Представлена динамічна математична модель безпроводової mesh-мережі, що реалізує принцип часового доступу до загального радіоресурсу. Модель орієнтована на спільне розв'язання задач розподілу слотів і маршрутизації із забезпеченням при цьому гарантій щодо швидкісних і часових показників якості обслуговування. Проведено аналіз міжкінцевих затримок з точки зору порядку розподілу слотів уздовж маршруту

Ключові слова: безпроводові mesh-мережі, якість обслуговування, множинний доступ з часовим розподілом, розподіл ресурсів

Представлена динамическая математическая модель беспроводной mesh-сети, реализующей принцип временного доступа к общему радиоресурсу. Модель ориентирована на совместное решение задач распределения слотов и маршрутизации с обеспечением при этом гарантий по скоростным и временным показателям качества обслуживания. Проведен анализ межконцевых задержек с точки зрения порядка распределения слотов вдоль маршрута

Ключевые слова: беспроводные mesh-сети, качество обслуживания, множественный доступ с временным разделением, распределение ресурсовулирование, рабочая характеристика

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MATHEMATICAL MODEL FOR RESOURCE ALLOCATION IN TDMA-BASED WIRELESS MESH-NETWORKS

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1. Introduction

Wireless networks are mobile (with mobile subscriber stations) or fixed networks (with fixed or low-mobile stations) which can work under two regimes, multipoint (Point-to-Multipoint, PMP) mode and mesh-mode. Under PMP-mode all flows, irrespective of the destination, pass through the base station. In the mesh-mode the stations of one network can interact with each other directly without involvement of the base station and thus perform the functions of routers. It has essential advantages, first of all, in efficiency of the channel resources use and area of coverage (service).

On specifications of technological bases for wireless mesh networks (WMNs) within next-generation communications systems different standardization organizations are working including IEEE 802.11, IEEE 802.15, IEEE 802.16 groups [1]. The standards and other described in literature approaches are based on different algorithms to share and compete in common resources. Depending on type of shared resources within WMNs time-division, code-division and space-division multiple access fashions (TDMA, CDMA and SDMA respectively) are used [1–3]. As it's shown in [2] TDMA scheduling scheme is more preferable from viewpoint of quality of service (QoS). Whereas problem of rendering of guaranteed quality of service is the urgent for next-generation communications systems, particularly for supporting real-time applications such as voice and video calls, in the article we'll focus on TDMA-based WMNs. Practical realization of TDMA-based wireless mesh-network is technology of wireless access WiMax (Worldwide Interoperability for Microwave Access), regulated by the IEEE standards of the 802.16 set [4].

2. Definition of resource allocation problem in TDMA-based WMNs and related work

Under TDMA an access to a common radio channel in wireless mesh-network is based on time division principle. Then the time slots, joined in the frames, act as a single resource of the link layer. In WiMax the slot is called minislot, and every such slot contains a particular amount of OFDM-characters as it is supposed to use the orthogonal frequency division multiplexing (OFDM) on a physical layer [4]. The amount of the user's data (byte), transmitted in one slot, depends on a common duration of the frame, on an amount of OFDM-characters in the slot, on a type of modulation and scheme of coding, which choice is, in turn, determined by the noise-to-signal conditions on each concrete site. As the frames duration of on the link layer of the WiMax mesh-network is noted by a service provider of the link, the number of OFDM-characters, fell within one slot and so the amount of bytes in each of them, are also known. Then the task of the link resources allocation represents the task of assignment of a particular set of time slots within the limits of one frame to each flow arriving for service. Thus, the resulting allocation

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of slots between stations of the mesh-network determines the sizes of link resources assigned each of them, i.e. the rate, at which the station can transmit the user's data flows. As a matter of fact the case in point is a reservation of resources, but, as is well known, the reservation should be carried out within the routes according to which the flow will be delivered to the end destination. Thus, the task of the link resources control of the mesh-network cannot be separated from the task of routing in it. Such cross-layered approach is demonstrated as more effective for network resource's utilization [5].

On the other hand resource allocation problem in TDMA-based WMNs must be solved with taking into account growing requirements of quality of service. For instance the WiMax technology is relatively new technology, it's designed in terms of the main tendencies of telecommunication systems and networks development: multiservice and multimedia nature of the transmitted traffic are provided in WiMax, the growing requirements of the users to the transfer rates and quality of service as a whole are taken into account. In general the 802.16 standard defines five classes (categories) of service quality for the WiMax net customers [6]:

– class of the guaranteed service "Unsolicited Grant Service" (UGS), oriented to the constant length packets transmission at a constant bit rate (the CRB service class analogue); it allows T1/E1 flows transmitting through IP;

– classes of the variable length packet transmission in a real time "Extended Real-time Polling Service" (ertPS) and "Real-time Polling Service" (rtPS), analogues of the VBR class are oriented to the VoIP traffic, MPEG video;

– class of the variable length packets transmission with a guaranteed minimum speed "Non-real-time Polling Service" (nrtPS) is oriented to the flows FTP, TFTP, HTTP;

– non-guaranteed Best Effort (BE) transmission applicable, for example, to the e-mail-traffic support.

Nevertheless, IEEE 802.16 standard defines the classes for PMP-mode but problems of the QoS support in the WiMax mesh-networks by the IEEE 802.16 standards are not touched upon. In the reason problem of rendering of guaranteed quality of service in the wireless mesh-networks stays an actual.

As the analysis has shown, the insufficiently complete elucidation of the QoS problems in the standard has given rise to a great number of publications on the given theme. Thus the main efforts of the developers are concentrated on the search of protocol solutions, for example, of the format of service messages and protocol of their exchange for different classes, or selection of configurable parameters for the flows of different classes in the process of "competition" for the right to begin transmission [7, 8]. In this case the principle of the slots allocation is saved on the basis of contention of neighboring stations for the right to utilize the particular slot (or group of slots).

Frequently the offered in literature protocol solutions for link resources allocation for the mesh-networks have a decentralized nature and are organized through the service messages exchange between adjacent stations, on the basis of which these stations "agree" about the order of the common link resources usage [5, 9]. The heuristic procedures of the link resources allocation should be changed for strictly formalized methods making it possible to provide the optimal controlling decisions with the aim to increase the efficiency of operation of the network and rendering of services of the guaranteed quality. In the reason we are offering following dynamic mathematical model of the TDMA-based mesh-network in the state space.

3. Mathematical model of the TDMA-based mesh-network

Thus in TDMA-based wireless mesh-network resources on link layer are represented by time slots with a fixed duration and capable to transmit a particular amount of the user's data bits. And task is time slots allocation, which provides the user's traffic delivery to the end recipient in the framework of the selected class of service.

Let us define as state variables values $q^{z}_{i,j}(k)$, representing the data volume in the i-th station and intended for transmission to the j-th station at the instant tk. Variables $\tau^{r,l,z}_{i,j}$ are considered as variables of control

$$\tau^{r,l,z}_{i,j}=\begin{cases}1, \text{ if the r-th slot is used in the link } (i,j) \\ \text{ for transmission of the flow addressed to l-st station} \\ \text{ in the frameworks of the z-th class of service,} \\ 0, \text{ otherwise,}\end{cases} \quad (1)$$

$$(i,j)\in E\,,\; r=\overline{1,N_F}\,,\; l=\overline{1,N_v}\,,\; l\neq i\,,\; z=\overline{1,N_{QoS}}\,,$$

where E – a set of links between stations of a mesh-network; N_F – an amount of slots in one frame used for transmission of a user's traffic; N_v – a total amount of stations of a mesh-network; N_{QoS} – an amount of QoS-classes supported by the network.

Such a choice of the state and control variables provides the flow nature for the model and makes it possible to inspect buffer and channel resources of the network. As the buffer capacity of the node is limited, the limitations are superimposed on the state variables:

$$q^{z}_{i,j}(k)\geq 0\,,\; \sum_{j} q^{z}_{i,j}(k)\leq q^{z\,\max}_{i}\,, \quad (2)$$

where $q^{z\,\max}_{i}$ – the maximal size of the queue of the z-th class of service at the i-th station.

It is supposed, that the packets of flows of different QoS-classes will be serviced in different queues, as it is implemented, for example, in the CBWFQ mechanism.

The control variables, according to the physical sense put in them, should meet one of the following conditions:

$$\sum_{z}\sum_{l}\sum_{(i,j)\in E}\tau^{r,l,z}_{i,j}(k)\leq 1\,, \quad (3)$$

$$\sum_{z}\sum_{l}\sum_{\substack{j,\\ j\neq i}}\tau^{r,l,z}_{i,j}(k)+\sum_{z}\sum_{l}\sum_{\substack{(g,j),\\ g\in S^2_i}}\tau^{r,l,z}_{g,j}(k)\leq 1\,, \quad (4)$$

where S^2_i – a set of stations experiencing an interference with the i-th station.

The expression (3) guarantees, that the r-th time slot in the k-th sampling interval will be utilized in the system only once. But the reuse of the slots can be applied to raise the efficiency of the link resources use in the mesh-network. Territorial separation of two stations, claiming for the same time slot, is a sufficient condition for this purpose and the distance between two stations must be more than two hops. In this case it is possible to consider, that the interference between stations is absent, and the control variables $\tau_{i,j}^{r,l,z}$ should be subordinated to the condition (4).

Then the dynamics of the queues of the z-th class of service at the i-th station of the mesh-network can be described as follows [10, 11]:

$$q_{i,j}^{z}(k+1)=q_{i,j}^{z}(k)-\sum_{\substack{v\in S_i^1,\\ v\neq i}}\sum_{r} m_{i,v}(k)\tau_{i,v}^{r,j,z}(k)n+$$

$$+\sum_{\substack{g\in S_i^1,\\ g\neq i,j}}\sum_{r} m_{g,i}(k)\tau_{g,i}^{r,j,z}(k)n+\xi_{i,j}^{z}(k)\Delta t, \qquad (5)$$

where $k=0,\ 1,\ 2\ldots$; $\Delta t=t_{k+1}-t_k$ – the sampling interval (period of re-computation and change of control variables $\tau_{i,v}^{r,j,z}(k)$); $m_{i,j}$ – amount of bits of the user's data that can be carried by one slot in link $(i,j)\in E$; S_i^1 – a set of stations of the mesh-network, adjoining with the i-th station; $\xi_{i,j}^{z}(k)$ – the intensity of the data arrival to the i-th station at the instant of time t_k in the frameworks of the z-th class of service addressed to the j-th station; n – the amount of the frames transmitted during time Δt, $n=\Delta t/T_F$; T_F – the frame duration.

Additional restrictions are imposed on the model with the purpose of providing the guaranteed quality services by the network:

$$\sum_{r} m_{i,j}(k)\tau_{i,j}^{r,l,z}(k)\le B_{req}^{z}$$

$$\text{or } B_{req\,min}^{z}\le\sum_{r} m_{i,j}(k)\tau_{i,j}^{r,l,z}(k)\le B_{req\,max}^{z}, \qquad (6)$$

where B_{req}^{z} – is the transmission rate (speed) of flow, required by the user, which can mean maximal or minimum rate depending on the requested class of service; $B_{req\,max}^{z}$ and $B_{req\,min}^{z}$ – are maximal and minimum values of the transmission rate of flow guaranteed by the network, for example, in case of the nrtPS class implementation.

The equation of state (5) can be written in the vector-matrix form

$$\vec{q}(k+1)=\vec{q}(k)+M(k)\vec{\tau}(k)n+\vec{\xi}(k), \qquad (7)$$

where $\vec{q}(k)$ – the mesh-network state vector at the k-th sampling interval of $N_{QoS}N_v(N_v-1)\times 1$ size, containing variables $q_{i,j}^{z}(k)$ and presenting the queues load in nodes; $\vec{\tau}(k)$ – the controlling vector of $N_F N_{QoS} N_e(N_v-1)\times 1$ size, which units the $\tau_{i,j}^{r,l,z}$ variables; N_e – the number of link in the network, $N_e=|E|$; $M(k)$ – the matrix of $N_v(N_v-1)\times N_F N_e(N_v-1)$ size, which units the $m_{i,j}(k)$ values at the k-th interval taken with regard to the sign (+ or -) in expression (5), and the principle of which creation is consistent with the order of elements in $\vec{\tau}(k)$; $\vec{\xi}(k)$ – the vector of a user's load of $N_{QoS}N_v(N_v-1)\times 1$ size, presenting the $\Delta t\xi_{i,j}^{z}(k)$ data volume, which come into the network from each station at the k-th interval.

Model (1)–(7) allows formulating the task of slot's allocation within the framework of the chosen class of service as an optimization one, where the Boolean variables $\tau_{i,j}^{r,l,z}(k)$ appear as the required variables, expressions (2)–(6) appear as limitations, and the cost functional can be offered as the goal function

$$J=\sum_{k=1}^{a}\Big[\vec{q}^{\,T}(k)W_q\vec{q}(k)+\vec{\tau}^{\,T}(k)W_\tau\vec{\tau}(k)-$$

$$-\vec{\tau}^{\,T}(k)W_{reuse}\vec{\tau}(k)\Big]\to\ \min, \qquad (8)$$

where a –the amount of intervals Δt, for which the control variables calculation is carried out; W_q, W_τ – the diagonal positive semidefinite weight matrices of buffer and link resources usage, respectively, W_{reuse} – the weight matrix presenting a gain at the cost of the slots reuse.

Thus, as a result of (8) solution we have the optimal strategy of dynamic routing and allocations of the link resources of the TDMA-based mesh-network.

4. Analysis of delays affected by an allocation of slots

Within offered model (1)–(8) conditions (6) guarantee delivering of user's traffic at required rate. But in general especially for voice and video-traffic there is important delay of delivering and the delay must satisfy

$$D_{\Sigma}^{z}\le D_{req}^{z}, \qquad (9)$$

where D_{Σ}^{z} – achieved total end-to-end delay of delivering within requested z-th class of service; D_{req}^{z} – required end-to-end delay within z-th class of service.

As compared to wired networks, in wireless multihop mesh-networks the situation with delays is much more complicated. It caused by two types of delays [2]. First type is conventional queuing delay that is related to difference between rates of packet arriving into node and packet transmission (service) through outgoing interface. In work [2] the delay is referred to as primary delay. But secondary delay is related to use of slots to carry packets. When new packet is arriving into node (mesh-station) it cannot be transmitted immediately; it should wait slot by which the packet can be carried to next node (station). Primary and secondary delays occur on every node in route from source to destination including node-source.

Thus total transmission delay D of packet in multihop network includes four components [2]. First component is the time between packet generation (moment when packet is arriving) and the end of current frame that is equal to $0.5\cdot T_F$, where T_F is the frame length. The result is based on assumption of uniform distribution of packet generation moments. And process of new packet arrival at the different nodes is assumed as independent. Second component of total transmission delay is the primary queuing delay D_q that can be calculated by using of queuing theory. Outgoing interface of Mesh Subsciber Stations (MSS) can be described as $M/D/1$ queuing system with the deterministic service time T_F [2]. Then

$$D_q=\frac{\rho}{2(1-\rho)}T_F, \qquad (10)$$

where $\rho=\lambda T_F$, λ – intensity of packet arriving, 1/s.

Third component of transmission delay is secondary delay described above. It's time interval between the first slot and the slot allocated to carry given packet for its MSS-destination. And last component is transmission time in both the source and destination MSSs that is equal to $T_s+T_s=2T_s$, where T_s is slot length.

From the components the third type delay (secondary delay) depends on number of hops in the source-destination route and order of slot allocation. To understand delay effect caused by slot allocation let us analyze two scenarios. In scenario # 1 shown in fig. 1 concerned topology is a four-node network where maximal distance between MSSs is three hops. MSS 1 is source of packets $\{y_1, y_2, \ldots\}$, MSS 4 is destination-station. Assume slots are allocated on the path MSS 1 – MSS 4 sequentially with no vacant slots between them (Fig. 1, *a*). For example in order to deliver packet y_1 source will used slot $\tau_{1,2}^{1,4}$, to transmit the packet to next station MSS 2 will use second slot $\tau_{2,3}^{2,4}$. So when MSS 2 will receive packet y_1 it can transmit the packet immediately. Similarly if station MSS 3 has allocated slot $\tau_{3,4}^{3,4}$ received packet can be transmitted in same frame. As a result the sum of third and fourth components in end-to-end delay is $3T_s$. As it's shown in Fig. 1, *b* if there is one vacant slot between used slots the sum becomes $4T_s$. An analytical expression for calculation of average value of secondary delay between source and destination was proved in work [2] and it's given by

$$D_{s-d} = \frac{\sum\limits_{j=h}^{M_F}\left[\binom{j-2}{h-2}\cdot(j-2)\right]}{\sum\limits_{j=h}^{M_F}\binom{j-2}{h-2}}T_s, \qquad (11)$$

where h – number of hops in the route between source and destination; M_F – number of slots per frame.

Thus total end-to-end delay of delivering can be given by

$$D_\Sigma = 0.5\cdot T_F + D_q + D_{s-d} + 2T_s. \qquad (12)$$

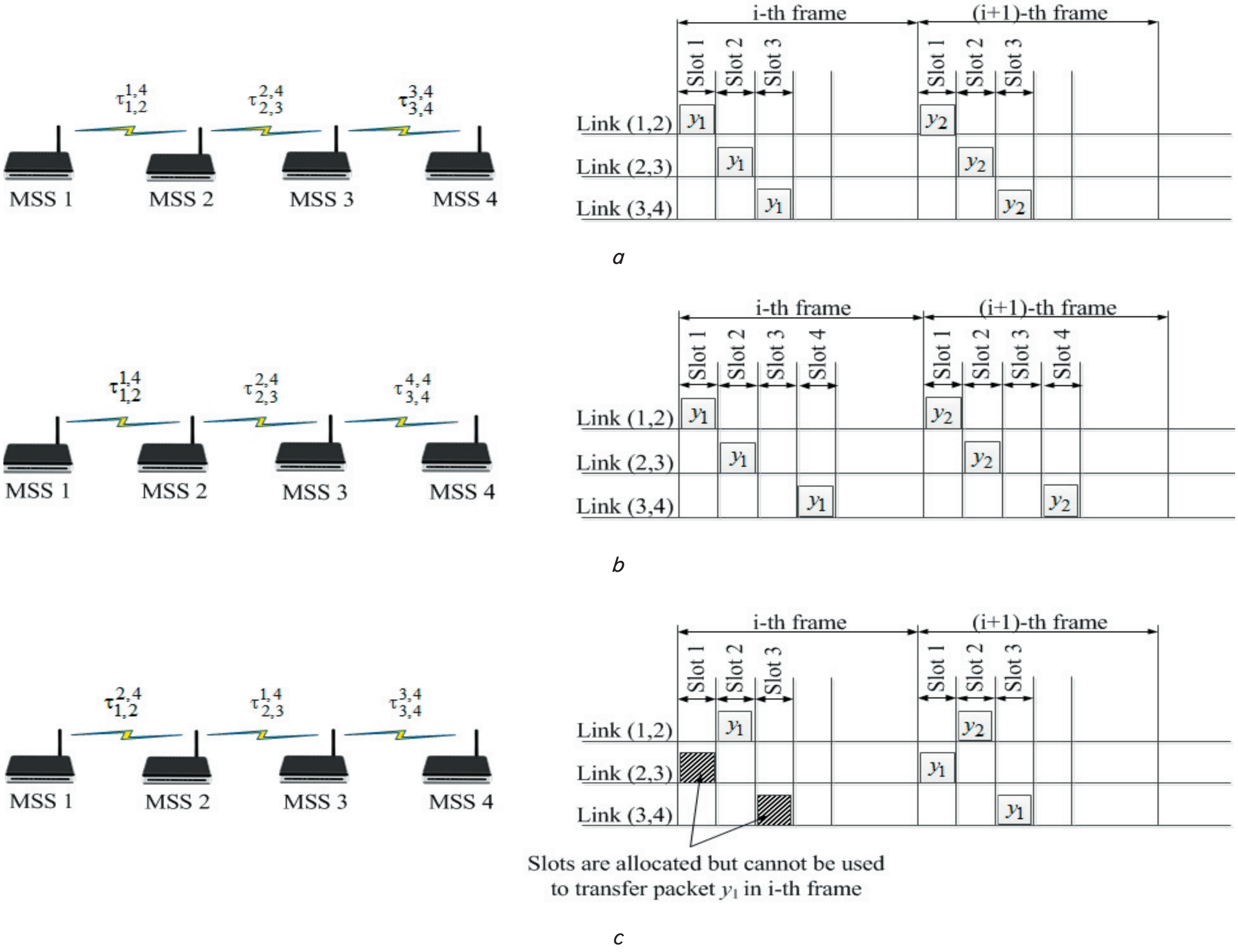
Equality (11) is related to case when all of slots along the path are used sequentially and defines low bound for secondary delay. If slots are allocated with breach of the order D_{s-d} is growing.

And every such "irregularity" adds time T_F to total delay. For example in Fig. 1, *c* secondary delay and transmission time in both the source and destination MSSs is equal to $(3T_s + T_F)$.

If order of slots on the path isn't controlled and is random then total end-to-end delay of delivering can be estimated by [2]

$$D_\Sigma = \frac{h+1}{2}T_F + T_s. \qquad (13)$$

In scenario # 2 topology has five nodes that allow reusing slots by stations MSS 1 and MSS4 (Fig. 2). If to compare scenarios without reuse of slots (Fig. 2, *a*) and with reuse (Fig. 2, *b*) we'll observe different delays.



a

b

c

Fig. 1. Examples of time slot allocation in WMN's topology with four nodes: *a* — with no vacant slots between used slots; *b* — with one vacant slot between used slots; *c* — with broken sequence of slots

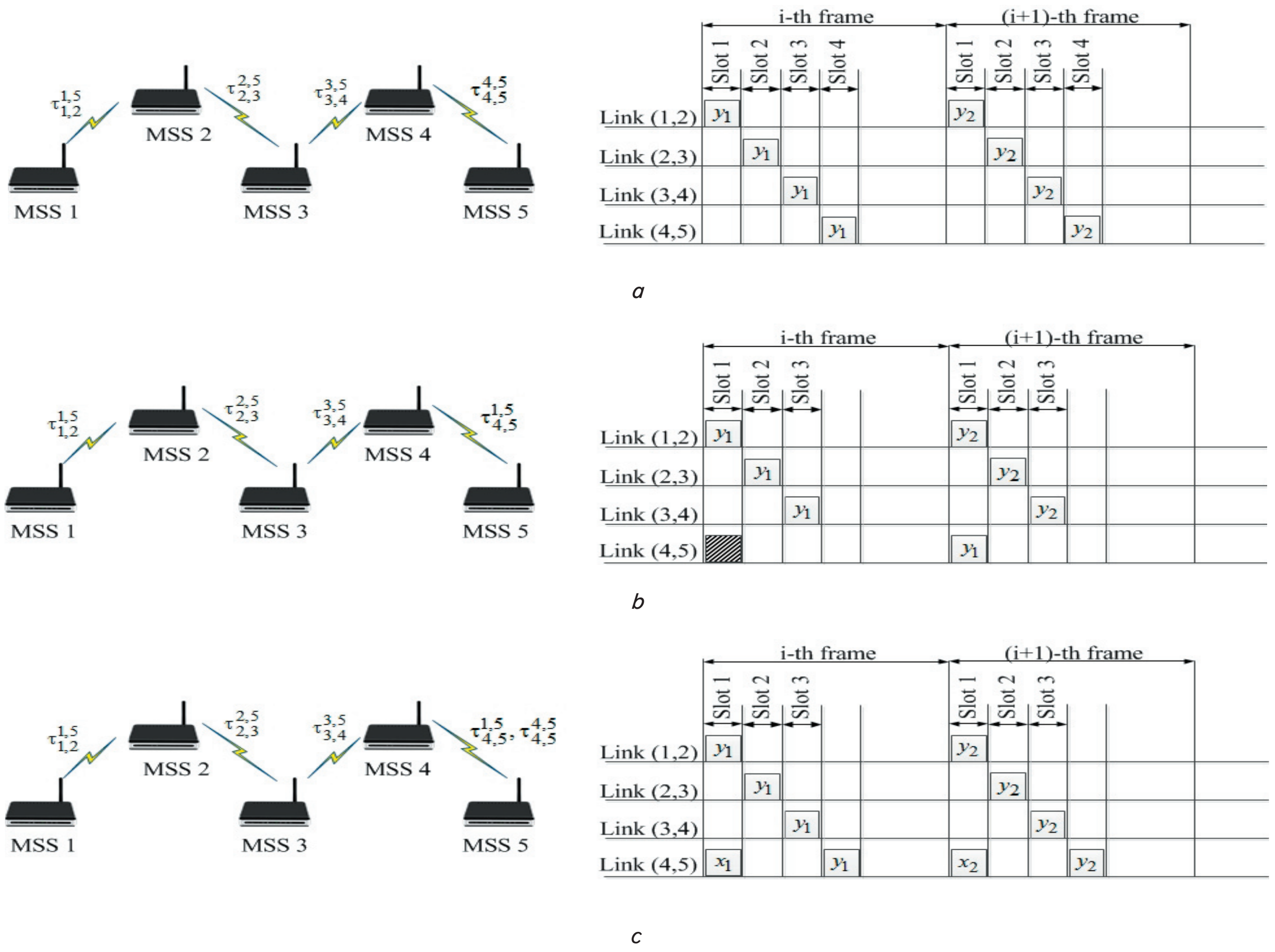


Fig. 2. Examples of time slot allocation in WMN's topology with five nodes: *a* — without reuse of slots; *b* — with reuse; *c* — with two flows and slot's reuse

In first case secondary delay including transmission time in both the source and destination is equal $4T_s$, but it becomes (T_F+T_s) in second case. In average every reused slot adds time T_F to total delay. Thus reuse of slots allows to save on link resources of WMN and to utilize them in effective way. But on the other hand it leads to quality loss through the delay's growing.

A way to avoid delay's growing under reuse of slots is shown in fig. 2, c according to which within some pair of source-destination all of slots are used sequentially without repeating. For example in order to carry flow of packets $\{y_1, y_2, \ldots\}$ from MSS 1 to MSS 5 following slots $\{\tau^{1,5}_{1,2}, \tau^{2,5}_{2,3}, \tau^{3,5}_{3,4}, \tau^{4,5}_{4,5}\}$ are allocated. But for transmitting packets $\{x_1, x_2, \ldots\}$ within pair of source-destination MSS 4 – MSS 5 slot $\tau^{1,5}_{4,5}$ is assigned again. So concerned WMN reuses first slot but for different flows.

Thus in order to avoid delay's growing especially for delay-sensitive traffic sequence of slots along the path must be controlled. Then cost function (8) can be rewritten by taking into account order of slots along the path:

$$J=\sum_{k=1}^{a}\Big[\vec{q}^{\,T}(k)W_q\vec{q}(k)+\vec{\tau}^{\,T}(k)W_\tau\vec{\tau}(k)-$$

$$-\vec{\tau}^{\,T}(k)W_{reuse}\vec{\tau}(k)+\vec{\tau}^{\,T}(k)W_{seq}\vec{\tau}(k)\Big]\to \min, \qquad (14)$$

where W_{seq} – the weight matrix presenting a breach of the order of slots along the path. Its element is 0 if sequence number of slot in a link is higher than sequence number of slot used in previous link and more the 0 if sequence of slots is broken.

5. Conclusions

Thus, offered mathematical model allows to describe TDMA-based wireless mesh-network from viewpoint of link layer where time slot is the main resource. Within the model the joint solution of the problems of routing and link resources allocation in the mesh-networks can be reduced to solution of the optimization task on minimization of a target cost function under a number of limitations, including QoS-conditions (6), (9) and the dynamic ones (5).

The main difference of the offered model from the earlier known ones consists in its belonging to the class of the dynamic models in the state space, that gives the following advantages: guaranteed quality of service; consideration of the dynamic nature of user's flows, arriving for service and the state of the network itself, including the structure dynamics and signal-to-noise conditions; the possibility to control both the link and buffer resources of the network; the possibility to apply developments in the theory of optimal control field to solving of the task of routing and control of the link resources, where the equation of the system behavior in the state space takes one of central places. On the other hand imposition of QoS-conditions leads to complication of the mathematical description and numerical calculation. So open issue for future study is related to method of applying the model where different heuristics and decompositions principles can be implemented.

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