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OPTIMIZATION OF MULTILAYERED ELECTRO-VISCOELASTIC PLATES

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ОПТИМІЗАЦІЯ ПАРАМЕТРІВ БАГАТОШАРОВИХ ПЛАСТИН З ЕЛЕКТРОВ'ЯЗКОПРУЖНИХ МАТЕРІАЛІВ

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ОПТИМИЗАЦИЯ ПАРАМЕТРОВ МНОГОСЛОЙНЫХ ПЛАСТИН ИЗ ЭЛЕКТРОВЯЗКОУПРУГИХ МАТЕРИАЛОВ

Mathematical modeling of vibrations of multilayered electro-viscoelastic material plates with passive energy dissipation and the optimal design problem of plates by maximum damping criterion using genetic algorithms are considered.

Key words: *multilayered plates, electro-viscoelastic materials, passive energy dissipation, vibration damping, optimization, genetic algorithms.*

Розглядаються методи математичного моделювання коливань пластин з шарами електров'язкопружних матеріалів з пасивним розсіянням енергії і задача оптимального проектування таких пластин за критерієм максимального демпфірування за допомогою методу оптимізації на основі генетичного алгоритму.

Ключові слова: *багатошарові пластини, електров'язкопружні матеріали, пасивне розсіяння енергії, демпфірування коливань, оптимізація, генетичні алгоритми.*

Рассматриваются методы математического моделирования колебаний пластин со слоями электровязкоупругих материалов с пассивным рассеянием энергии и задача оптимизации таких пластин по критерию максимального демпфирования с помощью метода оптимизации на основе генетического алгоритма.

Ключевые слова: *многослойные пластины, электровязкоупругие материалы, пассивное рассеяние энергии, демпфирование колебаний, оптимизация, генетические алгоритмы.*

Introduction. Thin-walled structural elements of viscoelastic composite materials are widely used in engineering, aerospace, shipbuilding and other fields of technology.

The advantages of fiber-reinforced composite materials in comparison with homogeneous ones are high specific stiffness, wide operating temperature range and the potential for significant variable damping, which is the determining factor for structures working under dynamic loads. The problem of vibration damping of structures with homogeneous structural materials, formulated by G. S. Pysarenko and his school [1, 2, 3], has received further development towards the using of composite materials [3, 4] and rational distribution of damping materials in structures to ensure maximum operational parameters, in particular, energy dissipation [4].

A potential for wide variation of physical and mechanical properties by changing the design parameters, including changes in fiber orientation and fiber concentration in order to obtain materials with desired properties was the main thing that attracted designers. This led to the possibility of design optimization by criteria of stiffness, mass, damping, and others [4].

The appearance of materials with special properties (piezoelectric materials, shape-memory alloys, etc.) intensified research towards the creation of structures with maximum passive and regulated energy dissipation [5]. The development of this line of investigations is associated primarily with research of V. H. Karnaukhov's school [6, 7] in Ukraine and Ural Branch of Russian Academy of Sciences under the direction of V. Matveyenko [8] in Russia.

Foreign scientists' research started to develop intensively several years earlier and at present have reached the level of practical use of active and passive damping in the special purpose structures [9].

Efficient use of materials with special properties is associated with development and application of the optimization techniques. An optimization problem involves finding the vector of design parameters that provides extreme objective function value under given constraints on the design parameters. This is a so-called conditional global optimization problem. Most optimization methods are aimed at searching for local extremum, but there are techniques that can increase the likelihood of determining the global extremum.

In this paper a method based on the use of the genetic algorithm [10] is applied.

Mathematical Model of a Multi-Layer Plate. This paper presents a method of selecting optimal parameters for multilayered plates with electro-viscoelastic material layers having modified damping properties due to external damping electrical circuits (shunts).

Dynamics variational equation [5, 11] of electro-viscoelastic solid of volume V, bounded by area S with the external forces p_S set on one part of S_1, and the electric charges q_S on the other part of S_2, was used to construct a mathematical model of the dynamics of the electro-viscoelastic material plate:

$$\rho\int_V \delta u^T \frac{d^2u}{dt^2} dV + \int_V \delta\varepsilon^T \sigma dV - \int_{S_1} \delta u^T p_S dS - \int_V \delta E^T D dV + \int_{S_2} \delta\varphi^T q_S dS = 0, \quad (1)$$

where u, ε, σ are displacements, strains and stresses respectively, E – electric field, D – electric displacement, φ – potential.

Dividing the plate by depth into n layers (Fig. 1a) we obtain the computational equations for the i-th layer.

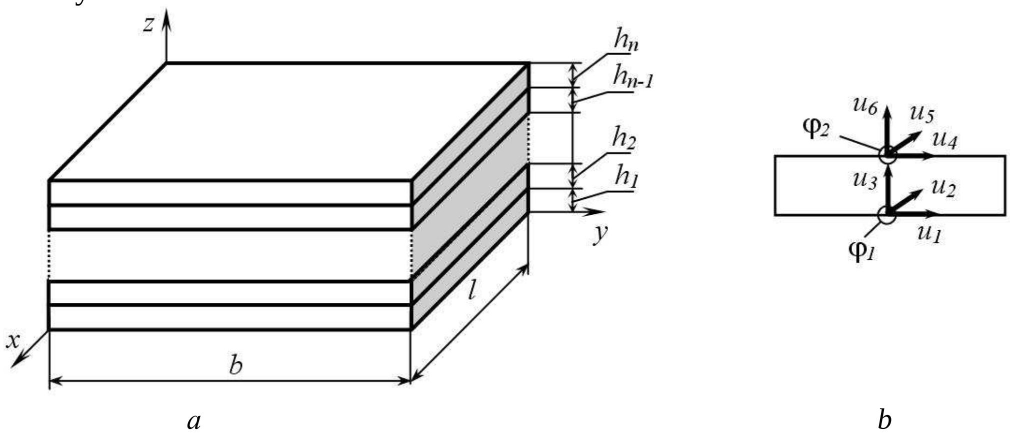

Fig. 1. Multi-layer plate
a – nodal planes, b – nodal displacements and potentials

To approximate the displacements and potentials by plate depth the linear Lagrange polynomial is used, and in the plane layers the globally defined functions of coordinates in compliance with the fixation conditions at the ends are used. The parameters of a layer, either viscoelastic or electro-viscoelastic one, are calculated according to the accepted synthesis method of multilayered plates [12], then the full package of layers is formed in a traditional way, using boundary conditions on the contact surfaces.

The linear approximation of the displacements u and the potential φ by depth in the i-th layer is accepted as:

$$u = N_u u_k, \quad \varphi = N_\varphi \varphi_k, \quad (2)$$

where N_u, N_φ are approximation function matrices, u_k, φ_k – nodal displacement and potential (Fig. 1b):

$$N_u = \begin{bmatrix} N_z^1 N_{xy}^u & N_z^1 N_{xy}^v & N_z^1 N_{xy}^w & 0 & 0 & 0 \\ 0 & 0 & 0 & N_z^2 N_{xy}^u & N_z^2 N_{xy}^v & N_z^2 N_{xy}^w \end{bmatrix},$$

$$N_\varphi = \begin{bmatrix} N_z^1 N_{xy}^\varphi & N_z^2 N_{xy}^\varphi \end{bmatrix}, \quad N_Z^1 = (1 - \frac{z}{h}), \; N_z^2 = \frac{z}{h}.$$

$$u = [u_1, u_2, u_3, u_4, u_5, u_6], \quad \varphi = [\varphi_1, \varphi_2]. \tag{3}$$

Functions $N_{xy}^u, N_{xy}^v, N_{xy}^w, N_{xy}^\varphi$ are selected according to the fixation conditions of the layer on the edges of the plate.

The strain tensor (in vector notation) and the electric field vector considering discretization are given as:

$$\varepsilon = A_u N_u u, \quad E = -A_\varphi N_\varphi \varphi, \tag{4}$$

where

$$A_u = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix}^T, \quad A_\varphi = -\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial z}{\partial} \end{bmatrix}. \tag{5}$$

As is known [11], most piezoelectric materials exhibit properties of linear hereditary environment at low strain and electric field intensity values.

Linear dependences of the hereditary theory of electro-viscoelastic material are given as:

$$\begin{aligned} \sigma(X,t) &= \int_0^t R_c(X, t-\tau)\dot{\varepsilon}(X,\tau)d\tau - \int_0^t R_e(X, t-\tau)\dot{E}(X,\tau)d\tau, \\ D(X,t) &= \int_0^t K_e(X, t-\tau)\dot{\varepsilon}(X,\tau)d\tau + \int_0^t R_k(X, t-\tau)\dot{E}(X,\tau)d\tau, \end{aligned} \tag{6}$$

where $R_c(X, t-\tau), R_e(X, t-\tau), R_k(X, t-\tau)$ are relaxation functions matrices of viscoelastic, piezoelectric and dielectric material properties respectively, $\sigma(X,t), D(X,t)$ – stress and electric displacement, $\dot{\varepsilon}(X,t), \dot{E}(X,t)$ – electric field change and deformation velocity respectively, X – coordinate axes vector.

We express the equation (6) using the concept of convolution functions a*b $= \int_0^t a(X, t-\tau)\dot{b}(X,\tau)d\tau$, and obtain

$$\sigma = R_c * \dot{\varepsilon} - R_e * \dot{E}, \quad D = R_e * \dot{\varepsilon} + R_k * \dot{E}. \tag{7}$$

After substituting physical dependences (7) in equation (1), we get the variational dynamic equations of the i-th electro-viscoelastic layer as:

$$\int_V (\delta u)^T \rho \ddot{u} dV + \int_V (\delta \varepsilon)^T \left(R_c * \dot{\varepsilon} - R_e * \dot{E} \right) dV - \int_{S_1} (\delta u)^T \ p_S dS - \\ - \int_V (\delta E)^T \left(R_e * \dot{\varepsilon} + R_k * \dot{E} \right) dV - \int_{S_2} (\delta \varphi)^T \ q_\varphi dS = 0. \qquad (8)$$

Further we use the displacements and potential approximations introduced above (2)-(4), and take the displacement and potential vector variations outside the integral sign, equating to zero the multipliers of variations. As a result, we obtain a system of integro-differential equations for the vector potential and displacements as:

$$M_{uu}\ddot{u}_k + K_{uu} * \dot{u}_k + K_{u\varphi} * \dot{\varphi}_k = F, \\ K_{\varphi u} * \dot{u}_k + K_{\varphi\varphi} * \dot{\varphi}_k = Q, \qquad (9)$$

where

$$M_{uu} = \int_V \rho N_u^T N_u dV, \ K_{uu} = \int_V B_u{}^T R_c B_u dV, \quad K_{u\varphi} = \int_V B_u{}^T R_e B_\varphi dV, \\ K_{\varphi u} = \int_V B_\varphi{}^T R_e B_u dV, \quad K_{\varphi\varphi} = \int_V B_\varphi{}^T R_k B_\varphi dV, \quad F = \int_S N_u{}^T p_S dS, \quad Q = \int_S N_\varphi{}^T q_S dS. \qquad (10)$$

We apply to (9) the direct Fourier transform [13] and obtain the equation system for the Fourier images of displacements $\tilde{u}$ and potential $\tilde{\varphi}$:

$$-\omega^2 M_{uu}\tilde{u} + \tilde{K}_{uu}\tilde{u} + \tilde{K}_{u\varphi}\tilde{\varphi} = \tilde{F} + f, \\ \tilde{K}_{\varphi u}\tilde{u} + \tilde{K}_{\varphi\varphi}\tilde{\varphi} = \tilde{Q}, \qquad (11)$$

where $f = j\omega M_{uu}\dot{u}(0) + M_{uu}u(0)$, $j = \sqrt{-1}$, and $u(0)$, $\dot{u}(0)$ are initial displacement and initial velocity of layer nodal points respectively,

$$M_{uu} = \int_V \rho N_u^T N_u dV, \ \tilde{K}_{uu} = \int_V B_u{}^T \tilde{c} B_u dV, \quad \tilde{K}_{u\varphi} = \int_V B_u{}^T \tilde{e} B_\varphi dV, \\ \tilde{K}_{\varphi u} = \int_V B_\varphi{}^T \tilde{e}^T B_u dV, \quad \tilde{K}_{\varphi\varphi} = \int_V B_\varphi{}^T \tilde{k} B_\varphi dV, \quad \tilde{F} = \int_S N_u{}^T \tilde{p}_S dS, \quad \tilde{Q} = \int_S N_\varphi{}^T \tilde{q}_S dS,$$

where M_{uu} – mass matrix, $\tilde{K}_{uu}, \tilde{K}_{u\varphi}, \tilde{K}_{\varphi u}, \tilde{K}_{\varphi\varphi}$ – stiffness, piezoelectric, coupling and capacitance complex matrices respectively, $\tilde{F}, \tilde{Q}$ – the external mechanical force and electric charge, $\tilde{c}, \tilde{e}, \tilde{k}$ – the viscoelasticity constants, the piezoelectric coupling coefficients, the dielectric constants complex matrices respectively.

The equation (11) differs from (9) by a reasonable potential for using frequency-dependent modules in analysis of non-stationary and multi-frequency vibrations [3, 4], and the capability to account initial conditions.

It is known [7] that the application of piezoelectric materials is caused not due to their vibration damping ability, which is comparably low against special viscoelastic materials, but due to the potential of increasing the passive damping by converting mechanical energy into electrical energy and then into heat (sensors), and by creating efforts opposite to structure deformation (actuators).

Without further analysis of costs and benefits of these two methods, in this paper we restrict ourselves to the passive damping analysis and obtain the piezoelectric layer vibration equations, where the damping increase is caused by the use of special devices (shunts).

Note that using electrical shunts makes possible the creating of special materials capable of reacting to the effect of an electric current by significant increase of damping properties.

Returning to equations (11), we consider the vibration of the plate with an attached RL-shunt (Fig. 2).

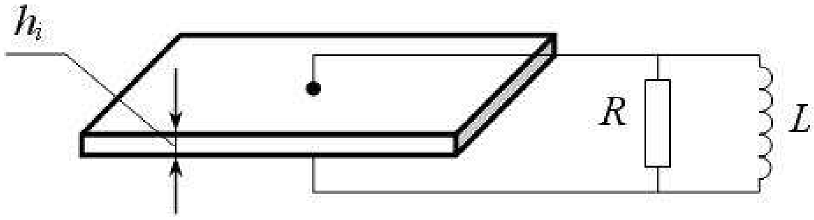

Fig. 2. Plate with an attached RL-shunt

As shown in [6, 9], vibration equation of the piezoelectric material plate can be used to analyze several cases of exploiting piezoelectric material properties. In particular, when the electrodes are supplied by the voltage (potential φ), the electromechanical load that makes the piezoelectric element function in the mode of excitation of oscillations (actuator) can be created. In this case the vibration equation is noted as:

$$-\omega^2 M\tilde{u} + \tilde{K}_{uu}\tilde{u} = \tilde{F} - \tilde{K}_{u\varphi}\tilde{\varphi}\,. \quad (12)$$

In an open circuit $Q = 0$, and the equation for the natural frequencies and forced vibration analysis is obtained as:

$$-\omega^2 M\tilde{u} + (\tilde{K}_{uu} - \tilde{K}_{u\varphi}\tilde{K}_{\varphi\varphi}^{-1}\tilde{K}_{\varphi u})\tilde{u} = \tilde{F}\,. \quad (13)$$

To account the attached external passive components, the equation (13) can be used, extending its matrix $\tilde{K}_{\varphi\varphi}$ with a matrix or a matrix sum that describes the attached external elements.

Specifically, for the parallel RL-shunt, matrix $\tilde{K}_{\varphi\varphi}$ is supplemented by matrices $\tilde{K}_{\varphi L}$ and $\tilde{K}_{\varphi R}$

$$-\omega^2 M\tilde{u} + (\tilde{K}_{uu} - \tilde{K}_{u\varphi}(\tilde{K}_{\varphi\varphi} + \tilde{K}_{\varphi R} + \tilde{K}_{\varphi L})^{-1} + \tilde{K}_{\varphi u})\tilde{u} = \tilde{F} + f\,, \quad (14)$$

$$\tilde{K}_{\varphi L} = \left(\frac{-1}{\omega^2 L}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \tilde{K}_{\varphi R} = \left(\frac{1}{i\omega R}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \quad (15)$$

The equation for the free vibration analysis considering additional damping and the initial conditions is obtained from (14) with $\tilde{F} = 0$ as:

$$-\omega^2 M\tilde{u} + (\tilde{K}_{uu} - \tilde{K}_{u\varphi}(\tilde{K}_{\varphi\varphi} - \tilde{K}_{\varphi R} + \tilde{K}_{\varphi L})^{-1}\tilde{K}_{\varphi u})\tilde{u} = j\omega M\dot{u}(0) + Mu(0). \quad (16)$$

Design of plate with maximum damping. Let us consider the example of calculation and parameters optimization of a three-layer plate with electro-viscoelastic outer layers and passive material internal layer (Fig. 3). To make the three-layer plate model we use the equations derived above for one layer, the constraints of displacement equality at the contiguous planes, as well as the constraints of potential equality at the connection nodes of external damping elements.

We use displacement and potential approximation functions in this form:

$$N_{xy}^{u} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\left(\frac{m\pi x}{l}\right)\cos\left(\frac{n\pi y}{b}\right), \quad N_{xy}^{w} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\left(\frac{m\pi x}{l}\right)\sin\left(\frac{n\pi y}{b}\right),$$

$$N_{xy}^{v} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos\left(\frac{m\pi x}{l}\right)\sin\left(\frac{n\pi y}{b}\right). \quad N_{xy}^{\varphi} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\cos\left(\frac{m\pi x}{l}\right)\sin\left(\frac{n\pi y}{b}\right).$$

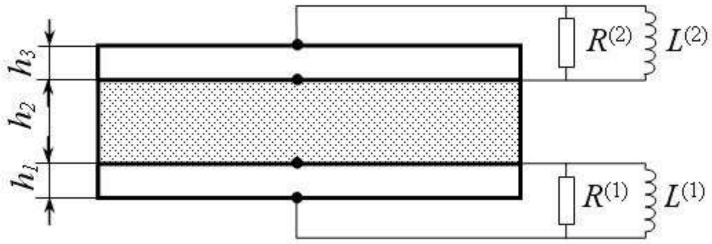

Fig. 3. Three-layer plate with parallel RL-shunts

The input parameters of the plate are dimensions, physical and mechanical properties of materials and shunt electrical parameters. Fundamentally all input parameters are subject to optimization, but are often limited to the most essential, including shunts parameters, layer thicknesses, and material properties. For multi-layered structures the quantity of optimization parameters can significantly increase.

In most cases to solve the problem of designing structures working under dynamic loads the nonlinear programming theory algorithms (NLP) are used.

With the increasing design complexity and the advent of new materials, classic NLP algorithms appeared ineffective due to the significant increase of the design parameters amount, complication of constraints and complex optimization criteria, defined in most cases by software. A significant drawback of classic NLP gradient algorithms is the complexity of calculating derivatives of objective functions and constraints, as well as fundamental disposition to determine only local extremum. These features interfered and continue to interfere with the broad implementation of classic NLP algorithms in the optimal structures design practice.

This led to active development of the search methods that use the principles of biology and genetics in recent years.

First of all, these are the genetic algorithms [14] with the basic idea to create a population of individuals in form of chromosomes, containing genes representing a set of heritable characteristics – design parameters. The best of the individuals is selected in an evolutionary search in accordance with the adopted fitness function. The evolutionary search process is implemented by using operators, similar to the hybridization, mutation, inversion biological processes. The population is updated with each by creating new and removing old individuals, resulting in ever-improving population in terms of compliance with the fitness function (objective function).

The justifications of using the binary alphabet to encode chromosomes, as well as many respective methods that use pre-encoding and binary chromosomes have appeared in the process of developing genetic (evolutionary) algorithms [14]. However, as it turned out, the binary coding has significant disadvantages when searching at continuous spaces with required accuracy, typical for problems of structural optimization. In this regard, the algorithms based on using chromosomes as a set of real numbers have been recently utilized for optimization in continuous space. Such algorithms are called RGA (real-coded GA) unlike BGA (binary-coded GA) [14].

In this work, the RGA algorithm described in [10] is used.

Examples of three-layer plate optimal parameters computation. We considered three-layer plate with input parameters:

– the plate dimensions $b = 0,4\,m, \quad l = 0,4\,m$;

– the outer layers thickness $h = 0,001\,m$.

Electro-viscoelastic material properties are given in Table.

Table

Material Properties

Coefficients	Outer Layer Material	Viscoelastic Middle Layer

	PZT-5	
Elastic moduli	$c11 = (1 + i0.001) \cdot 99.201 \cdot 10^9$	$c11 = (1 + i0.01) \cdot 183.443 \cdot 10^9$
	$c22 = (1 + i0.001) \cdot 99.201 \cdot 10^9$	$c22 = (1 + i0.01) \cdot 11.662 \cdot 10^9$
	$c33 = (1 + i0.001) \cdot 86.856 \cdot 10^9$	$c33 = (1 + i0.01) \cdot 11.662 \cdot 10^9$
	$c12 = (1 + i0.001) \cdot 54.016 \cdot 10^9$	$c12 = (1 + i0.01) \cdot 4.363 \cdot 10^9$
	$c13 = (1 + i0.001) \cdot 50.778 \cdot 10^9$	$c13 = (1 + i0.01) \cdot 4.363 \cdot 10^9$
	$c23 = (1 + i0.001) \cdot 50.778 \cdot 10^9$	$c23 = (1 + i0.01) \cdot 3.918 \cdot 10^9$
	$c44 = (1 + i0.001) \cdot 21.100 \cdot 10^9$	$c44 = (1 + i0.01) \cdot 2.870 \cdot 10^9$
	$c55 = (1 + i0.001) \cdot 21.100 \cdot 10^9$	$c55 = (1 + i0.01) \cdot 7.170 \cdot 10^9$
	$c66 = (1 + i0.001) \cdot 22.593 \cdot 10^9$	$c66 = (1 + i0.01) \cdot 7.170 \cdot 10^9$
Piezoelectric moduli	$e31 = -7.209 \cdot (1 + i0.001)$	$e31 = 0.0$
	$e32 = -7.209 \cdot (1 + i0.001)$	$e32 = 0.0$
	$e33 = 15.118 \cdot (1 + i0.001)$	$e33 = 0.0$
	$e24 = 12.322 \cdot (1 + i0.001)$	$e24 = 0.0$
	$e15 = 12.322 \cdot (1 + i0.001)$	$e15 = 0.0$
Permittivity	$z11 = (1 + i0.001) \cdot 153.0 \cdot 10^{-10}$	$z11 = 153.0 \cdot 10^{-9}$
	$z22 = (1 + i0.001) \cdot 153.0 \cdot 10^{-10}$	$z22 = 153,0 \cdot 10^{-9}$
	$z33 = (1 + i0.001) \cdot 153.0 \cdot 10^{-10}$	$z33 = 153.0 \cdot 10^{-9}$
Density	$g = 6.85 \cdot 10^3$	$g = 1.85 \cdot 10^3$

Some results of calculation and optimization of a three-layer plate are shown in Fig. 4-6.

We use the optimization method based on the genetic algorithm [10] to determine the optimal parameters of the plate by the maximum damping criterion. Design parameters vector is taken as

$$x = \left(h_2 \;\; R \;\; L\right),$$

where h_2 – viscoelastic middle layer thickness (m), R – resistance (Ω), L – inductance (henry).

Design parameters constraints are:

$$lb = \left[0,001 \;\; 10 \;\; 0,01\right],\; ub = \left[0,01 \;\; 500 \;\; 1\right].$$

The results of the calculation are: the objective function (decrement in the first mode) $\Delta = 0,5328$, frequency $\omega 1 = 1893,2$, maximum amplitude $a = 1,0921 \cdot 10^{-6}\, m$.

Optimal design parameter vector is $x = \left[0,0100 \;\; 500,0000 \;\; 0,3642\right]$.

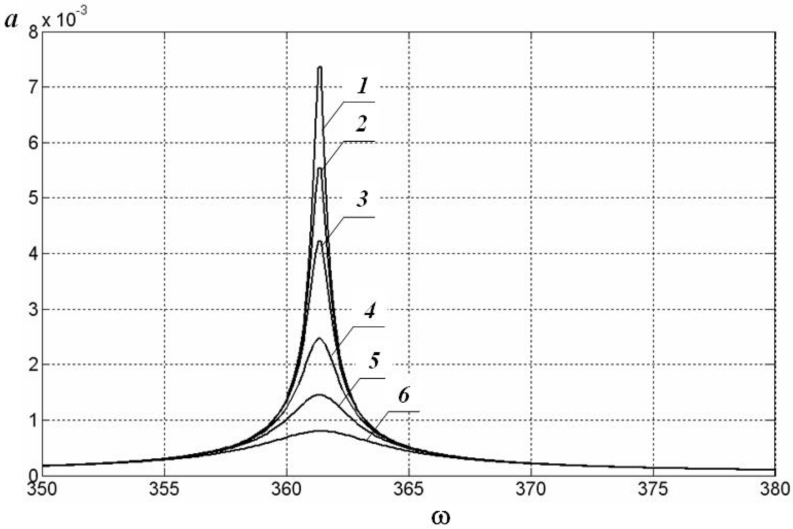

Fig. 4. Frequency response on the first vibration plate mode at zero inductance L and different resistance R:
1 – R=1 Ω ; 2 – R=5 Ω ; 3 – R=10 Ω ; 4 – R=25 Ω ; 5 – R=50 Ω ; 6 – R=100 Ω

In Fig. 5 the frequency responses for optimal design and an arbitrary vector of design parameters with decrement, first frequency and maximum amplitude $\Delta = 0,3075$, $\omega 1 = 995,8709,$, $a = 9,6087 \cdot 10^{-6}\ m$ respectively are shown.

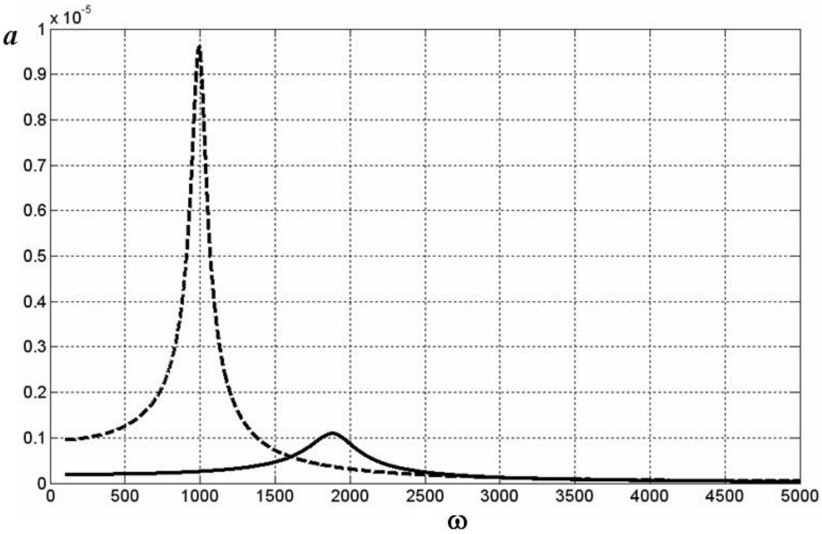

Fig. 5. Frequency response for optimal (solid) and random (dashed) design parameters values

In Fig. 6 damping vibrations graphs for two values of design parameter vectors – optimum and arbitrary are shown.

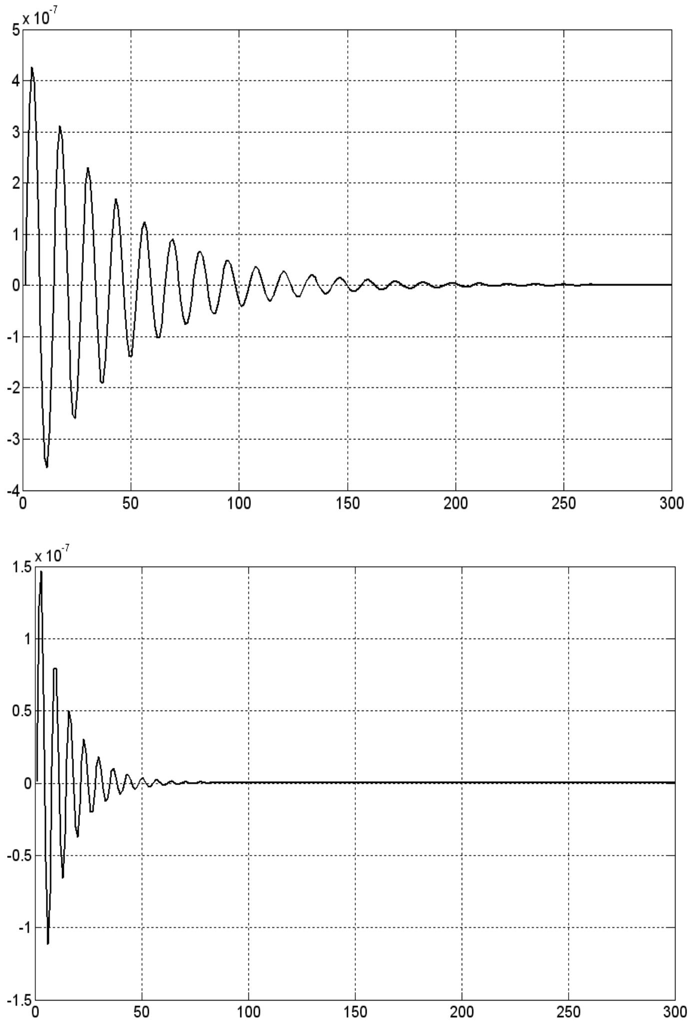

Fig. 6. Damping oscillogram for two values of design parameter vectors: a – arbitrary; b – optimal

Conclusions. Computational vibration equations of multilayer plates with layers of viscoelastic and electro-viscoelastic materials are obtained using the Fourier transform. Damping devices (shunts) are used for additional passive damping that can be employed to create special damping networks.

It is shown that using finite-element models of electro-viscoelastic material structures in the Fourier transform space allows to introduce justified complex moduli for non-stationary and multifrequency vibrations and to take the initial conditions into account correctly. It is necessary to use the optimal design methods to create effective vibration-proof structures.

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