

У роботі виконано порівняльний аналіз ефективності різних видів згорток у задачах умовної багатокритеріальної оптимізації. Дослідження проводилося для ряду інноваційних технологій формування біополімерних матеріалів. Отримані результати можуть бути використані для оптимізації аналогічних технологій формування біополімерних матеріалів

Ключові слова: згортка, багатокритеріальна оптимізація, фізико-хімічні процеси, біополімерні матеріали

В работе выполнен сравнительный анализ эффективности различных видов сверток в задачах условной многокритериальной оптимизации. Исследование проводилось для ряда инновационных технологий формирования биополимерных материалов. Полученные результаты могут быть использованы для оптимизации аналогичных технологий формирования биополимерных материалов

Ключевые слова: свертка, многокритериальная оптимизация, физико-химические процессы, биополимерные материалы

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COMPARATIVE ANALYSIS OF SOME COMPUTATIONAL SCHEMES FOR OBTAINING A COMPROMISE SOLUTION

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1. Introduction

The existing methods for searching a compromise solution to the problems of multi-criteria optimization are based on reducing the initial problem to one or more one-criterion optimization tasks. The so-called convolution of the partial criteria in the general one is used most often in the tasks of formation of multifunctional biopolymer materials [1–5]. However, the convolution, allowing finding the effective solutions to all the problems within one class may not be suitable for solving the tasks of another class. Therefore, the main issue a researcher faces at the stage of the generalized objective function is a preferable convolution method. The methods effectiveness of obtaining the generalized objective function, discussed below, is analyzed on the basis of the example of a number of models that describe various processes and stages of formation of multifunctional biopolymer materials.

2. Analysis of published data and problem statement

There are two basic approaches to the selection of the computational schemes of a compromise solution: the priority approach [1–3, 5, 6] and convolution of criteria [2–4, 7]. In multi-objective optimization tasks for physical and chemical processes of formation of the biopolymer materials, the second approach is mainly used.

The most commonly used types of convolution are the additive and multiplicative convolutions [3, 5], maximin contraction [2], the method of ideal point [2], Harrington's desirability function [4, 5], the analytic hierarchy process and some others [1–5]. The main difficulties encountered

by engineers using these methods – the selection of the convolution and justification of the weighting coefficients of the generalized objective function. The papers [1, 2] prove that a low score on one criterion is not always can be compensated by the higher one on the other criterion. Let us mention as well that the quality of the compromise solution decreases with the increase of the output variables vector length i.e. transitioning to the "big" systems. In this case, the Intellectual Methodology of «Big» System Research [8, 9] allows us to estimate the reserves of improving the efficiency of the studied objects by formalizing the procedures of criteria convolution.

The use of the additive and multiplicative types of convolutions requires a limitation: the set of feasible solutions must be convex, while in case of the multiplicative convolution it additionally requires concavity of all functions $\ln f_i(\bar{x})$, $i = 1, 2, \dots, k$ [1, 2]. The failure to comply with these conditions may lead to results that can not be realized in practice, since the obtained solutions do not meet the technological limitations. Some types of convolutions mean equal contribution of all partial objective functions that are quite rare in practical tasks. According to the Intellectual Methodology of «Big» System Research [8], the convolution in a generic criterion should be based on the regulatory requirements and other formal restrictions, and a compromise suboptimization is conducted in the range of the permissible values. The main problem of choosing the convolution method is a subjectivity of the decision made.

Considering the above stated, a comparative analysis of the effectiveness of various computational schemes for a compromise solution to the tasks of formation of the biopolymer materials is of practical interest to specialists

in this field. Let us formulate the problem of multi-objective optimization in the following way.

Let the quality of the optimization object is estimated by a vector function

$$f(\bar{x}) = (f_1(\bar{x}), f_2(\bar{x}), \dots, f_k(\bar{x})), \quad (1)$$

the components of which are the given functions $f_j(\bar{x})$ ($j=1,2,\dots,k$) of vector $\bar{x}=(x_1,x_2,\dots,x_n)$. Vector \bar{x} , thus, belongs to the set X of its possible values. On the variables x_i ($i=1,n$), as a rule, the constraint in the following form is imposed

$$l_j \leq x_j \leq u_j \quad (j=1,2,\dots,n), \quad (2)$$

or

$$q_i(\bar{x}) \leq b_i \quad (i=1,2,\dots,m). \quad (3)$$

In this case, when the set goal is characterized by several functions $f_i(\bar{x})$, the optimization task is to find the constrained minimum or maximum of all criteria. The solution of the formulated task requires finding a compromise between the criteria $f_1(\bar{x}), f_2(\bar{x}), \dots, f_k(\bar{x})$, which generally are contradictory.

We will call the limited and closed set D as a set of admissible values of the vector \bar{x} , which is formed by the limiting functions (2) and (3).

It is required to find such a point $\bar{x}^* \in D$, that will provide an optimal value of the functions $f_1(\bar{x}), f_2(\bar{x}), \dots, f_k(\bar{x})$ on the set D .

Analysis of publications devoted to the mathematical modeling of physical and chemical processes of the biopolymer materials formation showed that the authors mainly use the algebraic equations, resulting from the processing of the experimental data, as the objective functions [10–13]. This form of recording the mathematical models is the most convenient in case of having incomplete information about the specifics of the process.

Differential equations in publications devoted to the modeling and optimization of chemical technology processes describe the dynamic continuous-time processes [6]. Typically, these are the differential equations of the first or second order. The models in excess describe the processes with discrete time or established processes.

In [1, 2] it is shown that the solutions obtained for one class of tasks may not be suitable for another class of tasks. Thus, the comparison of the effectiveness of various computational schemes for obtaining a compromise solution for the tasks of multi-objective optimization for the formation of multifunctional biopolymer materials is an important task.

3. Purpose and objectives of the study

The purpose of this paper is a comparative analysis of the effectiveness of some of the most frequently used computational schemes for a compromise solution for the physical and chemical processes of the biopolymer materials formation.

In accordance with the set goal the following research objectives are identified:

- the formulation of the criteria to assess the effectiveness of various computational schemes;

- the allocation of a group of tasks specific to the production of multifunctional biopolymer materials;

- adaptation of the computational schemes for the tasks of the constrained optimization of the biopolymer materials.

4. Computational schemes for obtaining a compromise solution in the tasks of formation of multifunctional biopolymer materials

Analysis of the effectiveness of different schemes for obtaining a compromise solution is made for a number of physical and chemical processes occurring during the formation of the biopolymer materials such as leather and fur. In this paper we use a mathematical description of the main stages of production of leather and fur materials:

1. The stage of preparation of raw skin for further processing, such as structuring. In this paper, we consider two processes of soaking and liming:

Process 1.1. Dehairing and liming with hair recycling. As a result of the experimental data, three algebraic equations of the second order are obtained; each criterion is minimized.

Process 1.2. One-step dehairing-liming. The process statics is described by two algebraic equations of the second order, dynamics – by ordinary differential equations (ODE) of the first order; the first criterion is minimized, the second – is maximized.

2. The stage of stabilization and formation of the semi-finished product structure:

Process 2.1. Semi-finished product tanning. The mathematical description of the process is represented by four algebraic models of the second order; all objective functions are maximized.

Process 2.2. Wool sheepskin tanning-greasing. The mathematical description of the static process is obtained by processing the experimental data and is represented by five algebraic equations of the second order, the dynamics of the process is described by the first order ODE; all partial criteria are maximized.

3. Semi-finished product treatment stage:

Process 3.1. Formation of the lacquer coating. As a result of the experimental data processing, four algebraic equations of the second order are obtained; three obtained criteria are maximized, the fourth – is minimized.

Process 3.2. Formation of the emulsion coating. The mathematical description of the static process is represented by three algebraic equations; dynamics - three ODE; all objective functions are maximized.

Let us consider in more detail the mathematical description of the physical and chemical processes of a semi-finished product structure formation – pelt at the stage of cowhide dehairing-liming, the features of which are analyzed in detail in [10–12]. The development of a universal two-stage technology of dehairing-liming with hair recycling requires rapid dehairing of raw materials. This is facilitated by the use of effective surface and biologically active substances. This creates favorable conditions for the transport of chemicals, due to the removal of non-collagenous structures with animal skins, deep separation of fibrillar structures of the dermis, increased mobility of its structural elements, which contributes to the efficient use of raw materials.

The technology provides conduction all the soaking-ash processes at a rate of basic alkaline reagents and time course

of treatment presented in Table 1. The effectiveness of the process is determined by the consumption of raw materials to the 1 m² of skin (y₁, kg/m²), plumping extent due to excessive moisture content in the pelt (y₂, % mass of the fresh raw materials), and elongation at a load of 9,8 MPa (y₃, %) at minimal consumption of alkaline reactants and the duration of the process of preparation of raw hides to further technological processes.

Table 1

Experiment Plan Parameters

Factor	Symbol	Zero Level	Variability Interval
Consumption, % by weight of raw materials			
sulphides in recalculation to anion S ²⁻	x ₁	0,74	0,2
calcium hydroxide in recalculation to cation Ca ²⁺	x ₂	1,19	0,22
Liming Time Length, h	x ₃	12	4

The greatest influence on the output variables has the liming time, and the smallest – the consumption of calcium hydroxide and sodium sulfide [12 In order to obtain the mathematical description of the process of dehairing-liming, a series of experiments on the implementation of the central composite rotatable plan at the public Closed JSC Chinbar (Ukraine) using cowhides by green-salting cure. The results of the experiment are shown in Table 2.

Table 2

Experiment Results

#	1	2	3	4	5	6	7	8	9	10
y ₁	7,21	6,93	6,78	6,71	6,98	6,67	6,72	6,63	6,87	6,77
y ₂	14	19	17	23	20	25	24	29	17	26
y ₃	12	18	21	27	20	26	29	38	19	34
#	11	12	13	14	15	16	17	18	19	20
y ₁	6,74	6,45	7,03	6,46	6,57	6,44	6,5	6,45	6,55	6,59
y ₂	19	24	15	23	20,5	21,5	21	20	21,5	21
y ₃	21	39	17	38	34	35	37	35	32	33

As a result of the experimental data, the mathematical description of physical and chemical processes of soaking-liming of raw hides of cattle is obtained:

$$\begin{cases} y_1 = 6,51346 - 6,727833 \cdot 10^{-2}x_1 - 0,1053295x_2 - 0,116352x_3 + \\ + 0,127792x_1^2 + 0,0480732x_2^2 + 0,1012196x_3^2, \\ y_2 = 20,89248 + 2,647162x_1 + 1,714939x_2 + 2,817191x_3 + \\ + 0,3600024x_1^2 + 0,3600043x_2^2 - 0,5258x_3^2, \\ y_3 = 34,45781 + 3,825634x_1 + 5,074461x_2 + 5,150681x_3 - \\ - 3,56452x_1^2 - 2,324439x_2^2 - 3,210208x_3^2. \end{cases}$$

Restrictions on the variables

$$0,54 \leq x_1 \leq 0,94, \quad 0,97 \leq x_2 \leq 1,41, \quad 8 \leq x_3 \leq 16. \quad (5)$$

According to experts, all three indicators of quality y₁ – have about the same weight, but more important from an economic point of view is the first indicator – consumption of raw materials per 1 m² of skin.

The detailed description of the other processes is presented in [10–13].

In this paper, a compromise solution for each of the presented processes is obtained using the following types of convolutions: additive (linear and quadratic), multiplicative, maximin, ideal point method, the convolution based on Harrington's desirability function. Below we briefly consider computational scheme of each method and the results of solutions for the model (4) subject to the restrictions (5).

The additive method for constructing the objective function is most frequently used in multi-objective optimization tasks:

$$f(\bar{x}) = \sum_{i=1}^k \alpha_i f_i^p(\bar{x}), \quad (6)$$

where α_i – weight coefficients, α_i ≥ 0, $\sum_{i=1}^k \alpha_i = 1$, p = 1, 2, 3,...

In this paper, the weighting coefficients values are assumed equal for easier comparison of different methods of convolutions. Thus, the vector of the coefficients takes the form (0,33 0,33 0,33), and the generalized objective function at p = 1:

$$f(\bar{x}) = 0,33f_1(\bar{x}) + 0,33f_2(\bar{x}) + 0,33f_3(\bar{x}). \quad (7)$$

The search of the optimum function (7) is done by using the Box method; the algorithm of the method is discussed in detail, for example, in [7]. The following results are obtained.

Optimum coordinates: x₁ = 0,54 %, x₂ = 0,97 %, x₃ = 8,4 h.

The minimum values of the partial criteria: f₁(x̄) = 6,44 kg/m², f₂(x̄) = 26,24 % form weight of the fresh raw materials, f₃(x̄) = 40,02 % elongation under a load of 9,8 MPa. The minimum value of the generalized objective function f(x̄) = 3,522.

In case of the increase of the exponent of the function (6) to p = 2, we obtain the so-called additive quadratic convolution:

$$f(\bar{x}) = 0,33f_1^2(\bar{x}) + 0,33f_2^2(\bar{x}) + 0,33f_3^2(\bar{x}). \quad (8)$$

Let us note that the increase in the degree of partial criteria had virtually no impact on the results of optimization: the coordinate values of the optimum in this example differ only in the second and third place.

More versatile compared to the previous method is the maximin convolution [1, 2]. Let the optimization task be formulated as

$$f(\bar{x}) = \min_{i=1, \dots, k} \alpha_i f_i(\bar{x}),$$

then the optimal point is chosen:

$$(4) \quad \bar{x}^* = \max_{x \in X} \min_{i=1, \dots, k} \alpha_i f_i(\bar{x}).$$

In this example, the same values of the weighting coefficients are chosen the same: α₁ = α₂ = α₃ = 0,33. The calculation results: coordinates of the optimum – x₁ = 0,74 %, x₂ = 1,37 %, x₃ = 16 h, the minimum values of the partial criteria: f₁(x̄) = 6,44 kg/m², f₂(x̄) = 24,83 % from the weight of the fresh raw materials, f₃(x̄) = 38,99 % elongation at the load of 9,8 MPa. The minimum value of the generalized objective function f(x̄) = 0,479.

The multiplicative criteria convolution method envisages the acquisition of the generalized objective function in accordance with the following expression:

$$f(\bar{x}) = \sum_{i=1}^k f_i^{\alpha_i}(\bar{x}), \quad (9)$$

where α_i – weight coefficients, $\alpha_i \geq 0$, $\sum_{i=1}^k \alpha_i = 1$.

It was shown above that the convolution has the most stringent conditions of Paretooptimality: the convex set of feasible solutions and concavity of all the functions $\ln f_i(\bar{x})$, $i=1,2,\dots,k$ [2]. The results of the optimization process for the model with equal weighting factors: the coordinates of the optimum – $x_1 = 0,545\%$, $0,98\%$, $x_3 = 8h$, minimum values of the partial criteria: $f_1(\bar{x}) = 6,5 \text{ kg/m}^2$, $f_2(\bar{x}) = 28,17\%$ of the weight of fresh raw materials, $f_3(\bar{x}) = 39,42\%$ elongation at a load of 9,8 MPa. The minimum value of the generalized objective function $f(\bar{x}) = 1,56$.

In accordance with the method of an ideal point, in the criteria space an object which has the highest possible values for each of the criteria is selected:

$$b_i = \max_{\bar{x}} f_i(\bar{x}), \quad (10)$$

where b_i – vector estimation of the ideal point $\bar{x}^* \in D$ in the criterion space.

Let us define the distance $s(\bar{b}, f(\bar{x}))$ between the points \bar{b} and $f(\bar{x})$ as

$$s(\bar{b}, f(\bar{x})) = \left[\sum_{i=1}^k (b_i - f_i(\bar{x}))^p \right]^{\frac{1}{p}}, \quad (11)$$

where $p=1, 2, 3,\dots$

Then the search for the optimal solution is reduced to finding the point $\bar{x}^* \in D$, the closest one to the ideal point:

$$s(\bar{b}, f(\bar{x})) \rightarrow \min_{\bar{x} \in D}. \quad (12)$$

Let us consider this method on the example of a process 1.1 model, the mathematical description of which is represented by the equations (4) and constraints (5). We take $p=1$, then the distance between the points \bar{b} and $f(\bar{x})$ can be written as:

$$s = [b_1 - f_1(\bar{x})] + [b_2 - f_2(\bar{x})] + [b_3 - f_3(\bar{x})]. \quad (13)$$

Using the necessary extremum conditions of several variables function, we find the partial derivatives of the function (13) by x_1, x_2, x_3 and equate them to zero. We obtain a system of three equations with three unknowns:

$$\begin{cases} \frac{\partial f_1(\bar{x})}{\partial x_1} + \frac{\partial f_2(\bar{x})}{\partial x_1} + \frac{\partial f_3(\bar{x})}{\partial x_1} = 0, \\ \frac{\partial f_1(\bar{x})}{\partial x_2} + \frac{\partial f_2(\bar{x})}{\partial x_2} + \frac{\partial f_3(\bar{x})}{\partial x_2} = 0, \\ \frac{\partial f_1(\bar{x})}{\partial x_3} + \frac{\partial f_2(\bar{x})}{\partial x_3} + \frac{\partial f_3(\bar{x})}{\partial x_3} = 0. \end{cases} \quad (14)$$

After substituting the corresponding partial derivatives of the equations (4) to (14) and solving the resulting system by the known methods (iteration method, Newton's method, etc.), we find the vector of the optimal values $\bar{x}^* = (0,532 \ 0,80633 \ 7,68)$. In accordance with the ideal point method, the resulting coordinates of the optimum provide the shortest distance between the optimum and ideal point. Let us substitute the coded values of the vector \bar{x}^* in the equation (5) to obtain the optimal values of the partial criteria: $f_1(\bar{x}) = 7,295 \text{ kg/m}^2$; $f_2(\bar{x}) = 12,97\%$ of the mass of the fresh raw materials; $f_3(\bar{x}) = 1,386\%$ elongation at a load of 9,8 MPa.

When using the Harrington's desirability function [4] the quality of the process is assessed by the formula:

$$D = \sqrt[k]{\prod_{i=1}^k d_i}, \quad (15)$$

where k – the number of indicators of process quality, d_i – the private desirability function for the i -th quality index y_i , which is determined by the formula

$$d_i = \exp[-\exp(-y_i')], \quad (16)$$

where y_i' – dimensionless quality index y_i .

With the best value of the property $d_i=1$, the worst – $d_i=0$. In practical problems it is usually assumed that the value of $d_i=0,37$ corresponds to the lower boundary of the acceptable values, and $d_i=0,63$ – the lower limit of "satisfactoriness". In this paper, the values of the partial criteria corresponding to the worst and the best value, are the following: y_1 – 6,58 and 6,47 kg of raw material/m² of skin, y_2 – 21,5 and 20,5 % of raw materials weight, y_3 – 32 and 37 % of the initial sample length. The calculations showed the vector of optimal values for the process 1.1: $\bar{x}^* = (0,71 \ 1,25 \ 11,76)$. The value of the desirability function at the optimum point $D=0,62165$, and the corresponding partial criteria $y_1=6,51 \text{ kg of raw material/m}^2$ of skin, $y_2=20,8\%$ of the raw material weight, $y_3=34,6\%$ of the initial sample length.

The results of the experimental verification of the results of the optimization of different types of convolutions for the rest of the formation of the biopolymer materials are presented below.

5. Comparative analysis of the effectiveness of different types of convolutions

In order to compare and analyse the optimization results for all the models listed in p. 4, a software module in the VBA environment (Visual Basic for Application) was developed. It implements the considered computational schemes of a compromise solution. A user places initial data on the MS Excel sheet, and then can run the calculation of one of the computational schemes. When evaluating the different kinds of convolutions in the first place, the possibility of finding all effective solutions, the results of experimental verification of the solutions, simplicity of the algorithm, as well as the time it takes the researcher to carry out the calculations, including the time for the performance of mathematical operations required to form a generalized objective function were considered.

Optimum 1. 1 process parameters obtained using different types of convolutions are given in Table 3.

Table 3

The results of the optimization process of raw materials dehairing-liming with hair recycling

Type of Convolution	Optimum Coordinates			Objective Functions Value			Value of Generalized Objective Function
	x ₁	x ₂	x ₃	y ₁	y ₂	y ₃	
Additive	0,54	0,97	8,4	6,443	26,242	40,016	3,52238
Maximin	0,74	1,37	12,3	6,444	24,828	38,994	0,47871
Multiplivative	0,545	0,98	8	6,500	28,117	39,417	1,56192
Generalized Desirability Function	0,71	1,25	11,76	6,51	20,8	34,6	0,62165
Ideal Point Method	0,532	0,81	7,68	7,295	12,97	1,386	–

Compromise area for a mathematical model of the process 1.1 is shown in Fig. 1. Response surface of partial private functions are created when changing the coded values of the factors ranging within [-1; 1] in the increments of Δx_i=0,01.

Experimental verification of the optimization results shown in Table. 3 shows that the lowest consumption of raw materials in rational elongation under load of 9,8 MPa is achieved at moderate plumping is achieved at ashing process duration of about 12 hours; wherein the consumption of hydrosulfide, sodium sulfide and calcium hydroxide does not exceed 0,89; 1,19; 3,30 respectively % by weight of raw materials [10]. Thus, the best results are obtained using a maximin convolution and Harrington's desirability function. In accordance with the received data, raw material consumption can be reduced by 4.7 %, compared with other options, which differ by longer duration and a high content of environmentally hazardous substances in the waste solution. It should also be noted that the method of ideal point is the most cumbersome.

Optimum process parameters for 1. 2, process obtained using discussed in p. 4 convolutions types are presented in Table 4.

Table 4

The results of the optimization of one-step process of dehairing-liming of raw leather materials

Type of Convolution	Optimum Coordinates				Objective Functions Value		Value of Generalized Objective Function
	x ₁	x ₂	x ₃	x ₄	y ₁	y ₂	
Additive	5,4	10,2	13	32	631,74	83,09	49,19542188
Maximin	6	6	3,2	28	669,99	82,80	7,896796875
Multiplivative	6,8	10,6	13	32	631,74	83,09	36,20436238
Generalized Desirability Function	7,99	8,99	4	32	640,67	82,73	0,61425
Ideal Point Method	3,98	8,70	9,5	28,9	637,94	90,74	–

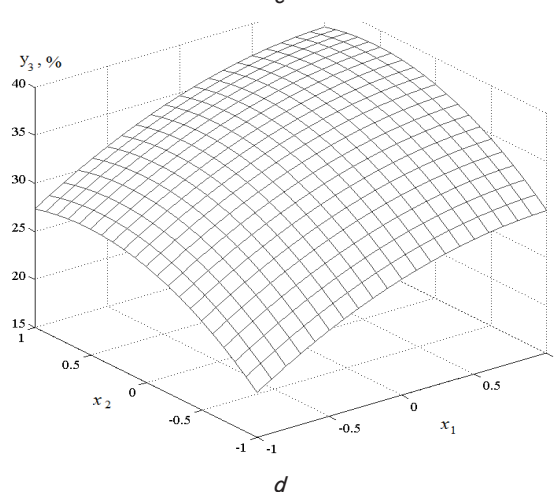
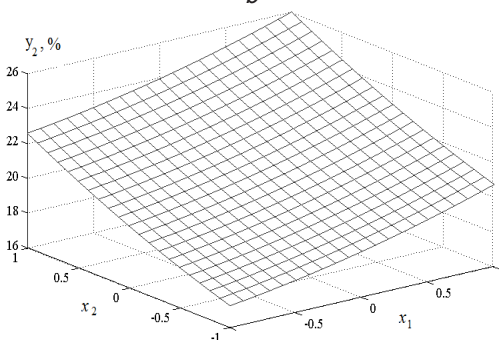
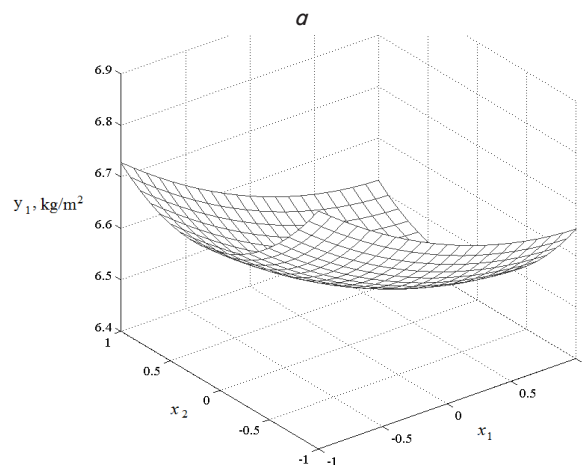
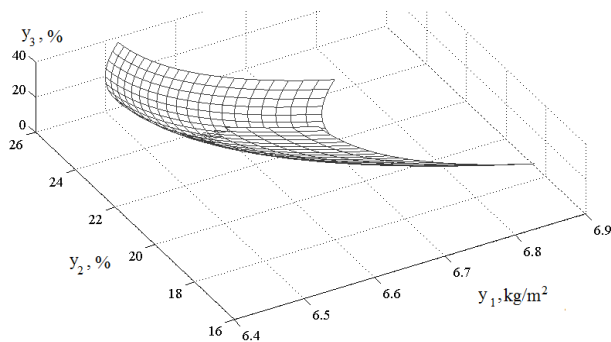


Fig. 1. The compromise area for a mathematical model of the process 1. 1: a – compromise area in the space of criteria y₁–y₃; b–d – response surface of the partial objective functions y₁=f(x₁,x₂), y₂=f(x₁,x₂), and y₃=f(x₁,x₂) accordingly

The compromise area for the mathematical model of the process 1. 2 is shown in Fig. 2. The response surface of the partial objective functions are created when changing the coded values of the factors ranging within [-1; 1] in increments of $\Delta x_i = 0,01$.

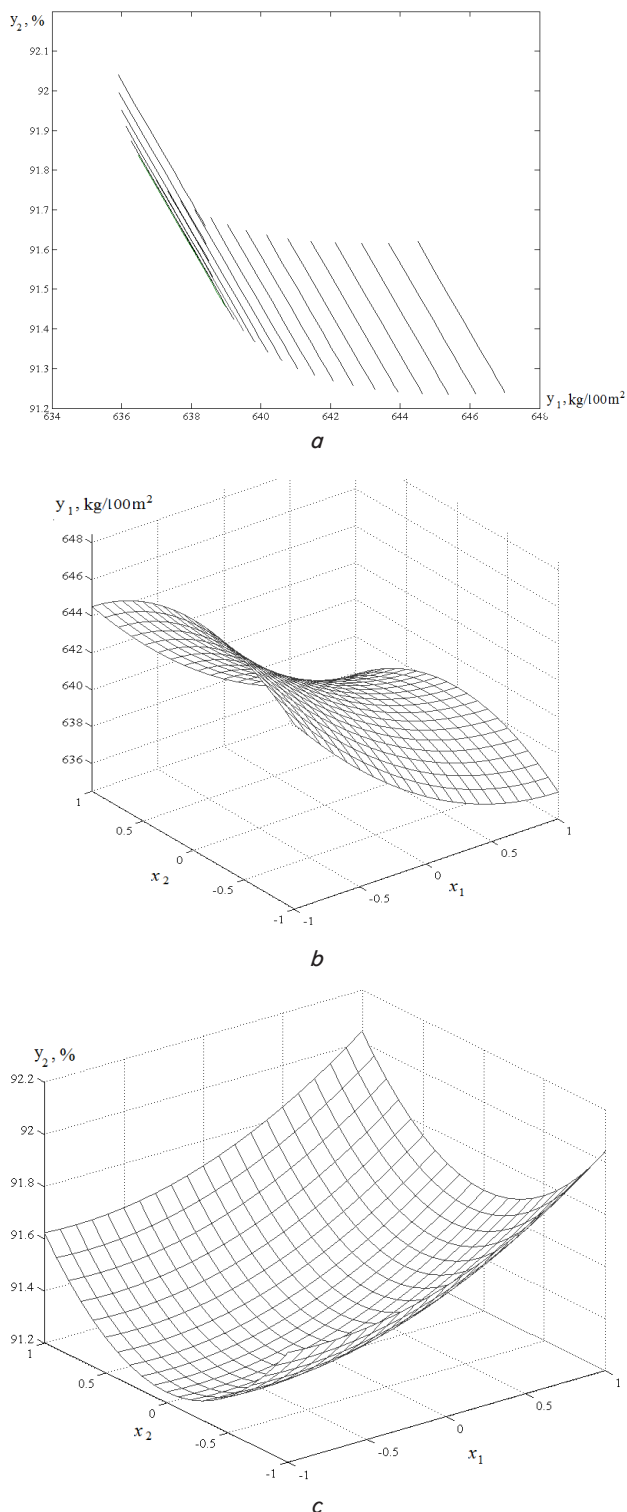


Fig. 2. Compromise area for a mathematical model of 1.2: a – compromise area in the space of criteria y_1 – y_2 ; b, c – the response surface of the partial objective functions $y_1=f(x_1,x_2)$ and $y_2=f(x_1,x_2)$ accordingly

The analysis of the information provided shows that the use of maximin convolution allowed to find all compromise tasks, unlike other methods of convolutions; while in the method of ideal point the value of the partial objective function y_2 is out of the bounds of the permissible area [82–85]. The conducted experiments confirmed the results of the calculations.

The optimal values of physical and chemical processes occurring at the stage of stabilization and structure formation of the semi-finished product are presented in Tables 5, 6.

Table 5

The results of the optimization process of semi-finished product tanning

Type of Convolution	Optimum Coordinates			Objective Functions Value				Value of Generalized Objective Function
	x_1	x_2	x_3	y_1	y_2	y_3	y_4	
Additive	1,288	33,8	0,68	87,74	108,5	243,1	-7,21	2,91136
Maximin	1,4	26	0,2	85,24	105,0	220,2	-7,69	1,10891
Generalized Desirability Function	1,2	34,17	0,68	99,27	106,6	239,6	-4,05	0,64523
Ideal Point Method	1,366	34,2	0,71	94,06	109,8	248,9	-3,70	-

The multiplicative convolution was not used for 2.1 process, since the value of the partial objective function y_4 is negative.

Table 6

The results of the optimization process of wool sheepskin tanning-greasing

Type of Convolution	Optimum Coordinates			Objective Functions Value				
	x_1	x_2	x_3	y_1	y_2	y_3	y_4	y_5
Additive	1,4	4,8	3,55	81,0	0,159	0,213	14,534	40,442
Maximin	0,6	3	3,25	62,8	0,089	0,065	12,703	30,546
Multiplicative	1,4	4,8	3,25	81,0	0,159	0,213	14,534	40,442
Generalized Desirability Function	1,01	3,91	3,55	75,8	0,064	0,041	14,5	43,0
Ideal Point Method	1,2	4,9	3,55	78,4	0,116	0,204	14,896	44,951

The results of the optimization processes at the stage of semi-finished product treatment are presented in Tables 7, 8.

Table 7

The results of the optimization process of the formation of the lacquer coating

Type of Convolution	Optimum Coordinates			Objective Functions Value				Value of Generalized Objective Function
	x_1	x_2	x_3	y_1	y_2	y_3	y_4	
Additive	189	9,4	52,5	256,02	54,10	342,2	9,82	2,5507
Maximin	246	8,4	26	290,00	66,09	263,1	10,98	0,7698
Multiplicative	193	10	50	256,02	54,10	342,2	9,82	2,2875
Generalized Desirability Function	197	11,25	55	347,7	74,8	320,1	7,5	0,74821
Ideal Point Method	197,25	11,55	51,4	368,12	73,34	331,1	10,11	-

Table 8

The results of the optimization of the formation of the emulsion coating process

Type of Convolution	Optimum Coordinates			Objective Functions Value			Value of Generalized Objective Function
	x ₁	x ₂	x ₃	y ₁	y ₂	y ₃	
Additive	28,6	10	1,85	226,23	272,78	25,90	3,1749
Maximin	26,6	6,2	1,5	240,24	336,59	22,96	1,2883
Multiplicative	28,8	9,6	1,85	226,23	272,78	25,90	3,1420
Generalized Desirability Function	29	9,25	1,75	248,5	342,7	22,9	0,74322
Ideal Point Method	28,1	9,05	1,86	248,89	343,01	22,77	-

Compromise area for a mathematical model of the process 3. 2 is shown in Fig. 3. Response surface of partial objective functions are constructed when changing the coded values in the range of factors [-1; 1] in increments of Δx_i=0,01.

The conducted studies and experimental verification of the obtained results showed that using the method of ideal point the worst vectors of the optimal values for processes 1. 2, 2. 1 and 3. 1, τ were found, as in this case some values of the partial objective functions are out of bounds of the admissible area. At the same time, the method of ideal point is the most time-consuming. The coordinates of the optimum, closest to the experimental, in most cases, are obtained using the maximin convolution. Note also that the use of maximin convolution allows you to find all efficient solutions.

6. Discussion of the results of studies on the effectiveness of different types of convolutions

The comparative analysis of the above mentioned computational schemes for a compromise solution in multiobjective constrained optimization tasks showed that the least convenient for practical use is the method of the ideal point.

Listed examples in p. 4 and 5 showed that the method of the ideal point is quite cumbersome, especially when p is enlarged. For example, for the process 1. 2 with p=2 the distance between the points \bar{b} and $f(\bar{x})$ makes

$$s = \sqrt{(b_1 - f_1(\bar{x}))^2 + (b_2 - f_2(\bar{x}))^2},$$

and the system (14) takes the form:

$$\begin{cases} ((b_1 - f_1(\bar{x}))^2 + (b_2 - f_2(\bar{x}))^2) \left(\frac{\partial f_1(\bar{x})}{\partial x_1} + \frac{\partial f_2(\bar{x})}{\partial x_1} \right) = 0, \\ ((b_1 - f_1(\bar{x}))^2 + (b_2 - f_2(\bar{x}))^2) \left(\frac{\partial f_1(\bar{x})}{\partial x_2} + \frac{\partial f_2(\bar{x})}{\partial x_2} \right) = 0, \\ ((b_1 - f_1(\bar{x}))^2 + (b_2 - f_2(\bar{x}))^2) \left(\frac{\partial f_1(\bar{x})}{\partial x_3} + \frac{\partial f_2(\bar{x})}{\partial x_3} \right) = 0, \\ ((b_1 - f_1(\bar{x}))^2 + (b_2 - f_2(\bar{x}))^2) \left(\frac{\partial f_1(\bar{x})}{\partial x_4} + \frac{\partial f_2(\bar{x})}{\partial x_4} \right) = 0. \end{cases} \quad (17)$$

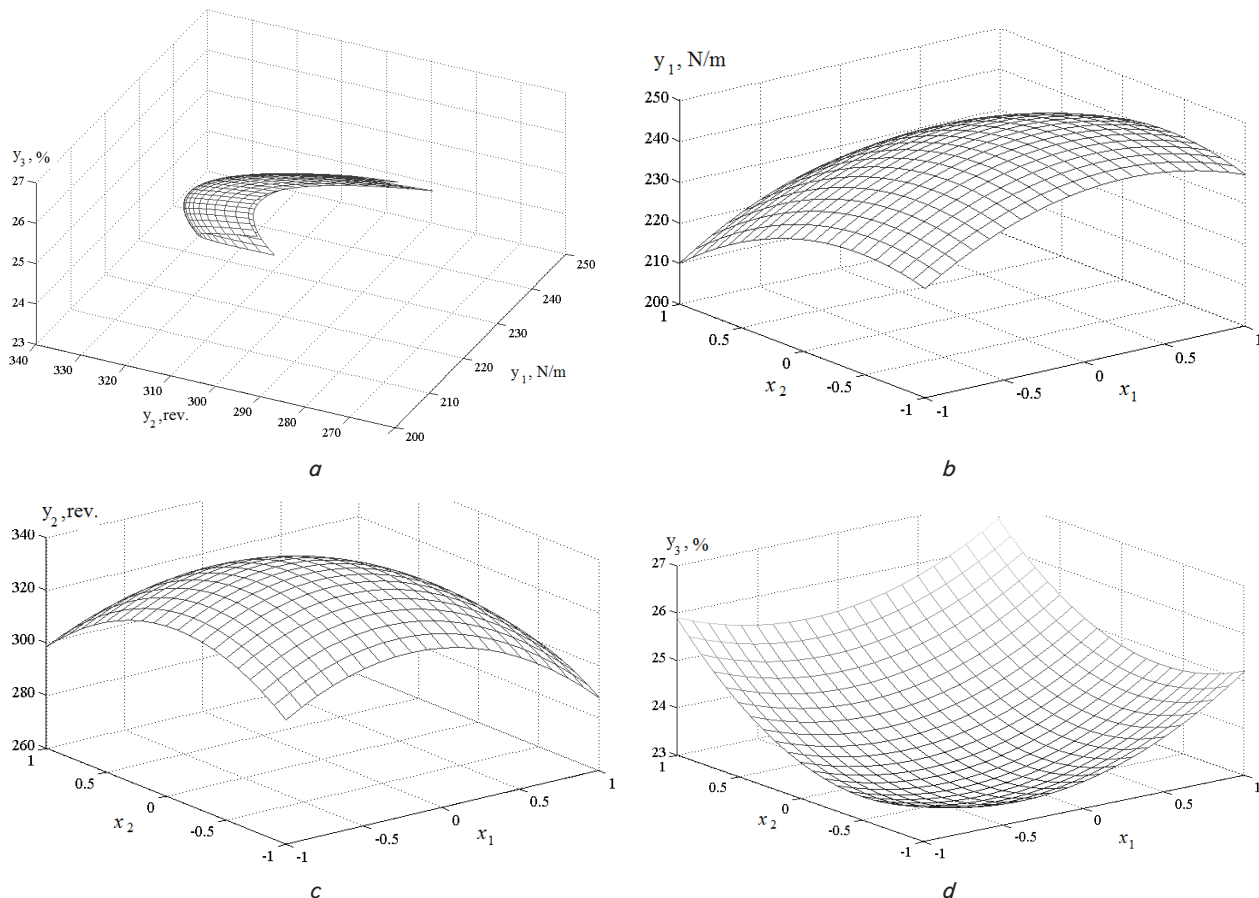


Fig. 3. Compromise area for a mathematical model of the process 3. 2: a – compromise area in the space of criteria: adhesion to wet skin y₁, N/m – resistance to wet friction of rotation y₂, revolutions - relative vapor permeability y₃, %; b–d– response surfaces of partial objective functions y₁=f(x₁,x₂), y₂=f(x₁,x₂), and y₃=f(x₁,x₂) accordingly

With the increase of the number of variables and partial objective functions the difficulty of obtaining a particular type of system (17) increases. The calculations showed that the method of an ideal point is the most time-consuming and cumbersome, as it provides for the determination of the values of the partial derivatives that are a part of the system (14) or (17). Calculation of derivatives manually requires appropriate mathematical skills of the researcher, that is not always possible in the real production, and the use of numerical methods leads to more errors. Furthermore, the system of the type (17) may either not have real solutions, or have their infinite number. It should also be noted that the method uses only the necessary conditions for an extremum, which explains the result of values of the partial objective functions beyond the feasible region in case of 1.2, 2.1 and 3.1 processes.

The rest of the considered computational schemes for compromise solutions are quite simple and easy to program. However, when using additive and multiplicative convolutions it should be noted that the set of feasible solutions must be convex. Solving convex programming problems usually complicates the optimization problem for a specialist in leather and fur production area. Note also that the main problems encountered by researchers for additive, multiplicative and maximin methods for generation of generalized objective function – the selection of criteria weight. This issue requires further research and is not considered in this paper.

The method multiplicative convolution is not applicable for processes in which one or more partial criteria take negative values. For example, in the case of the process 2.1 a special criterion $f_4(\bar{x})$ has negative values, and raising its value to 0,25 power is impossible. Also, the calculations showed that in the process of search for the optimum of the 2.3 process, the objective function takes negative values at several points, as the variation range is close to zero. Modification of some methods of search for the optimum, e.g. scanning method, would allow “skipping” negative values, but considerably limit the use of this type of convolution.

The disadvantage of the computational scheme using the Harrington's desirability function is the stiffness of the formula (15), which does not allow to use responses without additional distortion distributed under laws other than normal. Therefore, the use of this method of generation of the generalized function requires testing the hypothesis of normal distribution of experimental data. Great difficulties are caused by those quality indicators, the values of which increase at first, but after a certain value (or a range of values) begin to decrease. In addition, the results of the optimization affects the experience of a researcher, as the experimenter sets the best and worst values of quality randomly. The disadvantage of this

computational scheme should include the fact that all private quality indicators are recognized as equilibrium, although in practical problems this situation is rare.

In case of the maximin convolution the calculation is carried out for the worst case, and the result is influenced only by the partial criterion to which for the particular point $\bar{x}^{(i)}$, ($i=1,2,\dots,m$) the smallest value of the corresponding function $f_i(\bar{x}^{(i)})$ corresponds. This computational scheme is devoid of limitations associated with the convex set of vector assessments, making it convenient for practical use. The results presented in p. 4 and 5 proved that the maximin convolution is considered the most efficient among the convolutions described in this article.

7. Conclusion

Comparative assessment of the efficiency of five computation schemes for obtaining a compromise solution is made for six models describing the basic physical and chemical processes of formation of the biopolymer materials for various purposes. The mathematical description of the processes under study contains the most commonly used types of equations.

The criterion for assessing the various types of convolutions is formulated. It takes into account the possibility of finding all effective solutions, the simplicity of the algorithm, the time it takes a researcher to carry out the calculations, including the time to perform mathematical operations necessary to verify the possibility of using a particular type of a convolution and the formation of generalized objective function, as well as the results of the experimental verification of the obtained solutions.

It is shown that the maximin convolution is the most effective and allows to take into account the technological features of production of the biopolymer materials. The use of the additive and maximin convolutions is restricted by the convex programming problem; the generation of the generalized criterion using the Harrington's desirability function requires testing the hypothesis of normal distribution, and the ideal point method is the least effective and most time-consuming, and poorly programmed.

The software implementation of computational schemes of the additive, multiplicative, maximin convolutions and the convolution using the desirability function and the ideal point method is conducted; the developed software module is included in the application package COTSolution, developed at the Department of Cybernetics of Chemical-Technological Processes of the KPI. The results can be used to improve existing and develop innovative technologies forming biopolymer materials.

References

1. Taha, H. A. Operations Research: An Introduction (8th Edition) [Text] / H. A. Taha. – Upper Saddle River, NJ: Pearson Prentice Hall, 2007. – 819 p.
2. Steuer, R. E. Multiple Criteria Optimization: Theory, Computation and Application [Text] / R. E. Steuer. – New York: John Wiley, 1986. – 546 p.
3. Ostrovskiy, G. M. Optimizatsiya himiko-tehnologicheskikh protsessov. Teoriya i praktika [Text] / G. M. Ostrovskiy, T. A. Berezhinsekiy. – Moscow: Himiya, 1984. – 240 p.

4. Ahnazarova, S. L. Ispolzovanie funktsii zhelatelnosti Harringtona pri reshenii optimizatsionnykh zadach himicheskoy tehnologii [Text] / S. L. Ahnazarova, L. S. Gordeev. – Moscow: izd-vo RHTU, 2003. – 76 p.
5. Bandi, B. Metody optimizatsii. Vvodnyiy kurs [Text] / B. Bandi; perevod s angl. – Moscow: Radio i svyaz, 1988. – 129 p.
6. Sanginova, O. Multi-objective optimization in formation tasks of leather and fur materials [Text] / O. Sanginova, A. Danylkovych, S. Branovitskaja // ScienceRise. – 2014. – Vol. 2, Issue 2. – P. 43–50. doi: 10.15587/2313-8416.2014.27262
7. Danylkovych, A. G. Application of Box method for multi-objective optimization problems [Text] / A. G. Danylkovych, S. V. Branovitskaja, S. G. Bondarenko, O. V. Sanginova // Eastern-European Journal of Enterprise Technologies. – 2013. – Vol. 3, Issue 4 (63). – P. 4–8. – Available at: <http://journals.urau.ua/eejet/article/view/14743/12521>
8. Kats, M. A new method for solving the problems of identification, diagnosis, prognosis and optimization of complex systems [Text] / M. Kats // Eastern-European Journal of Enterprise Technologies. – 2011. – Vol. 3, Issue 12 (51). – P. 17–28. – Available at: <http://journals.urau.ua/eejet/article/view/2467/2268>
9. Zraychenko-Polozentsev, A. V. Evaluation of potential reserves of production for melting synthetic iron [Text] / A. V. Zraychenko-Polozentsev, O. S. Koval, D. A. Demin // Technology audit and production reserves. – 2011. – Vol. 1, Issue 1 (1). – P. 7–15. – Available at: <http://journals.urau.ua/tarp/article/view/4081/3747>
10. Ekologichno orientovani tehnologiyi virobnitstva shkiryanih ta hutrovih materialiv dlya stvorenniya konkurentospromozhnih tovariv. In 2 part.; Part I [Text] : monografiya / A. G. Danilkovich, V. I. Lischuk, V. P. Plavan, E. E. Kasyan, O. G. Zhigotskiy; A. G. Danilkovich (Ed.). – Kiev: FenIks, 2011. – 437 p.
11. Danylkovych, A. G. Innovatsiyni tehnologiyi virobnitstva shkiryanih i hutrovih materialiv ta virobiv [Text]: monografiya / A. G. Danylkovych, I. M. Grischenko, V. I. Lischuk et. al.; A. G. Danylkovych (Ed.). – Kiev: Feniks, 2012. – 344 p.
12. Lischuk, V. I. Viktoristannya bagatokriterialnoyi optimizatsiyi dlya poshuku kompromisnoyi oblasti protsesu zolinnya [Text] / V. I. Lischuk, T. G. Voytsehovska, A. G. Danylkovych // Legka promislolist. – 2007. – Vol. 1. – P. 37–39.
13. Danylkovych, A. G. Pidvischennya yakosti vtornnogo pokryttya shlyahom optimizatsiyi pokryvnoyi kompozitsiyi [Text] / A. G. Danylkovych, A. S. Brayilko, N. V. Omelchenko // Visnik HNU. – 2010. – Vol. 3. – P. 129–134.

Статтю присвячено прикладним аспектам інтервального математичного моделювання оптимізаційних задач розміщення геометричних об'єктів. Будується повний клас реалізацій інтервальної математичної моделі основної інтервальної оптимізаційної задачі розміщення геометричних об'єктів. Пропонуються інтервальні математичні моделі низки оптимізаційних задач розміщення та модифікації методів локальної та глобальної оптимізації для їх реалізації в інтервальних та евклідових просторах

Ключові слова: геометричне проектування, інтервальна геометрія, інтервальна математична модель оптимізаційної задачі розміщення

Статья посвящена прикладным аспектам теории интервального математического моделирования оптимизационных задач размещения геометрических объектов. Строится полный класс реализаций интервальной математической модели основной интервальной оптимизационной задачи размещения геометрических объектов. Предлагаются интервальные математические модели ряда оптимизационных задач размещения и модификации методов локальной и глобальной оптимизации для их реализации в интервальных и евклидовых пространствах

Ключевые слова: геометрическое проектирование, интервальная геометрия, интервальная математическая модель оптимизационной задачи размещения

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ЗАСТОСУВАННЯ ІНТЕРВАЛЬНИХ МАТЕМАТИЧНИХ МОДЕЛЕЙ ЗАДАЧ РОЗМІЩЕННЯ ГЕОМЕТРИЧНИХ ОБ'ЄКТІВ

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1. Вступ

На сучасному етапі стрімко зростає інтерес до ефективного розв'язання оптимізаційних задач геоме-

тричного проектування, зокрема, задач розміщення, що пояснюється розмаїттям практичних застосувань і надзвичайною складністю математичних моделей та методів їхнього розв'язання.