной безопасности и обороны Украины

С.В. ТИМКОВ, В.А. ЖУКОВ, К.Е. АФАНАСЬЕВ

Научно-производственное предприятие «ТЗЖК»

ФРАКТАЛЬНЫЙ ГЕНЕЗИС УГЛОВ МАТРИЦЫ СМЕШИВАНИЯ НЕЙТРИНО

Рассматривается гипотеза о фрактальном генезисе углов матрицы смешивания нейтрино (Pontecorvo – Maki–Nakagawa–Sakata matrix - PMNS matrix). Предполагается, что основой этого генезиса являются мантиссы золотых алгебраических фракталов: массы Планка, времени Планка и постоянной тонкой структуры, а именно: синусы углов $\Theta_{12}, \Theta_{23}, \Theta_{13}$ являются золотыми алгебраическими фракталами соответственно: постоянной тонкой структуры, массы Планка и времени Планка. Мантисса золотого алгебраического фрактала любого вещественного числа может быть представлена в виде аддитивного золотого алгебраического фрактала – бесконечной суммы степенного (более конкретно – геометрического) ряда с основанием в виде большего числа золотой пропорции. Дана оценка массы бозона Хиггса на основе аддитивного золотого алгебраического фрактала. Уточнено значение угла силы Кориолиса в модели вращающейся наблюдаемой Вселенной. Дана новая оценка вклада темной материи в общий энергетический баланс наблюдаемой Вселенной. Подтверждена универсальность констант Планка и постоянной тонкой структуры для всех структурных и пространственных уровней материи.

Ключевые слова: фрактальный генезис, матрица смешивания нейтрино, золотой алгебраический фрактал, константы Планка, постоянная тонкой структуры, бозон Хиггса, пространственные и структурные уровни материи.

1. Introduction.

It's known that main fundamental physical constants, for example the gravitational constant G , the reduced Planck constant \hbar , the speed of light in vacuum c , can be determined with Planck constants: the Planck length l_p , the Planck mass m_p , the Planck time t_p :

$$G = \frac{l_p^3}{m_p t_p^2}; \hbar = \frac{m_p l_p^2}{t_p}; c = \frac{l_p}{t_p}. \tag{1}$$

Using the “Planck universal proportions” [1], the Planck constants values [2]:

$$l_p = 1.616229(38) \cdot 10^{-35} m; t_p = 5.39116(13) \cdot 10^{-44} s; m_p = 2.176470(51) \cdot 10^{-8} kg; \tag{2}$$

and the Hubble constant [3]:

$$H_0 = 66.93 \pm 0.62 (km / s) / Mpc, \tag{3}$$

it's possible to evaluate the main spatial characteristics of the observable Universe [4]:

mass M_U , radius R_U , rotation period T_U [5], which is equal to the light signal propagation delay at a distance equal R_U :

$$T_U = t_p \frac{M_U}{m_p} = t_p \frac{R_U}{l_p} = \frac{1}{H_0} = 4.431534683326 \cdot 10^{17} s, \tag{4}$$

$$M_U = m_p \frac{T_U}{t_p} = m_p \frac{R_U}{l_p} = \frac{c^3 T_U}{G} = 1.789337036792 \cdot 10^{53} \text{ kg}, \quad (5)$$

$$R_U = l_p \frac{M_U}{m_p} = l_p \frac{T_U}{t_p} = c T_U = 1.328540675427 \cdot 10^{26} \text{ m}. \quad (6)$$

As well, in accordance with the law “Planck universal proportions” – for every celestial body of the observable Universe (including the observable Universe), mass which is m , the curvature of space radius is S , which created by gravitational field mass which is m , with the light signal propagation delay is t_{dm} at distance equal to S , the gravitational constant G , and Planck force F_p , true:

$$G = \frac{l_p^3}{m_p t_p^2} = \frac{R_U^3}{M_U R_U^2} = \frac{S^3}{m t_{dm}^2} = 6.673045869 \cdot 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}, \quad (7)$$

$$F_p = m_p \frac{l_p}{t_p^2} = M_U \frac{R_U}{T_U^2} = m \frac{S}{t_{dm}^2} = 1.21048301950 \cdot 10^{44} \text{ kg}^1 \text{ m}^1 \text{ s}^{-2}. \quad (8)$$

Energy of body E which mass m , a and also the observable Universe energy E_U are:

$$E = m c^2 = h_e t_{dm} = F_p S; E_U = M_U c^2 = h_e T_U = F_p R_U, \quad (9)$$

where: $h_e = \frac{E_p}{t_p}$ – is the quantum of the Planck energy, where E_p – the Planck energy: $E_p = m_p c^2$.

It is evident:

$$h_e = \frac{E_p}{t_p} = \frac{m_p c^2}{t_p} = \frac{m_p l_p^2}{t_p^3} = 3.628372528 \cdot 10^{52} \text{ J}^1 \text{ s}^{-1}. \quad (10)$$

It's known [6], that charge is function of the moment of the mass. Then for the elementary charge e is true:

$$e = \sqrt{10^7 \alpha m_p l_p} \left[\text{kg}^{\frac{1}{2}} \text{m}^{\frac{1}{2}} \right], \quad (11)$$

where α – is the fine-structure constant, it's value is [2]:

$$\alpha = 7.2973525664(17) \cdot 10^{-3}. \quad (12)$$

From formula (11), considering [6]:

$$c^2 = \frac{10^7}{4\pi\epsilon_0}, \quad (13)$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2} = \frac{4\pi\epsilon_0 m_p^2 l_p^4}{10^7 \alpha m_e m_p l_p t_p^2} = \frac{m_p l_p}{\alpha m_e}, \quad (14)$$

where: a_0 – the Bohr Radius, m_e – the electron mass, ϵ_0 – electric constant, it follows:

$$a_0 \alpha m_e = m_p l_p; \alpha = \frac{m_p l_p}{a_0 m_e}; m_p = \frac{a_0 \alpha m_e}{l_p}; l_p = \frac{a_0 \alpha m_e}{m_p}; m_e = \frac{m_p l_p}{a_0 \alpha}. \quad (15)$$

Considering the formula (11) the Coulomb's law for two elementary charges in gravitational form is:

$$F = k_e \frac{e^2}{r_{12}^2} = G \alpha \frac{m_p^2}{r_{12}^2}. \quad (16)$$

In the formula (16) k_e – is the proportionality factor (Coulomb's constant, or the electric force constant), r_{12} – is a distance between two elementary charges.

From formulas (11 – 16) it follows then that measurement units of electromagnetic interaction on the basis of constants l_p, m_p, t_p, α can be expressed with units of length, of mass and of time [6].

For instance:

Coulomb:

$$[C] = k_C \left[kg^{\frac{1}{2}} \cdot m^{\frac{1}{2}} \right], \tag{17}$$

Ampere:

$$[A] = k_A \left[kg^{\frac{1}{2}} \cdot m^{\frac{1}{2}} \cdot s^{-1} \right], \tag{18}$$

Volt:

$$[V] = k_V \left[kg^{\frac{1}{2}} \cdot m^{\frac{3}{2}} \cdot s^{-2} \right], \tag{19}$$

impedance:

$$[\Omega] = k_{\Omega} \left[m^1 \cdot s^{-1} \right], \tag{20}$$

electric capacitance:

$$[F] = k_F \left[m^{-1} \cdot s^2 \right], \tag{21}$$

inductance:

$$[H] = k_H \left[m^1 \right], \tag{22}$$

magnetic induction:

$$[Tl] = k_{Tl} \left[kg^{\frac{1}{2}} \cdot m^{-\frac{1}{2}} \cdot s^{-1} \right], \tag{23}$$

where: $k_C, k_A, k_V, k_{\Omega}, k_F, k_H, k_{Tl}$ – are dimensionless coefficients of proportionality between units of measurements. Given that units of electromagnetic interaction basis is a moving charge that is the function of moment of the mass, then electromagnetic interaction is particular case of gravitational interaction that also is confirmed by formulas (17 – 23).

Therefore in certain conditions exists gravitational-electromagnetic resonance [1,7,8]. The parameters of such resonance are defined by constants: l_p, m_p, t_p, α .

In [9] was shown that between fundamental constants exists fractal connectivity and basic characteristics of a muon are gold algebraic fractals of the Planck mass and of the Planck length.

Based on formulas (4 – 16) can be concluded that fundamental physic constants and spatially-energetic characteristics of the observable Universe – are multiplicative gold algebraic fractals of a muon.

Taking into account universal character of constants l_p, m_p, t_p, α , for gravitational and electro-magnetic interactions, i.e. for matter macro world, unity and the interrelationship of all spatial and structural levels of matter, it can be assumed that constants l_p, m_p, t_p, α are also take part in formation of micro world patterns but given it specific and the influence of scale. It is handier to look for relationship of constants representation l_p, m_p, t_p, α on macro and micro levels of matter along the lines of gold algebraic fractals, more particularly – on the basis of gold algebraic fractals mantissas for the physical constants case and mantissas functions for physical processes case.

2. Fractal genesis of the angles of the neutrino mixing matrix.

The angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$ of neutrino mixing matrix have the following values [10]:

$$\sin^2(\Theta_{12}) = 0.307 \pm 0.013, \tag{24}$$

$$\sin^2(\Theta_{23})_{nm} = 0.51 \pm 0.04 \text{ (normal mass hierarchy),} \tag{25}$$

$$\sin^2(\Theta_{23})_{im} = 0.50 \pm 0.04 \text{ (inverted mass hierarchy),} \tag{26}$$

$$\sin^2(\Theta_{13}) = (2.10 \pm 0.11) \cdot 10^{-2}, \tag{27}$$

or:

$$\sin(\Theta_{12}) = 0.55407580708780 \pm 0.114, \tag{28}$$

$$\sin(\Theta_{23})_{nm} = 0.714142842854284 \pm 0.2, \tag{29}$$

$$\sin(\Theta_{23})_{im} = 0.70710 \pm 0.2, \tag{30}$$

$$\sin(\Theta_{13}) = 0.144913767461894 \pm 0.03316624790, \tag{31}$$

then:

$$\Theta_{12} = 33.64708224^\circ, \Theta_{12+} = 34.44990199^\circ, \Theta_{12-} = 32.83473313^\circ, \quad (32)$$

$$\Theta_{23nm} = 45.572996^\circ, \Theta_{23nm+} = 47.86958524^\circ, \Theta_{23nm-} = 43.28009362^\circ, \quad (33)$$

$$\Theta_{23im} = 45^\circ, \Theta_{23im+} = 47.29428287^\circ, \Theta_{23im-} = 42.70571713^\circ, \quad (34)$$

$$\Theta_{13} = 8.33228569^\circ, \Theta_{13+} = 8.54931942^\circ, \Theta_{13-} = 8.10961446^\circ. \quad (35)$$

Let's say that sines of the angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$ are gold algebraic fractals of constants l_p, m_p, t_p, α . Represent the constants l_p, m_p, t_p, α , numerical values for whom are defined by the formulas (2, 12), in form of gold algebraic fractals (in GAF form) [4,9,11]:
mantissa of the Planck length:

$$l_p^m = 1.2865859866, \text{ then: } l_p = l_p^m \cdot f_g^{167} m, \quad (36)$$

mantissa of the Planck mass:

$$m_p^m = 1.1756969040, \text{ then: } m_p = m_p^m \cdot f_g^{37} kg, \quad (37)$$

mantissa of the Planck time:

$$t_p^m = 1.588954250534220, \text{ then: } t_p = t_p^m \cdot f_g^{208} s, \quad (38)$$

mantissa of the fine-structure constant:

$$\alpha^m = 1.4522098299, \text{ then: } \alpha = \alpha^m \cdot f_g^{11}. \quad (39)$$

In formulas(36 – 39) f_g - is large number of golden ratio: $\frac{1}{f_g} = \frac{f_g}{f_g^2}, f_g + f_g^2 = 1$, is equal:

$$f_g = \frac{\sqrt{5}}{2} - 0.5 = 2 \sin(18^\circ) = 2 \cos(72^\circ) = 0.61803398874989484820...$$

(40)

Some constants can be used in two positions: as direct or inverse values. For example, the inverse value of the fine-structure constant:

$$\frac{1}{\alpha} = 137.03599913815450, \quad (41)$$

in the form of gold algebraic fractals is:

GAF mantissa of the inverse value of the fine structure constant:

$$\alpha^{mi} = 1,11418746481018987150, \text{ then: } \frac{1}{\alpha} = \alpha^{mi} \cdot f_g^{-10}. \quad (42)$$

It is obvious that for all constants mantissas and for their inversions, on example of fine-structure constant is true:

$$\frac{1}{f_g} = \alpha^{mi} \cdot \alpha^m = 1.61803398874989484820... \quad (43)$$

Define first five levels of gold algebraic fractals of the constants: l_p, m_p, t_p, α .

For this sequentially multiply five times the mantissas of this constants by f_g . The results are presented in the table 1:

Table1

Mantissa name	Level1	Level2	Level3	Level4	Level5
Planck`s length	0.79515387	0.49143212	0.30372175	0.1877103	0.11601
Planck`s mass	0.726620	0.44907	0.27754439	0.17153187	0.106012
Planck`s time	0.982028	0.6069265	0.3751012	0.23182529	0.14327590
Fine structure constant	0.89751503	0.55469479	0.3428202	0.21187456	0.13094568

Believing that numbers in the table 1 – are the sines of some angles will present their values in table 2:

Table2

Constant name	Level1	Level2	Level3	Level4	Level5
Planck`s length	52.669791°	29.4347540°	17.681277°	10.819194°	6.6619637°
Planck`s mass	46.6038340°	26.684433°	16.11370°	9.876896°	6.08550°
Planck`s time	79.1209132°	37.3675974°	22.03057°	13.404558°	8.2374534°
Fine structure constant	63.833328°	33.68969567°	20.048792°	12.232228°	7.5242428°

If the hypothesis under consideration is right then the true values of the angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$, the following:

$$\Theta_{12} = 33.68969567^\circ [\text{fine - structure constant, level 2}], \tag{44}$$

$$\Theta_{23} = 46.6038340^\circ [\text{Planck`s mass, level 1}], \tag{45}$$

$$\Theta_{13} = 8.2374534^\circ [\text{Planck`s time, level 5}]. \tag{46}$$

Angles represented in expressions (44-46) are within tolerance limits of the angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$, which are given in the expressions (32 – 35).

In [9] is shown that the main characteristics of a muon are gold algebraic fractals of the fundamental constants: of the Planck mass and of the Planck length. Considering:

$$c = \frac{l_p}{t_p},$$

it could be argued that main characteristics of a real elementary particle – muon, underlies the fractal genesis of the angles (44, 45, 46).

In [5] was stated that the angle Θ_{13} is close to the integral angle of the Carioles force (which is equal to 9.79°) in the model of rotating observable Universe. According to updated data, this angle is closer to the angle: $\angle[\text{Planck`s mass, level 4}] = 9.876896^\circ$. Considering that genesis if the angles $\Theta_{12}, \Theta_{23}, \Theta_{13}$, and also the integral angle of the Carioles force and of their fractals is determined by Big Bang, it is conceivable that the integral angle of the Carioles force and the angle $\angle[\text{Planck`s mass, level 4}] = 9.876896^\circ$ – is the one and the same angle. Then contribution of dark matter in general energetic balance of Universe will be not 26.8%, but 26.88%, including contribution of the Carioles force constitutes 6.88%.

Conclusions. On the basis of gold algebraic fractals it is possible to research physical processes at different structural and spatial levels of matter. If the hypothesis under consideration is right then gold algebraic fractals of neutrino mixing matrix are related to the gold algebraic fractals of Planck constants, of a muon main characteristics and of the fine-structure constant.

3. Additive gold algebraic fractals.

In [4,9] were examined simple and multiplicative gold algebraic fractals. On example of the abstract physical constant $0 < A \leq 1$, consider additive gold algebraic fractals.

For this represent the constant A in the form of gold algebraic fractal:

$$A = m^A \cdot f_g^n, \tag{47}$$

where: m^A – GAF mantissa, n – integer number, value of structure level of GAF in doing so:

$$1 \leq m^A \leq \frac{1}{f_g}, 0 \leq n < \infty. \tag{48}$$

It is evident that: $\frac{1}{A}$ can be represented as: $\frac{1}{A} = m^{Ai} \cdot f_g^{-(n-1)}$, (49)

where m^{Ai} – is inversion of the mantissa m^A , wherein under the condition of formula (43):

$$\frac{1}{f_g} = m^A \cdot m^{Ai}. \tag{50}$$

Explore the mantissa m^A . For this we'll look at the power series as follows:

$$f_g^1 + f_g^2 + f_g^3 + \dots + f_g^k + \dots = \sum_{k=2}^{\infty} f_g^{k-1}. \tag{51}$$

It is obvious that the power series (51) – is geometrical progression given that it's every element z can be computed under formula:

$$f_g^z = f_g^1 \cdot f_g^{z-1}, 2 \leq z < \infty. \tag{52}$$

The series (51) makes sense as much as the following conditions are met:

- 1) the necessary test for the convergence: $\lim_{d \rightarrow \infty} f_g^d = 0, d = 1, 2, 3, \dots$
- 2) the sufficient test for the convergence:

$$\text{a) } d^{\text{Alambert's}}: \lim_{d \rightarrow \infty} \frac{f_g^{d+1}}{f_g^d} = f_g^1 < 1,$$

$$\text{b) } \text{the Cauchy's radical test: } \lim_{d \rightarrow \infty} \sqrt[d]{f_g^d} = f_g^1 < 1.$$

From the formula (52) follows that the number $f_g^1 = f_g$ – is denominator of the series (51), because $f_g^1 < 1$, than the series (51) – is decreasing geometrical progression, then the sum of the series is:

$$\sum_{k=2}^{\infty} f_g^{k-1} = \frac{f_g^1}{1-f_g} = \frac{f_g^1}{f_g^2} = f_g^{-1} = \frac{1}{f_g} = 1,61803398874989484820... \quad (53)$$

For the series:

$$f_g^2 + f_g^3 + f_g^4 + \dots + f_g^k + \dots = \sum_{k=3}^{\infty} f_g^{k-1}, \quad (54)$$

amount is:

$$\sum_{k=3}^{\infty} f_g^{k-1} = \frac{f_g^2}{1-f_g} = \frac{f_g^2}{f_g^2} = f_g + f_g^2 = 1. \quad (55)$$

For the series:

$$f_g^3 + f_g^4 + f_g^5 + \dots + f_g^k + \dots = \sum_{k=4}^{\infty} f_g^{k-1}, \quad (56)$$

amount is:

$$\sum_{k=4}^{\infty} f_g^{k-1} = \frac{f_g^3}{1-f_g} = \frac{f_g^3}{f_g^2} = f_g = 0,61803398874989484820... \quad (57)$$

For series:

$$f_g^4 + f_g^5 + f_g^6 + \dots + f_g^k + \dots = \sum_{k=5}^{\infty} f_g^{k-1}, \quad (58)$$

amount is:

$$\sum_{k=5}^{\infty} f_g^{k-1} = \frac{f_g^4}{1-f_g} = \frac{f_g^4}{f_g^2} = f_g^2 = 0,381966011250105151795... \quad (59)$$

For series:

$$f_g^5 + f_g^6 + f_g^7 + \dots + f_g^k + \dots = \sum_{k=6}^{\infty} f_g^{k-1}, \quad (60)$$

amount is:

$$\sum_{k=6}^{\infty} f_g^{k-1} = \frac{f_g^5}{1-f_g} = \frac{f_g^5}{f_g^2} = f_g^3 = 0,23606797749978969640... \quad (61)$$

Let:

$$i = 1, 2, 3, \dots \rightarrow \infty,$$

then in general case for considered series is true:

$$f_g^i + f_g^{i+1} + f_g^{i+2} + \dots + f_g^k + \dots = \sum_{k=i+1}^{\infty} f_g^{k-1} = f_g^{k-3}. \quad (62)$$

For example consequently for $k = 7, 8, 9, 10, 11, 12$, will get:

$$f_g^4 = 0,145898033750315455386..., \quad (63)$$

$$f_g^5 = 0,09016994374947424102..., \quad (64)$$

$$f_g^6 = 0,055728090000841214..., \quad (65)$$

$$f_g^7 = 0,0344418537486330267..., \quad (66)$$

$$f_g^8 = 0,02128623625220818770..., \quad (67)$$

$$f_g^9 = 0,01315561749642483896... \quad (68)$$

It is evident that at $k \rightarrow \infty$, is true:

$$f_g^i + f_g^{i+1} + f_g^{i+2} + \dots + f_g^k + \dots = \sum_{k=i+1}^{\infty} f_g^{k-1} = f_g^{k-3} \rightarrow 0. \tag{69}$$

GAF mantissa of any physical constant consisted of two parts: first part – is constant, which inherent for all mantissas – this is a number 1, wherein:

$$f_g^0 = f_g + f_g^2 = 1, \tag{70}$$

second part – is constant, let's define it as: m^{Ac} , which inherent only for particular GAF. It's characteristic number reflecting the specificities of the given fractal:

$$0 \leq m^{Ac} \leq f_g, \tag{71}$$

that is GAF m^A can be represented as:

$$m^A = 1 + m^{Ac} = f_g + f_g^2 + m^{Ac}. \tag{72}$$

On the basis of the formulas (51 – 69) it can be argued that an any characteristic number m^{Ac} can be represented with specified accuracy \mathcal{E} in the form of series sum of the pattern (62). Let S – is the series sum of the form (62), representing number m^{Ac} with accuracy \mathcal{E} , i.e.:

$$|m^{Ac} - S| \leq \mathcal{E}. \tag{73}$$

Algorithm of the amount finding S is following:

- 1) find the amount S_1 , as series of (62) type, which value as close as possible to the number m^{Ac} , but is less than it on modulus;
- 2) define the accuracy \mathcal{E}_1 :

$$m^{Ac} - S_1 = \mathcal{E}_1, \tag{74}$$

- 3) verify the ratio of accuracy \mathcal{E} and \mathcal{E}_1 , if: $\mathcal{E} < \mathcal{E}_1$, then:
- 4) define the amount S_2 , as series of (62) type, which value as close as possible to the number \mathcal{E}_1 , but is less than it on modulus;
- 5) define the accuracy \mathcal{E}_2 :

$$m^{Ac} - (S_1 + S_2) = \mathcal{E}_2, \tag{75}$$

- 6) verify the ratio of accuracy \mathcal{E} and \mathcal{E}_2 , if: $\mathcal{E} < \mathcal{E}_2$, then:
- 7) define the amount S_3 , as series of (62) type, which value as close as possible to the number \mathcal{E}_2 , but is less than it on modulus;
- 8) define the accuracy \mathcal{E}_3 :

$$m^{Ac} - (S_1 + S_2 + S_3) = \mathcal{E}_3, \tag{76}$$

- 9) verify the ratio of accuracy \mathcal{E} and \mathcal{E}_3 , and then:
- 10) do iterations of (74 – 76) type until the condition (73) will be met, then:

$$S = S_1 + S_2 + \dots + S_b, \tag{77}$$

where b – is the iterations quantity of (74 – 76) type, required to met the (73) condition.

N.B.: if characteristic number of the mantissa m^{Ac} is close to (59 – 68) or multiply to it, then the algorithm of it representation as power series is simplified.

From (70, 72, 77) it follows that any physical constant's mantissa can be represented as additive gold algebraic fractal with any specified accuracy.

Let's see the examples. Represent first ten numbers of a number sequence as additive gold algebraic fractals:

$$1 = (f_g + f_g^2) \cdot f_g^0, \tag{78}$$

$$2 = (f_g + f_g^2 + f_g^3) \cdot f_g^{-1}, \tag{79}$$

$$3 = (f_g + f_g^2 + f_g^4) \cdot f_g^{-2}, \tag{80}$$

$$4 = (f_g + 2 \cdot f_g^2 + f_g^4) \cdot f_g^{-2}, \tag{81}$$

$$5 = (f_g + f_g^2 + 2 \cdot f_g^5) \cdot f_g^{-3}, \tag{82}$$

$$6 = (f_g + 2 \cdot f_g^2 + f_g^7) \cdot f_g^{-3}, \quad (83)$$

$$7 = (f_g + f_g^2 + f_g^8) \cdot f_g^{-4}, \quad (84)$$

$$8 = (f_g + f_g^2 + f_g^4 + f_g^8) \cdot f_g^{-4}, \quad (85)$$

$$9 = (f_g + f_g^2 + f_g^3 + f_g^6 + f_g^8) \cdot f_g^{-4}, \quad (86)$$

$$10 = (f_g + 2 \cdot f_g^2 + f_g^6 + f_g^8) \cdot f_g^{-4}, \quad (87)$$

where $f_g^i, i = 1, 2, \dots, 8$ – determined by (57 – 67). Represent as additive gold algebraic fractals – the mass of some elementary particles: leptons (except for neutrino): a electron e , a muon μ , a tau τ and bosons: Higgs boson H^0 , Z^0 boson, W^\pm boson. Wherein will use not the energetic but the gravitational equivalent of a mass. Mass values [12] of the above-mentioned elementary particles:

$$\text{electron mass } m_e : m_e = 9.10938356(11) \cdot 10^{-31} \text{ kg}, \quad (88)$$

$$\text{muon mass } m_\mu : m_\mu = 1.883531594(48) \cdot 10^{-28} \text{ kg}, \quad (89)$$

$$\text{tau mass } m_\tau : m_\tau = 3.16747(29) \cdot 10^{-27} \text{ kg}, \quad (90)$$

$$\text{Higgs boson mass } m_{H^0} : m_{H^0} = 2.2299(43) \cdot 10^{-25} \text{ kg}, \quad (91)$$

$$Z^0 \text{ boson mass } m_{Z^0} : m_{Z^0} = 1.625567(37) \cdot 10^{-25} \text{ kg}, \quad (92)$$

$$W^\pm \text{ boson mass } m_{W^\pm} : m_{W^\pm} = 1.43299(27) \cdot 10^{-25} \text{ kg}. \quad (93)$$

Mass values of elementary particles (88 – 93) in the form of gold algebraic fractals:

mantissa of the electron mass:

$$e^m = 1.13164233168890, \text{ then: } m_e = e^m \cdot f_g^{144} \text{ kg}, \quad (94)$$

mantissa of the muon mass:

$$\mu^m = 1.175788105912, \text{ then: } m_\mu = \mu^m \cdot f_g^{133} \text{ kg}, \quad (95)$$

mantissa of the tau mass:

$$\tau^m = 1.10190162111, \text{ then: } m_\tau = \tau^m \cdot f_g^{127} \text{ kg}, \quad (96)$$

mantissa of the Higgs boson mass:

$$H^{0,m} = 1.02054727240, \text{ then: } m_{H^0} = H^{0,m} \cdot f_g^{118} \text{ kg}, \quad (97)$$

mantissa of the Z^0 boson mass:

$$Z^{0,m} = 1.20374351254, \text{ then: } m_{Z^0} = Z^{0,m} \cdot f_g^{119} \text{ kg}, \quad (98)$$

mantissa of the W^\pm boson mass:

$$W^{\pm,m} = 1.0611412325, \text{ then: } m_{W^\pm} = W^{\pm,m} \cdot f_g^{119} \text{ kg}. \quad (99)$$

Represent mantissas from formulas (94 – 99) as of gold algebraic fractals.

Mantissa of the electron mass: $e^m = 1.13164233168890$, is close to multiply value of $f_g^9 = 0.01315561749642483896\dots$, therefore approximately it can be represented:

with accuracy 10^{-3} :

$$e^m \approx f_g + f_g^2 + 10 \cdot f_g^9, \quad (100)$$

with accuracy 10^{-5} :

$$e^m \approx f_g + f_g^2 + 10 \cdot f_g^9 + f_g^{20} + f_g^{23} + f_g^{26} = 1.1316415716769572.$$

(101)

Approximation representation with accuracy 10^{-5} mantissa of the electron mass: $e^m = 1.13164233168890$

In the form of additive algebraic gold fractal on the algorithm (74 -77):

$$e^m \approx f_g + f_g^2 + f_g^5 + f_g^7 + f_g^{11} + f_g^{13} + f_g^{20} + f_g^{23} + f_g^{26} = 1.1316415716769572.$$

(102) Approximation representation of the electron mass in the form of additive gold algebraic fractal:

$$m_e \approx (f_g + f_g^2 + f_g^5 + f_g^7 + f_g^{11} + f_g^{13} + f_g^{20} + f_g^{23} + f_g^{26}) \cdot f_g^{144} \text{ kg}. \quad (103)$$

Mantissa of the muon mass: $\mu^m = 1.175788105912$ can be represented approximately as follows:

with accuracy 10^{-3} :

$$\mu^m \approx \sqrt{f_g + 2 \cdot f_g^2} = 1.175570504585, \tag{104}$$

with accuracy 10^{-5} :

$$\mu^m \approx f_g + f_g^2 + f_g^4 + f_g^8 + f_g^{10} + f_g^{16} + f_g^{24} = 1.175783598349. \tag{105}$$

Approximation representation of the muon mass in the form of additive gold algebraic fractal:

$$m_\mu \approx (f_g + f_g^2 + f_g^4 + f_g^8 + f_g^{10} + f_g^{16} + f_g^{24}) \cdot f_g^{133} \text{ kg}. \tag{106}$$

Mantissa of the tau mass: $\tau^m = 1.10190162111$ can be represented approximately with 10^{-5} in the form of additive gold algebraic fractal:

$$\tau^m \approx f_g + f_g^2 + f_g^5 + f_g^{10} + f_g^{12} + f_g^{16} + f_g^{21} = 1.1019001427232. \tag{107}$$

Approximation representation of the tau mass in the form of additive gold algebraic fractal:

$$m_\tau \approx (f_g + f_g^2 + f_g^5 + f_g^{10} + f_g^{12} + f_g^{16} + f_g^{21}) \cdot f_g^{127} \text{ kg}. \tag{108}$$

Mantissa of the Higgs boson mass: $H^{0,m} = 1.02054727240$ can be represented in the form of additive algebraic gold fractal:

$$H^{0,m} = f_g + f_g^2 + f_g^8 = 1.02128623625220. \tag{109}$$

Value of the Higgs boson mass in energetic equivalent is [12]:

$$m_{H^0} = 125.09 \pm 0.24 \text{ GeV}. \tag{110}$$

Value of the Higgs boson mass in energetic equivalent with (109) represented in the form of additive algebraic gold fractal is:

$$m_{H^0} = (f_g + f_g^2 + f_g^8) \cdot f_g^{118} = 125.18057589962250 \text{ GeV}, \tag{111}$$

what is within tolerance on formula (110). Should note that the Higgs boson mass and number 7 (formula (84)) – are one and the same additive algebraic gold.

Mantissa of the Z^0 boson mass: $Z^{0,m} = 1.20374351254$ can be represented approximately with accuracy 10^{-6} in the form of additive gold algebraic fractal:

$$Z^{0,m} \approx f_g + f_g^2 + f_g^4 + f_g^6 + f_g^{13} + f_g^{20} + f_g^{22} = 1.203743823360. \tag{112}$$

Approximation representation of the Z^0 boson mass in the form of additive gold algebraic fractal:

$$m_{Z^0} \approx (f_g + f_g^2 + f_g^4 + f_g^6 + f_g^{13} + f_g^{20} + f_g^{22}) \cdot f_g^{119} \text{ kg}. \tag{113}$$

Should note that characteristic number inversion of the mantissa of the Z^0 boson mass approximately with accuracy 10^{-3} multiply to the value f_g^7 by formula (66), therefore inversion of mantissa's Z^0 boson mass approximately in the form of additive gold algebraic fractal:

$$Z^{0,mi} \approx f_g + f_g^2 + 10 \cdot f_g^7. \tag{114}$$

Mantissa of the W^\pm boson mass: $W^{\pm,m} = 1.0611412325$ can be represented approximately with accuracy 10^{-5} in the form of additive gold algebraic fractal:

$$W^{\pm,m} \approx f_g + f_g^2 + f_g^6 + f_g^{12} + f_g^{13} + f_g^{17} + f_g^{19} = 1.0611400856. \tag{115}$$

Approximate value of the W^\pm boson mass in the form of gold algebraic:

$$m_{W^\pm} \approx (f_g + f_g^2 + f_g^6 + f_g^{12} + f_g^{13} + f_g^{17} + f_g^{19}) \cdot f_g^{119} \text{ kg}. \tag{116}$$

Conclusions. Formulas (78 – 87) and (100 – 116) demonstrate that any physical constants which have any numerical value can be represented with specified accuracy in the form of additive gold algebraic fractals. Probably the Higgs boson mass and number 7 are one and the same gold algebraic fractal. In this case the Higgs boson mass can be defined accurately.

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