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WORK ANALYSIS OF ARCH FRICTIONAL JOINT LOADED WITH THE IMPACT MASS

In the article there is presented analysis of the state of stress and deformation of the arch frictional joint, which was carried out on the model of these joints using finite element method. Joints were loaded dynamically with the impact of freely falling mass. Arch joints with and without passive pressure were submitted to an axial compression. Physical model of the frictional joint was developed on the basis of system applied during the stand tests. To solve mathematical model an explicit integration method was used. In result of analyses carried out, temporal courses of force transmitted through the frictional joint, and displacements of section sliding down were determined. Distributions of reduced stresses in elements of frictional joint were also determined and the state of deformation was described.

Keywords: yielding support, mining support, arch friction joint.

1. Introduction

In yielding support of dog headings, there are two types of frictional joints, namely: straight and arched. The base for this division is radius of curvature of sections forming the frictional joint. In case when the joint is made of rectilinear sections of the profile, we are dealing with a straight joint, which is used mainly in the friction props. In steel frames of yielding support we handle with an arch frictional joints, in which contact between the cooperating sections take place along an arc line.

Despite the widespread use of arch frictional joints, studies which concern researches and analyses of their operation are rarely met. Practically, beside the papers [1, 3] there are no elaborations, concerning arch frictional joints studies.

It seems that the cause of that are problems resulting from their geometry, which has influence on more complex distribution of internal forces in relation to a straight joint. It causes difficulties to carry out researches of these joints, coming from necessity of having a specialized test stand, what is related to high cost of these studies.

Stand and underground tests, despite many advantages have also other limitations regarding mainly range of changes in parameters describing the frictional joint and the way of its loading. These tests unable also determination of distributions of stresses and deformations of particular elements of frictional joint during its operation.

Possibilities of such an analysis create model studies carried out on the structural model of frictional joint, and in some range could be an alternative and complement to stand tests and tests in real conditions.

Such a way of modeling was used at development of arch frictional joint model in order to carry out simulation studies on this model, loaded with an impact of freely falling mass.

The fundamental source of information, necessary for the development of physical and mathematical models of arch frictional joint, were stand tests of joints loaded with an impact of freely falling mass [1].

A mathematical model of continuous medium, which is frictional joint after spatial discretization using the finite element method, is an initial-value problem described by system of ordinary differential equations with the appropriate initial conditions.

The equations of motion are determined based on second order Lagrange's equation, indirectly derived from the second law of Newton's dynamics. General equation of motion describing the mechanical system in matrix form is [5, 6]:

$$M \cdot \ddot{y} + C \cdot \dot{y} + K \cdot y = P(t) \qquad (1)$$

where: M – matrix of inertia,
C – damping matrix,
K – rigidity matrix,
$P(t)$ – generalized forces vector,
$\ddot{y}$ – generalized accelerations vector,
$\dot{y}$ – generalized velocity vector,
y – generalized displacements vector.

Equation (1) is the matrix form of ordinary differential equations, linear with constant coefficient. In the present paper to solve these equations an explicit integration method was used [2, 4, 5].

This method consists in the fact, that an equation of the motion is integrated over step by step, which means that it have to be satisfied only in the selected moments, and not in the entire range of integration. Relatively short time of determination the following unknown values in subsequent moments of time is an advantage of the explicit integration method. Application of mass matrix diagonalization caused that there is not necessity to invert matrix in order to solve the system of algebraic equations. Calculations time depends only on the number of model's degrees of freedom. The disadvantage of this method is necessity to apply a short time step at the integration, because too long time step causes occurrence of numerical instabilities.

Taking into account the specificity of operation of arch frictional joint, and difficulties with its stand and underground tests, in the present paper an analysis of operation of this joint, using the finite element method was performed [2, 6]. This analysis included the case of dynamic loading of a joint resulting from the action of freely falling impact mass.

2. Performance characteristic of arch frictional joint

Specificity of arch frictional joint's operation in respect to straight joint results from its different geometry, which has significantly impact on a distribution of internal forces in this joint. Besides longitudinal compressive force, which predominates in straight joints, there are also shearing force and bending moment [1].

The difference between those joints within the internal and external passive forces (of the reaction) is particularly visible in the case of their axial compression. In the case of a straight joint one can assume, that during the axial compression of the cooperating sections, an axial force acts, causing their compression. A geometric shift of the centre of cross-section between the cooperating sections causes that the bending moment and shearing force occur. However, their values are small, and the influence on the state of deformation and stress is negligible, in consequences their influence is not taken into account during the calculations of straight frictional joint.

For arch frictional joint axially loaded (along the straight line which connecting its ends) the influence of bending moment and shearing force is very relevant.

In a Figure 1 there is presented distribution of internal forces in arch frictional joint cross-section at point D, loaded with an external force P. The joint is attached at the ends and is also supported in the center of the arch at the point C. Reaction R, occurring at this point, is called a passive pressure.

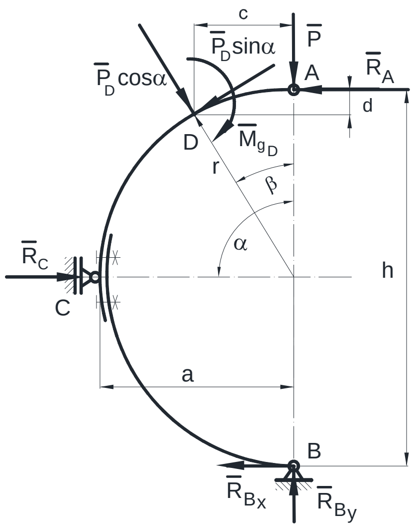


Fig. 1. Scheme of the loading with a distribution of internal forces in the cross-section of arch frictional joint with passive pressure

The complex loaded state of particular cross-section of arch frictional joint has a significant influence on the nature of its work, deciding together with the values of friction forces between cooperating sections, whether a yield in a joint will occur, or it will be working as rigid. A very significant impact on the values of the internal forces has also a way of supporting the joint, which should reflect the rock mass interaction to the steel frames. It regards particularly taking into account the side thrust, which impacts on arch joint. In practice, this side thrust could be passive force (reaction) or an active force resulting from the rock mass interaction on the steel frames, and it is dependent on the way of assembly the steel frames in dog heading and on rigidity of liner.

A very significant impact on the distribution of internal forces and the values of reaction has also a geometry of the arch frictional joint.

Using of adopted scheme (Fig. 1) one can determine the impact of chord of arch (h) and rise of arch (a) on the value of the bending moment and the transverse and longitudinal forces operating in the cross-section of the arch.

Equation of bending moment acting at point D of cross-section of the arch will have a form:

$$M_{g\beta} = -P \cdot c + R_A \cdot d \tag{2}$$

Whereas the angle β will vary in range: $0 \le \beta \le \alpha$

Using the geometrical dependences one can determine the maximum value of the bending moment acting on the considered arch. This value will be reached at point C ($\alpha = \beta$) and will amount:

- for $\alpha = \beta \Rightarrow M_{g\beta\,max} = -P \cdot \{ r[cos(\alpha - \alpha) - 1] + a \} + R_A \cdot [-r\, sin(\alpha - \alpha) + \frac{h}{2}] \Rightarrow$

$$M_{g\beta\,max} = -P \cdot a + R_A \cdot \frac{h}{2} \tag{3}$$

On the basis of dependencies derived above, one can conclude that very significant impact on the value of the bending moment acting on the arch frictional joint, beside the value of external force, have a length of chord of arch (h) and rise of arch (a).

Assuming, that there are two main parameters which characterize the geometry of the arch, and that the radius of both sections forming the arch are the same, one can define a geometric coefficient of the arch frictional joint in a form:

$$k_l = \frac{a}{h} \tag{7}$$

In case when a rise of arch (a) equals zero, we have a case of simple frictional joint ($k_l = 0$), and when a rise of arch equals half-value of the chord of arch ($k_l = 0{,}5$), then we take into consideration an arch in the form of a semicircle (Fig.1).

In order to determine the influence of passive pressure on distributions and maximum values of bending moment and transverse force operating on arch frictional joint, the calculations for arch frictional joints with and without passive pressure were performed. For each of these cases the active loading was constant and amounted to 100kN. The length of the arch chord was also constant and amounted to 1 m.

In a case of calculations of arch frictional joint with passive pressure, there were determined dependences between the geometric coefficient of the arch frictional joint and the maximum value of the bending moment, and the value of reaction (value of passive pressure) operating at point C in an arch frictional joint with passive pressure (Fig. 2).

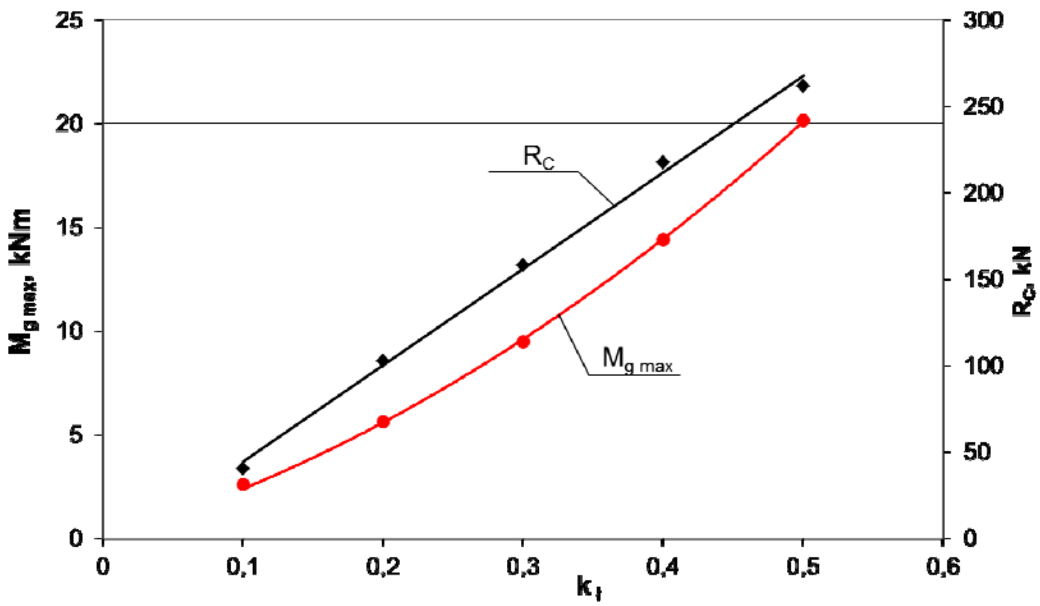


Fig. 2. Influence of the k_l coefficient value on the maximum value of the bending moment ($M_{g\ max}$) and the value of the reaction in passive pressure (R_C) for a friction joint with passive pressure

Based on the obtained results one can conclude, that together with increasing values of geometric coefficient of the arch frictional joint, maximum bending moment and reaction in a passive pressure in arch joint increase.

It has very significant meaning for the operation of frictional joint, because the higher value of bending moment, the more difficult yield occurs in arch frictional joint, and its bending occurs easier.

In a case of analysis of arch frictional joint without passive pressure, determined values of bending moment are very high, which considerably limits the possibilities of occurrence of yield within the arch frictional joint.

Also distribution of the transverse force indicates greater possibility of occurrence of bending process than a yield within joint [1].

3. Axial compression of arch frictional joint

A discrete spatial model of joint was developed in order to analyze the operation of arch frictional joint subjected to the axial compression with the force arises from an impact of freely falling mass along the straight line connecting the arch's ends. The base to formulate the model was arch frictional joint made of V29 sections with two SDO29 stirrups. Model of the joint consisted of 14 independent solid elements, for which defined 36 potential contact surfaces. For the materials from which the joints are made, elastic-plastic characteristics were assumed. Model of the joint with an impact mass is presented in a Figure 3. In this model it was assumed, that initial value of axial force in each of bolts of stirrups amounted to 80 kN.

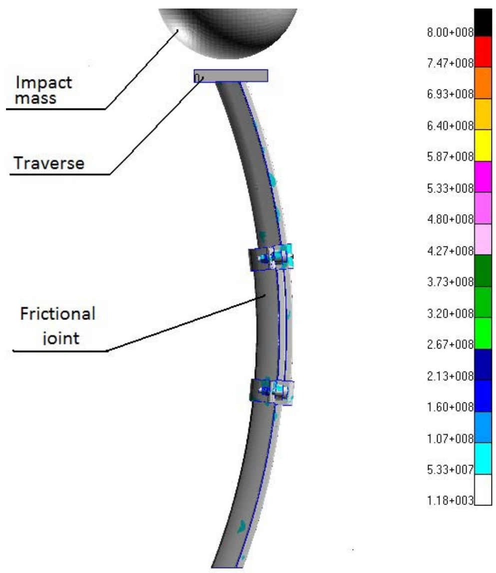


Fig. 3. Discrete model of the arch frictional joint and the way of its loading with an impact of the mass

Two arch frictional joints with different supporting way were subjected to analysis. In a first case, the joint was without passive pressure, in a second with a rigid passive pressure. In both cases, the joint was loaded by the impact of the mass of 1000 kg in perfectly spherical rigid form. Traverse was modeled in a form of an ideal rigid plate of mass 1500 kg.

So modeled arch frictional joint was subjected to numerical analysis, and in result of this analysis there were determined the time course of the force loading joint and distributions of constituents stress state, and deformations of particular elements of joints during its testing.

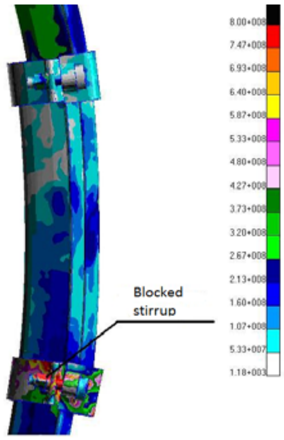
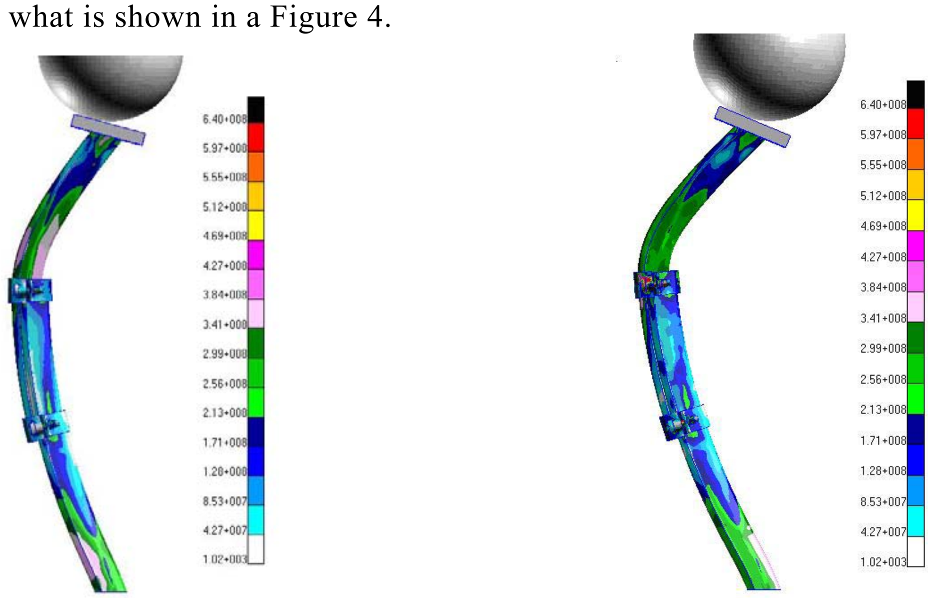


Fig. 4. State of stress and deformation of an arch frictional joint at the moment of its locking

In a Figure 4 there is presented state of stress and deformation of an arch frictional joint at the moment of its locking. Joint was loaded with an impact of freely falling mass of impact energy amounted to 9.81 kJ.

During this test, a yield of 0.26 m occurred in the joint, and maximum value of the force transmitted through the joint amounted to 435 kN (at the moment of its locking).

Analyzing stress and deformation distributions of joint's elements in particular phases, one can observe their changes depending on magnitude of yield. Particularly visible is an increase of value of stresses in the bottom stirrup of joint. In this phase of yield the locking lower stirrup occurred, what reveals as particularly apparent deformation and an increase of stresses in its bolts. Farther yield within the joint causes its unlocking, what is shown in a Figure 4.



a) b)

Fig.5. State of stress and deformation of arch frictional joint without passive pressure, for which the initial value of axial forces in bolts of stirrups amount to 100 kN in each subsequent phases of its operation

Tests were performed for the joints with different initial axial forces in bolts of stirrups. In a Figure 5 there is presented state of joint's stress and deformation, for which the values of initial axial forces in each of bolts of stirrups amounted to 100kN.

Analyzing obtained distributions one can conclude, that at higher initial axial forces in the bolts of stirrups, yield had not occurred in the joint. The high value of the force, with which the cooperated sections was pressed, caused that instead of yield, before the beginning of an overlap, the bending of upper section occurred. It caused a significant increase of the value of the axial forces in bolts of upper stirrup.

The second system, which was submitted for analysis in the range of the axial compression of arch frictional joint, was the joint with rigid passive pressure.

In a Figure 6 there are shown initial and terminal states of stress and deformation in arch frictional joint with passive pressure, for which an initial axial forces in the bolts of stirrups amounted to 100 kN each. The initial state is determined at the moment of contact of an impact mass with a joint, what causes the loading of upper section (Fig. 6a). Terminal phase determines the end of yield within the joint (Fig. 6b).

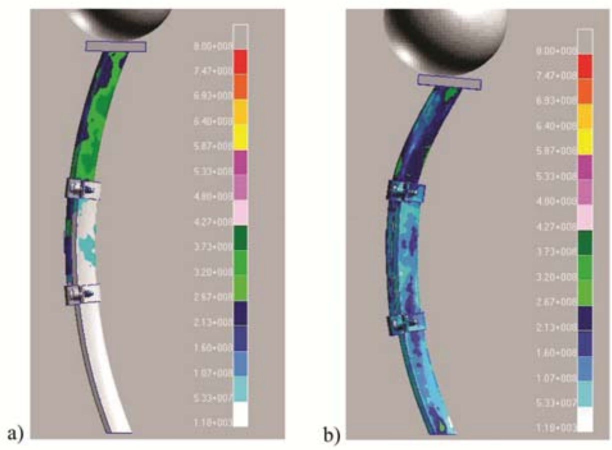


Fig. 6. The initial and terminal state of stress and deformations in arch frictional joint with passive pressure for which the initial value of axial forces in each of bolts stirrups amount to 100 kN each

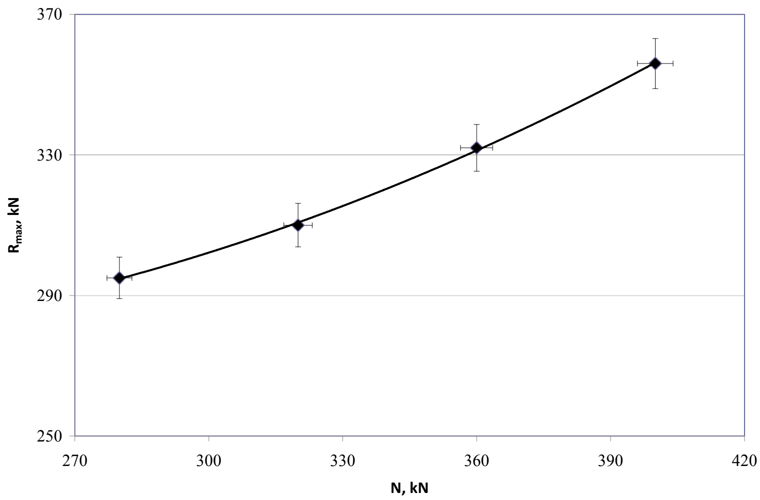


Fig. 7. Dependence between maximum value of force transmitted through the arch frictional joint with a rigid passive pressure and total value of initial axial forces in the bolts of stirrups at constant impact energy

On the basis of obtained operation characteristics, there was determined dependence between maximum value of force transmitted through the arch frictional joint with a rigid passive pressure (maximum load capacity of a joint) and total value of initial axial forces in the bolts of stirrups at constant impact energy (Fig. 7).

On the basis of obtained results one can state that with an increase of total value of initial axial forces in the bolts of stirrups, an increase of loading capacity of arch frictional joint takes place. Presence of passive pressure causes, that yield occurs in a joint at high value of initial axial forces in the bolts of stirrups, what in a case of lack of pressure is impossible.

Comparing the operation of an arch frictional joint with and without the rigid passive pressure, one can conclude that the passive pressure caused the occurrence of a yield within the joint. At the same parameters in the joint without passive pressure, a yield did not occur, whereas in the joint with passive pressure, a yield of 0.22 m occurred. The maximum value of the force transmitted through this joint amounted to 320 kN, at the impact energy equal to 9.81 kJ.

4. Summary and Conclusions

Results of tests presented in the article clearly indicate, that geometry and the way of supporting of the frictional joint have very significant influence on value of internal forces and components of the state of stress and deformation in particular elements of frictional joint.

Arch frictional joints subjected to the analysis, due to their geometry characterize with complex loading state, in which a bending moment has very significant role. Depending on the geometric parameters of the arch frictional joints, the way of its supporting and loading, the value of the bending moment can decide, whether in arch frictional joint a yield will occur, or it will be working as a rigid system.

Application of FEM for the analysis of operation of arch frictional joint enabled to determine distributions of state of stress and deformation in its particular elements during the arbitrary instant of loading.

Thanks to that, the state of an effort of these elements could be observed, and their weakest position could be finding. It is impossible to perform such an analysis on the base of stand or underground tests. Obtained results should be used at design works on the improvement and development of new solutions of frictional joints.

Results clearly indicate that the most strenuous of arch frictional elements, are bolts of stirrups.

Analyses carried out, confirm also a significant influence of liner on the operation of arch frictional joints. Liner in the form of passive pressure, which was simulated in the developed model, had a very significant impact on the occurrence or lack of the yield within the joint. Assuming, that the fundamental advantage of yielding support of dog headings is its yielding capacity, in the context of in the obtained results, a well-done liner could have a decisive impact on the correct operation of a support.

Summing up, one can conclude that numerical tests should be complement, and in some cases an alternative in relation to stand and underground tests of the elements of yielding support of dog headings.

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