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NONSTATIONARY VIBRATIONS OF A BEAM WITH ELECTRO-VISCOELASTIC DISSIPATIVE PATCHES

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НЕСТАЦІОНАРНІ КОЛИВАННЯ БАЛКИ З ЕЛЕКТРОВ'ЯЗКОПРУЖНИМИ ДИСИПАТИВНИМИ НАКЛАДКАМИ

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НЕСТАЦИОНАРНЫЕ КОЛЕБАНИЯ БАЛКИ С ЭЛЕКТРОВЯЗКОУПРУГИМИ ДИССИПАТИВНЫМИ НАКЛАДКАМИ

Passive damping of non-stationary vibration in a beam with piezoelectric patches and RL-shunts was investigated. The finite-element modeling technique of smart structures in Fourier transform frequency space is proposed.

Key words: *nonstationary vibrations, passive damping, smart constructions, Fourier transformation.*

Досліджено пасивне демпфірування нестаціонарних коливань балки з п'єзоелектричними накладками та підключеними RL-шунтами. Запропоновано методику скінченно-елементного моделювання smart-конструкції у частотному просторі інтегральних перетворень Фур'є.

Ключові слова: *нестаціонарні коливання, пасивне демпфірування, smart-конструкція, перетворення Фур'є.*

Исследовано пассивное демпфирование нестационарных колебаний балки с пьезоэлектрическими накладками и подключенными RL-шунтами. Предложена методика конечно-элементного моделирования smart-конструкции в частотном пространстве интегральных преобразований Фурье.

Ключевые слова: *нестационарные колебания, пассивное демпфирование, smart-конструкция, преобразование Фурье.*

Introduction. The problem of mechanical vibration damping in structures has always been and now remains one of the most important and most complex issues of the modern science and technology. For a long time the passive damping methods, based on the use of viscoelastic materials with high internal energy dissipation of mechanical vibrations, were applied to reduce the vibration of structures with different loads [1; 2].

Recently, according to foreign publications [3; 4] the so-called smart-materials and designs based on them, are increasingly spreading in various areas, particularly in aviation and space technology. The appearance of such materials and structures is due to the successes in science of materials and attempts to approximate the living organism functioning principles.

The most effective smart-materials include piezoelectric materials that after pre-polarization have the ability to respond to mechanical strains on electrode surfaces by voltage appearance, and to deform when applying the voltage. The appearance of voltage makes possible to use it in sensor devices or convert electrical energy into heat [5]. The reduction of the mechanical strain energy, particularly under dynamic loading, leads to vibration amplitude decrease (damping). Such methods are mainly based on including piezoelectric patches or layers in the structure for conversion of the mechanical vibration energy to electrical energy and heat.

The vibrations damping methods by means of piezoelectric energy conversion are divided into active and passive. Active control devices have some restrictions in application caused by complexity of their implementation and sensitivity to changes and uncertainty of the system parameters.

However, passive methods are simpler. They are based on the vibration energy dissipation in special electrical circuits called shunts that are attached to electrode surfaces of a piezoelectric element [6]. A shunt with piezoelectric elements forms a kind of a resonant circuit in which the energy is dissipated. The vibration energy dissipation occurs due to the electromagnetic and thermal radiation in the shunt elements and in the main structure’s viscous layers and patches.

Passive damping of vibrations in the structure made of aluminum alloy, with patches of viscoelastic piezoelectric PVDF elements, polarized by depth (Fig. 1), is consider in this paper. Electric energy dissipation mainly occurs in electric circuits with resistors R and induction coils L that form an RL-shunt, connected to the piezoelements.

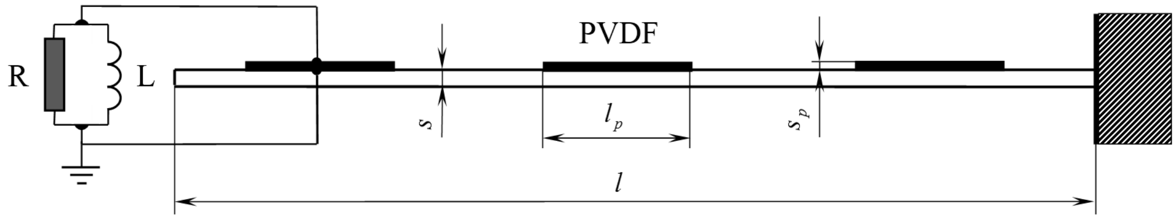


Fig. 1. A beam with connected shunts

The shock or impulse loads of thin-walled structures cause non-stationary vibrations with amplitude and duration which can exceed the permissible limits.

The state of research in non-stationary vibrations of piezoelectric structures can be estimated as basic. These studies are mostly limited to mono-harmonic vibration cases which are not always true for this structure type. It is not ascertained how effective is using piezoelectric materials to reduce the vibration amplitude after impulse and shock loads.

The idea of using such materials seems attractive, considering that the electromagnetic fields distribution is much faster compared to the mechanical strain fields [7].

Hence the non-stationary vibration problem for smart-structures requires detailed study and development of appropriate mathematical modeling methods for piezoelectric materials with attached electric energy dissipation elements.

Investigation Method. The complexity of tasks requires the use of approximate methods.

The finite element method is the most common method for the complex composite structure synthesis, but the use of this method for solving the imperfectly-elastic structures dynamic problems requires selection of the appropriate physical dependences.

It is shown in [1; 2] that the frequency finite element method (FFEM) [8], in which the design synthesis and vibration analysis is performed in the integral Fourier transform space, can be effectively used for the non-stationary vibration analysis of imperfectly-elastic structures. The advantages of using this method are the ability taking into account the dependencies of the linear theory of hereditary environments, including correct input of the frequency-dependent complex modules, and the ability to analyze non-stationary vibration with given initial conditions. The synthesis of structures with piezoelectric materials is also significantly facilitated in frequency space.

According to the standard finite element method algorithm [9], we perform structure discretization (Fig. 1) by plane six-node finite elements (Fig. 2) with the corresponding interpolation functions. The node "movement" approximation for an electro-elastic finite element, which is a model of the piezoelectric material, is carried out by mechanical and electrical degrees of freedom. Therefore, this finite element (Fig. 2, *b*) has six additional degrees of freedom – electric potentials at each node, as compared with a mechanical finite element (Fig. 2, *a*), which approximates the mechanical displacement.

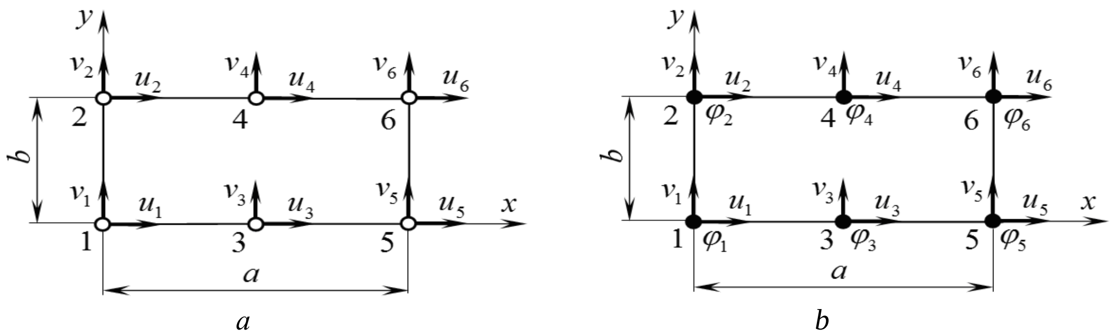


Fig. 2. Six-node finite mechanical (a) and electro-elastic (b) elements

The identical interpolation functions are taken for the six-node mechanical and electro-elastic finite elements [2]:

$$N_1=\left(1-\frac{y}{b}\right)\left(1-3\frac{x}{a}+2\frac{x^2}{a^2}\right),\ N_2=\frac{y}{b}\left(1-3\frac{x}{a}+2\frac{x^2}{a^2}\right),$$
$$N_3=\left(1-\frac{y}{b}\right)\left(4\frac{x}{a}-4\frac{x^2}{a^2}\right),\ N_4=\frac{y}{b}\left(4\frac{x}{a}-4\frac{x^2}{a^2}\right), \tag{1}$$
$$N_5=\left(1-\frac{y}{b}\right)\left(-\frac{x}{a}+2\frac{x^2}{a^2}\right),\ N_6=\frac{y}{b}\left(-\frac{x}{a}+2\frac{x^2}{a^2}\right).$$

Finite element strain $\mathbf{e}$ is determined by the movement of nodal points 1-6:

$$\mathbf{e}=\mathbf{A}\mathbf{N}_u\mathbf{u}, \tag{2}$$

where $\mathbf{A}$ – differential operator matrix, $\mathbf{N}_u$ – mechanical displacement interpolation function matrix:

$$\mathbf{A}=\begin{bmatrix}\frac{\partial}{\partial x} & 0\\ 0 & \frac{\partial}{\partial y}\\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x}\end{bmatrix},\ \mathbf{N}_u=\begin{bmatrix}N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6 & 0\\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 & N_5 & 0 & N_6\end{bmatrix}. \tag{3}$$

For a piezoelectric material, the potential distribution by depth is assumed linear in the direction of polarization.

The electric field vector $\mathbf{E}$ is related with the potential φ by a well-known relation, which in the finite-element version will look as:

$$\mathbf{E}=-\nabla\mathbf{N}_\varphi\varphi, \tag{4}$$

where ∇ and $\mathbf{N}_\varphi$ are differential operator and electric potential interpolation functions matrix respectively:

$$\nabla=\begin{bmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\end{bmatrix}^T,\ \mathbf{N}_\varphi=\begin{bmatrix}N_1 & N_2 & N_3 & N_4 & N_5 & N_6\end{bmatrix}. \tag{5}$$

The system of linear differential equilibrium equations and quasi-static linear electro-elasticity equations for a piezoelectric body disregarding temperature, relative to the mechanical displacement and electric potential under the influence of external mechanical and volumetric forces and electrical charges are discussed in detail in [10].

Linear physical dependencies for a plane stress of a perfectly elastic piezoelectric material in a constant electric field are defined as:

$$\mathbf{y} = \mathbf{C}\cdot\dot{\mathbf{e}} - \mathbf{e}^{\mathrm{T}}\cdot\dot{\mathbf{E}},\ \ \mathbf{D} = \mathbf{e}\cdot\dot{\mathbf{e}} + \mathbf{k}\cdot\dot{\mathbf{E}}, \tag{6}$$

where $\mathbf{C} = \begin{pmatrix} C_{11} & C_{13} & 0 \\ C_{31} & C_{33} & 0 \\ 0 & 0 & C_{55} \end{pmatrix}$ – matrix of elastic moduli; $\mathbf{y} = \begin{pmatrix} \sigma_x & \sigma_z & \tau_{xz} \end{pmatrix}$ – mechanical stresses; $\mathbf{e} = \begin{pmatrix} 0 & 0 & e_{15} \\ e_{31} & e_{33} & 0 \end{pmatrix}$ – matrix of piezoelectric moduli; $\mathbf{E} = \begin{pmatrix} E_x & E_z \end{pmatrix}^T$ – the electric field; $\boldsymbol{\kappa} = \begin{pmatrix} -\kappa_{11} & 0 \\ 0 & -\kappa_{33} \end{pmatrix}$ – matrix of dielectric moduli; $\mathbf{D} = \begin{pmatrix} D_x & D_z \end{pmatrix}^T$ – electric displacement; $\mathbf{e} = \begin{pmatrix} \varepsilon_x & \varepsilon_z & \gamma_{xz} \end{pmatrix}^T$ – elastic strains; points mark differentiation by time.

Differential equations of the electrical shunt with parallel connection of elements [11] can be written as:

$$\ddot{Q} = \frac{1}{R}\dot{\varphi} + \frac{1}{L}\varphi, \tag{7}$$

where $1/R$ – electrical conductivity; $1/L$ – inverse coil inductance; points mark differentiation by time.

In the finite-element version the electric "stiffness" matrix for an RL-shunt (Fig. 3a) has the form:

$$\ddot{\mathbf{Q}} = \mathbf{K}_R\dot{\varphi} + \mathbf{K}_L\varphi, \tag{8}$$

where

$$\mathbf{K}_R = \frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},\ \mathbf{K}_L = \frac{1}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{9}$$

$\mathbf{Q}$ – nodal charges, based on the law of charge conservation:

$$\mathbf{Q} = \int_0^a\int_0^b \mathbf{N}_\varphi{}^T q\,dy\,dx; \tag{10}$$

q – charge density on the electrode surface of a piezoelectric element.

The synthesis of a finite element with RL-shunts (Fig. 3) is performed according to the standard finite element method procedure [9]. The boundary conditions in the "metal base / piezoelectric element" contact (metal / dielectric) are ensured by using the approximation functions (3) and (5). The grounding condition of the shunt terminals is taken into account by assigning zero rows and columns to the corresponding "grounded" shunt nodes.

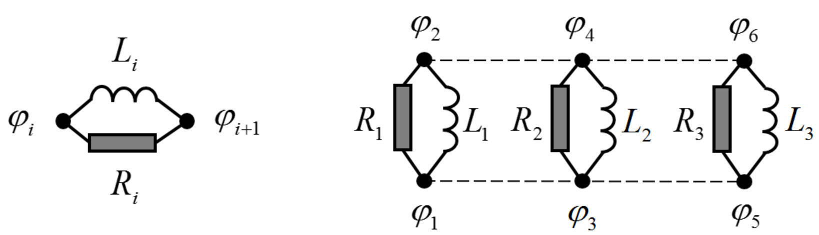


Fig. 3. Shunt finite elements

Mathematical model of non-stationary vibrations of a beam. The physical dependences for piezoelectric materials having small dynamic deformations can be written using the integral operators of the linear hereditary theory as for a linear viscoelastic body [10]. However, directly using integral dependences in dynamic problems causes significant difficulties associated with the experimental determination of physical parameters [12] and the solution of integro-differential equations.

Let's write the finite-element version of differential equations (6) in the frequency space and use an approximate variant related by the direct application of integral Fourier transform with the equations derived by the principle of Hamilton-Ostrogradski. The inadmissibility and nonrigorousness of this variant was specified in [13].

Dynamic equations derived by the variational principle of Hamilton-Ostrogradski after integral Fourier transform will have the form of the linear elasticity theory equations with comprehensive modules:

$$(i\omega)^2\mathbf{M}\tilde{\mathbf{u}}+\tilde{\mathbf{K}}_u\tilde{\mathbf{u}}+\tilde{\mathbf{K}}_{u\varphi}\tilde{\varphi}=\tilde{\mathbf{F}}(i\omega)+\mathbf{f}\,,$$

$$\tilde{\mathbf{K}}_{u\varphi}^T\tilde{\mathbf{u}}+\tilde{\mathbf{K}}_{\varphi}\tilde{\varphi}=\tilde{\mathbf{Q}}(i\omega)\,, \tag{11}$$

where $\mathbf{M}$ – mass matrix, $\tilde{\mathbf{K}}_{\varphi}$ – electric "stiffness" matrix, $\tilde{\mathbf{K}}_{u\varphi}$, $\tilde{\mathbf{K}}_{\varphi u}$ – matrices corresponding to the direct and the reverse piezoelectric effect respectively:

$$\mathbf{M}=h\int_0^a\int_0^b\mathbf{N}_u{}^T\rho\,\mathbf{N}_u\,dydx\,,\quad \tilde{\mathbf{K}}_u(i\omega)=h\int_0^a\int_0^b(\mathbf{A}\mathbf{N}_u)^T\,\tilde{\mathbf{C}}(i\omega)\,\mathbf{A}\mathbf{N}_u\,dydx\,,$$

$$\tilde{\mathbf{K}}_{u\varphi}(i\omega)=h\int_0^a\int_0^b(\mathbf{A}\mathbf{N}_u)^T\,\tilde{\mathbf{e}}(i\omega)^T\,\nabla\mathbf{N}_{\varphi}\,dydx,\quad \tilde{\mathbf{K}}_{\varphi u}(i\omega)=\tilde{\mathbf{K}}_{u\varphi}^T(i\omega),$$

$$\tilde{\mathbf{K}}_{\varphi}(i\omega)=h\int_0^a\int_0^b(\nabla\mathbf{N}_{\varphi})^T\,\tilde{\boldsymbol{\kappa}}(i\omega)\nabla\mathbf{N}_{\varphi}\,dydx\,,\quad \tilde{\mathbf{F}}(i\omega)=\int_0^\infty\int_0^a\int_0^b\mathbf{N}_u^T\mathbf{p}(x,y,t)\exp(-i\omega t)dydxdt,$$

$$\tilde{\mathbf{Q}}(i\omega)=\int_0^\infty\int_0^a\int_0^b\mathbf{N}_{\varphi}^T\mathbf{q}(x,y,t)\exp(-i\omega t)dydxdt,$$

where $\tilde{\mathbf{C}}(i\omega)=\mathbf{C}'(\omega)+i\mathbf{C}''(\omega)$ – matrix of frequency-depended complex elastic moduli; $\tilde{\mathbf{e}}(\omega)=\mathbf{e}'(\omega)+i\mathbf{e}''(\omega)$, $\tilde{\boldsymbol{\kappa}}(\omega)=\boldsymbol{\kappa}'(\omega)+i\boldsymbol{\kappa}''(\omega)$ – complex matrix of piezoelectric and dielectric modules respectively; ρ – material density; h – finite element width; $\mathbf{p}(x,y,t)=\left(p_x\ p_y\right)^T$ – external loads; $\tilde{\mathbf{F}}(x,y)$ – Fourier images of the external mechanical loads; $\tilde{\mathbf{Q}}(x,y)$ – images of nodal charges; $\mathbf{f}=i\omega\mathbf{M}\dot{\mathbf{u}}(0)+\mathbf{M}\mathbf{u}(0)$, $\dot{\mathbf{u}}(0)$, $\mathbf{u}(0)$ – nodal initial velocity and displacements respectively.

We exclude the electric potential φ in order to find the solution of the linear equations system (11) for displacements.

After a simple transformation, we define the Fourier image of the nodal potential values from the second equation:

$$\tilde{\varphi}=\left(\tilde{\mathbf{Q}}-\tilde{\mathbf{K}}_{\varphi}\right)^{-1}\tilde{\mathbf{K}}_{u\varphi}^T\tilde{\mathbf{u}}\,. \tag{13}$$

The computed value of the image potential (13) is substituted in the first equation of system (11), taking into account the material tensors complex components. Thus the equation for mechanical displacement images is obtained:

$$\tilde{\mathbf{Z}}(i\omega)\tilde{\mathbf{u}} = \tilde{\mathbf{F}}(i\omega), \qquad (14)$$

where $\tilde{\mathbf{Z}}(i\omega)$ – the dynamic stiffness matrix

$$\tilde{\mathbf{Z}}(i\omega) = \tilde{\mathbf{K}}_u + \tilde{\mathbf{K}}_{u\varphi}\left(\tilde{\mathbf{Q}} - \tilde{\mathbf{K}}_\varphi\right)^{-1}\tilde{\mathbf{K}}_{u\varphi}^T - \omega^2\mathbf{M}. \qquad (15)$$

The solution of linear algebraic equations (11) for displacements in frequency space has the form:

$$\tilde{\mathbf{u}} = \tilde{\mathbf{Z}}(i\omega)^{-1}\tilde{\mathbf{F}}. \qquad (16)$$

The return to the time space after determining displacements is conducted through numerical (discrete) inverse Fourier transform, namely the fast Fourier transform (FFT):

$$\mathbf{u} = \mathrm{FFT}^{-1}\left(\tilde{\mathbf{Z}}(i\omega)^{-1}\tilde{\mathbf{F}}\right). \qquad (17)$$

To analyze the vibration energy dissipation in a structure it is required to determine the eigenvectors and the eigenvalues of the dynamic stiffness matrix:

$$\left|\tilde{\mathbf{Z}}(i\omega)\right| = 0. \qquad (18)$$

According to [2], one can use a numerical simple iteration method for determining the eigenvectors and eigenvalues.

Computation of a beam with piezoelectric patches. Let's consider an example of computing the non-stationary vibration of a beam (Fig. 1) with viscoelastic piezoelectric patches.

Beam parameters are:

- length $l = 0.7\,m$;
- width $b = 20 \cdot 10^{-3}\,m$;
- the main bearing structure thickness $s = 2 \cdot 10^{-3}\,m$;
- material density $\rho = 2.7 \cdot 10^3\,\frac{kg}{m^3}$;
- modulus of elasticity of the bearing layer material

$E = 6.71 \cdot 10^{10} \cdot (1 + i \cdot 0.025)$ *Па.*

The properties of patches of viscoelastic piezo-composite material based on PVDF and PZT-2 [6]:

- patch length $l_p = 0.1$ m;
- width $b_p = 20 \cdot 10^{-3}$ m;
- thickness $s_p = 2 \cdot 10^{-3}$ m;
- material density $\rho_p = 1.75 \cdot 10^3\,kg/m^3$;
- real and imaginary components of the piezoelectric material elastic moduli matrix:

$C_{11} = 15.7 \cdot 10^9 \cdot (1 + i \cdot 0.064)\ Pa$, $C_{31} = 9.30 \cdot 10^9 \cdot (1 + i \cdot 0.098) Pa$,

$C_{33} = 13.6 \cdot 10^9 \cdot (1 + i \cdot 0.069) Pa$, $C_{55} = 2.52 \cdot 10^9 \cdot (1 + i \cdot 0.014) Pa$;

– piezoelectric moduli:

$e_{31} = -1.0 \cdot \left(1 - i \cdot 8.3 \cdot 10^{-3}\right) C/m^2$, $e_{33} = 1.5\ C/m^2$, $e_{15} = 1.13 \cdot \left(1 - i \cdot 2.1 \cdot 10^{-3}\right) C/m^2$;

– dielectric properties matrix components:

$\kappa_{11}/\kappa_0 = 12.7\cdot\left(1 - i\cdot 4.7\cdot 10^{-3}\right)$, $\kappa_{11}/\kappa_0 = 11.8\cdot\left(1 - i\cdot 1.2\cdot 10^{-3}\right)$, $\kappa_0 = 8.85\cdot 10^{-12}\ F/m$.

Fast Fourier transform parameters: number of points $N = 2^{12}$ on time interval $T = 4$; maximum frequency spectrum $f_{\max} = 6432\,Hz$. Impulse and shock loads are applied in the direction of the generalized coordinate 4.

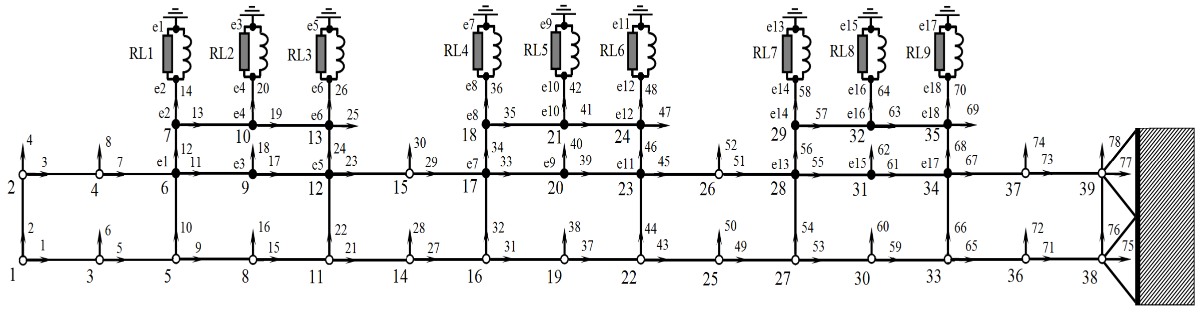


Fig. 4. Finite-element model of structure with RL-shunts

The problem solution was carried out for three cases:

1) for a connected parallel shunt (8). The dynamic electromechanical stiffness matrix in this case is written as follows:

$$\tilde{\mathbf{Z}}_{sh}(i\omega) = \tilde{\mathbf{K}}_u + \tilde{\mathbf{K}}_{u\varphi}\left(\frac{1}{i\omega}\tilde{\mathbf{K}}_R - \frac{1}{\omega^2}\tilde{\mathbf{K}}_L + \tilde{\mathbf{K}}_\varphi\right)^{-1}\tilde{\mathbf{K}}_{u\varphi}^T - \omega^2\mathbf{M}; \quad (19)$$

2) for a disconnected shunt – with open electrodes ($\tilde{\mathbf{Q}} = 0$):

$$\tilde{\mathbf{Z}}_{open}(i\omega) = \tilde{\mathbf{K}}_u + \tilde{\mathbf{K}}_{u\varphi}\tilde{\mathbf{K}}_\varphi^{-1}\tilde{\mathbf{K}}_{u\varphi}^T - \omega^2\mathbf{M}; \quad (20)$$

3) mechanical problem solution – without electric component:

$$\tilde{\mathbf{Z}}(i\omega) = \tilde{\mathbf{K}}_u - \omega^2\mathbf{M}. \quad (21)$$

The results of determining the beam response to an impulse load (Fig. 5) for the three cases shown in Fig. 6.

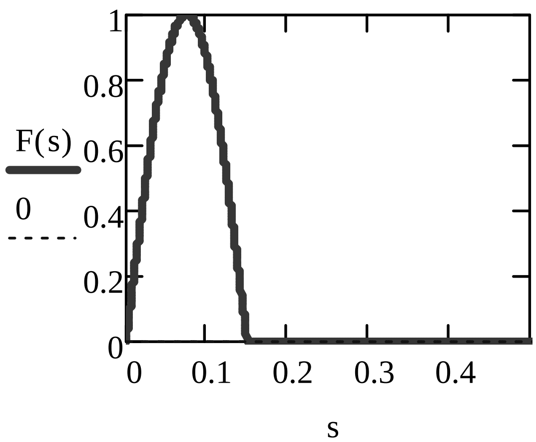


Fig. 5. Impulse load shape in the direction of the generalized coordinate 4

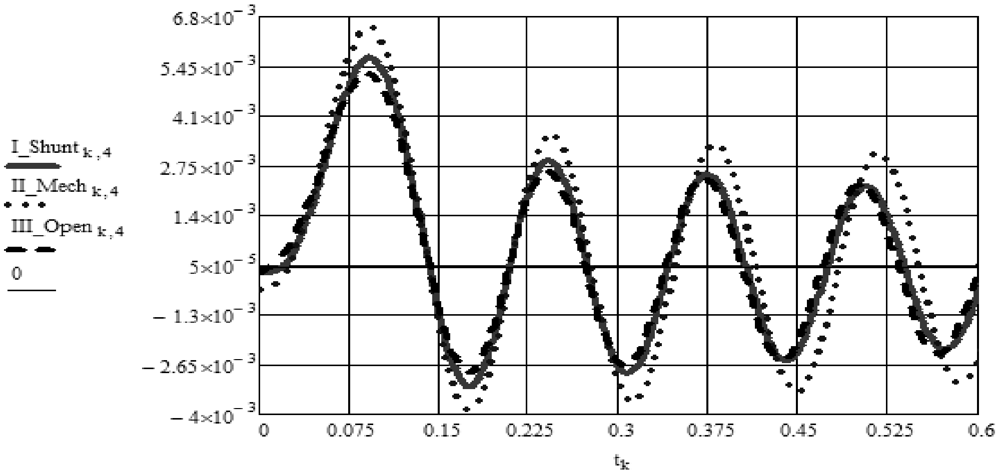
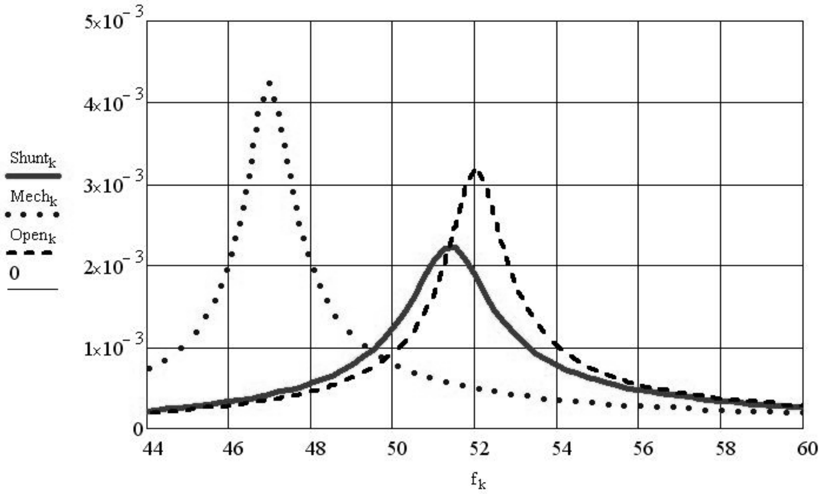
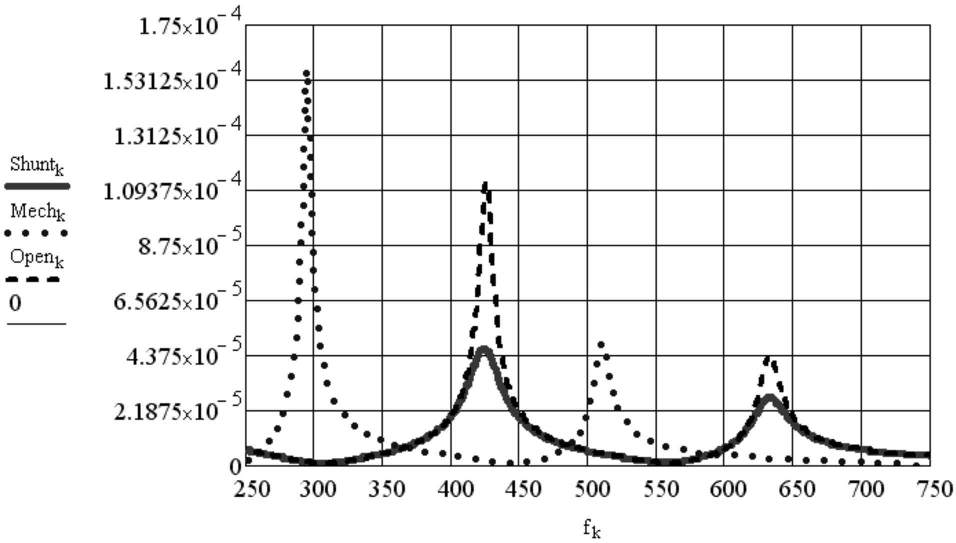


Fig. 6. The beam response to an impulse load in the direction of coordinate 4:
I – with connected shunt (19); II – without shunts (21); III – with open electrodes (20)

Gain-frequency characteristics for the first, second and third vibration modes are shown in Fig. 7 *a*, *b*.



a



b

Fig. 7. Frequency response to the first (a), second and third (b) frequencies:
I – with a connected shunt (19); II – without shunts (21); III – with open electrodes (20)

Conclusions. Finite element analysis in Fourier space can be used for the computation of piezoelectric structures and composites under impulse load.

According to the computation results, the use of piezoelectric elements with an attached *RL*-shunt reduces the magnitude of reaction of considered structures under non-stationary loads. The largest decrement and the smallest displacement amplitude from the impulse and shock loads are observed in a structure with shunts. The design vibration decrement can be significantly increased by adjusting electrical resistance and inductance of the shunt. The advantages of using this technique are the ability to define responses to an external load of an arbitrary spectral composition and taking into account the dependencies of physical material characteristics to frequency. The return to the temporal space is carried out only at the last step of calculations.

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