Досліджено вплив зміни параметрів п'єзокерамічного елемента на процес формування ультразвукового імпульсу. Установлено, що для створення імпульсу напруженість електричного поля перетворюється в електродинамічне зусилля, котре викликає пружну деформацію мембранного блоку. Визначено вплив деформації на форму ультразвукового імпульсу. Отримані аналітичні моделі на основі теорії незворотних реологічних перетворень

Ключові слова: п'єзокераміка, ультразвук, імпульс, реологія, газ, рідина, мембрана, імпульс, інтенсивність

Исследовано влияние изменения параметров пьезокерамического элемента на процесс формирования ультразвукового импульса. Установлено, что для создания импульса напряженность электрического поля преобразуется в электродинамическое усилие, которое вызывает упругую деформацию мембранного блока. Определено влияние деформации на форму ультразвукового импульса. Получены аналитические модели на основе теории необратимых реологических преобразований

Ключевые слова: пьезокерамика, ультразвук, импульс, реологія, газ, жидкость, мембрана, импульс, интенсивность

UDC 681.586.773
DOI: 10.15587/1729-4061.2016.74844

RESEARCH INTO RHEOLOGICAL TRANSFORMATIONS IN A PIEZOCERAMIC ULTRASONIC SENSOR OF FLUID LEVEL CONTROL

I. Stencel
Doctor of Technical Sciences,
Professor, Head of Department*
E-mail: kafedraKISU@gmail.com
K. Litvinov
Postgraduate student*
E-mail: LitvinovK@yandex.ru
*Department of Computer Integrated Management Systems
Eastern-Ukrainian National University Volodymyr Dahl
Tsentral'nyi ave., 59-a,
Severodonetsk, Ukraine, 93400

1. Introduction

The ultrasonic method of measuring control (USMC) is widely used in various sectors of the national economy. It is most widely used to control the level, volume and mass of fluids in containers [1, 2], material balance of fluids in technological devices (evaporators, adsorption and distillation columns, liquid reactors, steam boilers, etc.), volume rate of gas flows [3] in non-destructive testing of materials [4], etc. The essence of USMC is that the pulse of ultrasonic energy is transferred to gas medium, in which its physical parameters change: speed, amplitude and phase [5]. In most cases USMC is based on ultrasonic impulse formation (ultrasound) by piezoelectric generator [6]. In many cases, to create ultrasound, the piezoceramic elements are used (PCE) [7]. These ultrasonic transducers (UST) are mounted in the wall of the container (e.g., on its lid) or the pipeline. The principle of work of ultrasonic sensors of measuring control (USML) of fluid level or the volume rate of gas in pipelines is based on measuring the ultrasound transit time through gas medium (GM). In many cases, these tools are used for commercial metering (custody-transfer) not only of the volume of liquid, but the gas volume rate, too [8]. The level refers to some of the key parameters, it is subject to measurement control in technological processes of chemical, oil, food and other industries, as well as storage tanks. Modern USML are required to meet high requirements regarding their precision, measurement range control (MRC), accuracy, linearity of static characteristics. Modern USML are equipped with measurement data processing units (MDPU) and control units (CU). They are intended for introduction of amendments to the measurement result and displaying information on the screen in real time. These units contain controllers, which have recorded programs for converting and processing of measurement data. The main is the program of normalized static characteristics, by which a comparison of measurement data and calculation of deviation from the normalized value is performed. In all cases, the normalized static characteristic is taken as linear and calculated by the formula, in accordance with which the time of passage of the thickness of the gas is proportional to the velocity of ultrasound [9]. As the experimental studies show, MRC USML are as a rule limited by a maximum (zone of "insensitivity") and a minimum level of fluid, which is selected by linearity of gas ultrasonic energy absorption. Nonlinearity error of real static characteristics is taken into account, which reduces the accuracy of measurement control. In addition, non-linear are the changes of physical parameters of PCE of membrane unit (MU) at the temperature deviation from its normalized value and rejecting the gas when deviation from its temperature, pressure and composition. These factors cause additional errors of measuring control (EMC), which cannot be significantly reduced by compensating devices (thermometers, pressure gauges, etc.). Therefore, the actual problem is the development of mathematical models of ultrasound emitter, analysis of parameters, affecting its performance and optimization of static characteristic.

2. Analysis of scientific literature and the problem statement

In all cases where the fluid level measurement or the volume rate of GM is associated with ultrasonic radiation, a property either of change in the velocity of ultrasonic vibrations is used (USV) at a constant length of its movement (for example, flow volume of gas rate) [10], or the time at which the ultrasonic pulse undergoes a certain distance in GM (eg, level gauges) [11]. For a mathematical description of the motion of ultrasound in the GM they usually use the main law of light absorption of Lambert-Bouguer-Beer, whereby the current intensity I of ultrasonic radiation is proportional to the initial intensity I_0 and exponential decrease depending on the molecular composition (concentration of Q) of gas, the absorption coefficient and thickness $\mathfrak{R}$ of GM layer. As the main parameter of such measurements is the thickness of the GM layer, then it is determined by the ratio of the current and the initial intensity. As the thickness of the GM is the product of the velocity of ultrasound at the time of its motion, we have:

$$t=(1/d\varepsilon Q)\ln\left[I(\mathfrak{R})/I_0\right],$$

where $I(\mathfrak{R})$ is the current radiation intensity which is a function of the thickness $\mathfrak{R}=\text{var}$ GM. This principle is widely used for building ultrasonic USML, which measure the time from the moment of the original ultrasound I_0 mprior to its aceptance with the ultimate intensity $I(\mathfrak{R})$. Volumetric rate of gas flow in the pipeline is defined by the formula [12]:

$$F_O=V/t=Sd/t=(S/t)(1/\varepsilon Q)\ln(I/I_0)$$

or

$$t=(S/F_O\varepsilon Q)\ln\left[I(\mathfrak{R}_0)/I_0\right],$$

where V is the volume of a given length of pipeline; S is the cross-sectional area of the gas flow; $\mathfrak{R}_0=\text{const}$. The last equation shows that the flow rate of gas is inversely proportional to the measured time.

Thus, based on the above-mentioned, for USML, based on measuring the transit time of ultrasound, the distance $\mathfrak{R}$ is determined by the formula:

$$\ln I(\mathfrak{R})=k_P t_P+\ln I_0,$$

where $k_P=(S/Q\varepsilon)\ln\left[I(\mathfrak{R})/I_0\right]$ is the conversion coefficient; t_P is the measured time. Analysis of the literature cited below shows that the study of USML, built on the PCE, focuses on the behavior of ultrasound in GM and it is accepted that the main source of EMC is only this medium. There were no studies found of the piezoceramic probes that are used to build gauges and flow meters. The source of errors is not only the gas medium, but the own probes as well, which include the PCE, a membrane unit and the supply system of electric excitation pulse (EEP) and output of the electromotive force (EMF) from PCE. In the cited scientific literature it is assumed that the errors caused by PCE are minor and can be ignored. But the lack of knowledge of electrodynamic processes that take place in PCE, their role in the formation of ultrasound under real conditions of USML determine the need for further research into this area.

3. The purpose and objectives of the study

The aim of the work is to develop mathematical models of piezoceramic ultrasonic transducer element for fluid level gauges with a circular motion of the ultrasonic pulse [13] and optimization of its static characteristic.

To achieve the set goal, it is necessary to solve the following problems:

– to develop physical models of irreversible rheological transitions (IRT) for a measuring channel of a level gauge;

– to describe IRT by nonlinear differential equations of the transfer of energy, mass and momentum with the dissipative function of flow rate;

– to find on the basis of zero gradient method an analytic solution of nonlinear differential equations of the speed transfer of energy, mass and momentum in the ultrasonic emitter;

– to optimize the transient response of the ultrasonic emitter.

4. Research results of piezoceramic USE

USML of fluids with a circular motion of ultrasound has one piezoceramic ultrasonic emitter (USE) 5 (Fig. 1), which is designed to excite ultrasound by electric exciting pulse (EEP).

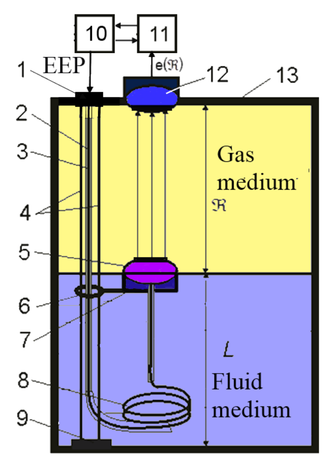

Fig. 1. Scheme of USL with a circular motion of ultrasound: 1 is the adapter; 2 is the cable; 3 is the polymer coating; 4 are the guiding rods; 5 is the ultrasonic emitter; 6 is the collar; 7 is the float; 8 is the spring twist; 9 is the sinker; 10 is the control unit; 11 is the measurement data processing unit; 12 is the ultrasonic receiver; 13 is the lid

Typically, the ultrasound emitter (USE) 5 is located above the float 7, which floats on the fluid surface. Ultrasound pulse goes through the layer of gas with the thickness $\mathfrak{R}$ and is perceived by ultrasonic receiver (USR) 12, which converts the pulse to the electromotive force (EMF) $e(\mathfrak{R})$. The electrical signal $e(\mathfrak{R})$ arrives to the measurement data processing unit (MDPU) 11. Proccessed by the corresponding algorithm, the measuring information is sent to the control unit (CU). By separating the perceiving and emitting channels, a conventional circular form is created that allows us to receive the following results:

– to reduce the zone of insensitivity of USML almost by 5 times due to the lack of a reference device;

– the MRC of fluid level increases by 2 times by a single passage of ultrasound through a GE layer;

– the USML accuracy increases almost by two times due to: a decrease in the distance of ultrasound passage in GM up to 2 times, decrease of the secondary ultrasound effects inside the vessel due to the lack of a reference device.

USE and UST have the same type of performance design, so electrodynamic processes are considered identical. Under these assumptions, the major transformational nodes are membrane units of USE, UST and GM. Let us consider the process of fluid level measurement with one USE, placed on the surface of floating float. This process is accompanied by rheological transformations of electrical and mechanical energy. For the proposed USML, the following rheological transformations occur:

– the EEP electric energy to mechanical movement PCE (in the form of a circular thin plate), which is rigidly fixed to a metal membrane;

– mechanical energy of membrane unit (MU) to USV, which are distributed in the GM;

– USV energies that passed GM, to the mechanical energy of MU UST;

– mechanical energy of MU UST to electrical energy of PCE and creation of EMF $e(\Re)$.

During the activation of PCE, energy of EEP reduces to zero while the energy accumulated by this element increases from zero to some maximum value. The interaction of two energies under the law of Ampere creates electrodynamic force (EDF), which leads to the displacement of the center of the membrane unit USE to the distance x_0. Physical model and graphics of IRT are shown in Fig. 2. Fig. 2, *a* shows the physical model of PCE, which consists of two IRTs. At the first transition, the electrical energy of EEP is converted to electric field intensity (EFI), as the PCE is conductive and is characterized by active resistance R, electric capacity C and low inductance L. At the second transition, EFI is converted to electrodynamic force (EDF), which leads to displacement of PCE to the distance x_0. On the first IRT the EEP energy reduces owing to its losses for heating and creating of EDF. At the second IRT the EDF is spent on mechanical deformation of PCE. Thus, the dissipative losses occur on IRT, which we will call further as the functions of flow rate of energy transfer, the mass or momentum. Structural and logical model (Fig. 2, *b*) of EEP electric power change is a step function that can be described by the equation:

$$E_I(t)=\begin{cases}1, & t\le t_{11}; \quad t_{11}<t_{12};\\ 0, & t\ge t_{12}; \quad t_{11}<t_{10}<t_{12}.\end{cases}$$

Structural-logical model (Fig. 2, *c*) of the growth of EFI also represents a step function, which can be described by the following equation [14]

$$H_\Pi(t)=\begin{cases}1, & t\ge t_{12},\\ 0, & t<t_{11}.\end{cases}$$

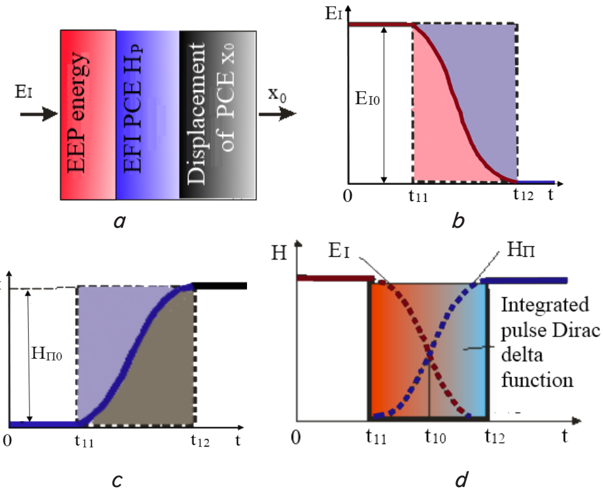

Fig. 2. Structural-logical schemes of IRT of EEP energy in the intensity of electric field: a is the physical model of IRT of EEP energy to the MU displacement; b is the schedule of the conversion process of electrical energy; c is the schedule of the conversion process of the intensity of electric field; d is the integrated pulse Dirac delta IRT function

The duration of IRT phase will depend on the electrical parameters of the PCE. According to the theory of generalized functions, the derivative of a step function results in symbolic equality:

$$\delta_1(t)=\frac{d}{dt}f_1(t),$$

where $\delta_1(t)$ is the Dirac delta function; $f_1(t)$ is the certain arbitrary function. If the function $f_1(t)$ describes the process of EFI transfer with heredity, then the Dirac function is asymmetrical and is described by the following equation (Fig. 2, *d*):

$$\int_{t_{11+0}}^{t_{12}} f_1(\xi)\delta(\xi-t_{10})d\xi=\begin{cases}0, & t_{10}<t_{11}, \quad t_{10}\ge t_{12},\\ f_1(t_{10}+0), & t_{11}<t_{10}<t_{12},\end{cases}$$

where ξ is the certain variable; $\delta(\xi-t_{10})$ is the core of linear integral transformation; t_{10} is the time of phase IRT.

Under the action of the electric field EEP, current occurs in PCE, which causes a decrease of the electrical energy E_I.

The rate of EEP energy transfer is described by the following differential equation:

$$\frac{\partial E_I}{\partial\theta}=-\operatorname{div}(D_E\nabla E_I)+\gamma_E(t), \tag{1}$$

where D_E is the coefficient of EEP energy transfer towards the $\overline{r}_n$, m^2/s; ∇E_I is the gradient of EFI change by the x, y, z coordinates; θ is the time of EEP energy transfer; $\gamma_E(t)$ is the dissipative function of change of electrical energy; t is the flow time.

The flow rate of electricity, which is spent on creating the EDF $F_B(\overline{r}_n,t)$, is described by the following equation:

$$\gamma_E(t)=\tau_E\frac{d^2F_B(t)}{dt^2}+\frac{dF_B(t)}{dt}, \tag{2}$$

where $\tau_E=RC$ is the PCE time constant.

If the transfer of EDF is performed only at the time t, then substituting (2) to (1), we obtain the following nonlinear differential equation:

$$\frac{\partial E_I(\theta,z)}{\partial\theta}+D_E\frac{\partial^2 E_I(\theta,z)}{\partial z^2}= \\ =k_I\left[\tau_E\frac{d^2F_B(t)}{dt^2}+\frac{dF_B(t)}{dt}\right]. \quad (3)$$

In the equation (3) k_I is the coefficient of conversion of electrical energy of EEP to EDF, and z is the linear dimension of PCE, by which the energy of electric field is distributed. According to a zero gradient method [16], the equation (3) is divided into the following system:

$$\frac{\partial E_I(\theta,z)}{\partial\theta}+D_E\frac{\partial^2 E_I(\theta,z)}{\partial z^2}=0; \quad (4)$$

$$\tau_E\frac{dF_B(t)}{dt^2}+F_B(t)=k_E E_I(\theta,x). \quad (5)$$

Under zero initial conditions, solution of the equation (4) will take the form:

$$E_I(\theta,z)=E_{I0}\operatorname{erf}\left(z/2\sqrt{D_E\theta}\right), \quad (6)$$

where E_{I0} is the maximum value of EEP energy.

Substituting (6) to the equation (5), we get:

$$\tau_E\frac{dF_B(t)}{dt}+F_B(t)=k_E E_{I0}\operatorname{erf}\left(z/2\sqrt{D_E\theta}\right). \quad (7)$$

Solution of the equation (7) under initial conditions will take the form:

$$F_B(t)=k_E E_{I0}\operatorname{erf}\left(z/2\sqrt{D_E\theta}\right)\left[1-\exp(-t/\tau_E)\right]. \quad (8)$$

As the function $\operatorname{erf}\left(z/2\sqrt{D_E\theta}\right)$ for the linear part of characteristic is insignificant, so, limited by its linear part, and assumed that z=z0, where z_0 is the PCE thickness, the equation (8) is simplified to the following:

$$F_B(t)=k_{E0}E_{I0}x_0\left[1-\exp(-t/\tau_E)\right]. \quad (9)$$

A multiplier in the equation (9) $k_{E0}=k_E z_0/2\sqrt{D_E\theta}$ is the coefficient of the first IRT transfer. Electrodynamic force $F_B(t)$ leads to mechanical deflection of MU from the initial state at the distance r. The process of transfer of the effort $F_B(t)$ can be described by the following differential equation:

$$\frac{\partial F_B(\bar r,\xi)}{\partial\xi}+\operatorname{div}\left(F_B(\bar r,\xi),\bar v\right)=\operatorname{div}\left(k^2\nabla F_B(\bar r,\xi)\right)+\gamma_x(t), \quad (10)$$

where $F\ (\bar r,\ )$ is the distribution of EDF by the linear vector $\bar r$ and its action time ξ; $\bar v$ is the vector of rate change of EDF; $k^2=j\omega_0\mu_a\sigma$ is the complex parameter that characterizes electrodynamic properties of PCE; ω_0 is the angular frequency of oscillations of MU; μ_a is the absolute magnetic susceptibility; σ is the specific electrical conductivity, $\gamma_x(t)$ is the flow rate of EDF per time unit t.

Let the speed of the MU motion is negligible compared to the rate of change in EDF. Then the equation (10) takes the following form:

$$\frac{\partial F_B(\bar r,\xi)}{\partial t}=\operatorname{div}\left(k^2\nabla^2 F_B(\bar r,\xi)\right)+\gamma_x(t). \quad (11)$$

If the parameter k^2 is insignificantly dependent on the transfer process and $\bar r\to x,$ then (11) is simplified and takes the form:

$$\frac{\partial F_B(x,\xi)}{\partial\xi}=k^2\frac{\partial^2 F_B(x,\xi)}{\partial x^2}+\gamma_x(t). \quad (12)$$

The flow of this IRT is the velocity of the MU moving under the influence of EDF. Then for the free component of this movement we have:

$$\gamma_x(t)=\tau_M\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}, \quad (13)$$

where $\tau_M=\sqrt{mk_1/D_M}$ is the constant of time of displacement of MU; m is its mass, k_1 is the damping factor; D_M is the stiffness of the membrane.

Substituting (13) to the equation (12), we obtain the following nonlinear differential equation:

$$\frac{dF_B(x,\xi)}{d\xi}+k^2\frac{d^2F_B(x,\xi)}{dx^2}=k_M\left(\tau_M\frac{d^2x(t)}{dt^2}+\frac{dx(t)}{dt}\right). \quad (14)$$

The factor k_M is the conversion factor of IRT. According to the method of zero gradient, the nonlinear differential equation (14) is divided into the following system of equations:

$$\frac{\partial F_B(x,\xi)}{\partial\xi}+k^2\frac{\partial^2 F_B(x,\xi)}{\partial x^2}=0, \quad (15)$$

$$\tau_M\frac{dx(t)}{dt}+x(t)=k_x F_B(x,\xi), \quad (16)$$

where k_x is the flow rate of EDF.

Solution of the equation (15) under initial conditions takes the form:

$$F_B(x,\xi)=F_{B0}\operatorname{erf}\left(x/2\sqrt{k^2\xi}\right), \quad (17)$$

where F_{B0} is the maximum EDF.

Solution of the equation (16) under initial conditions will take the form:

$$x(t)=k_{H0}F_B(x,\xi)\exp(-t/\tau_M), \quad (18)$$

where k_{H0} is the coversion factor of EDF to the displacement of PCE.

Substituting the equation (17) to (18), limited by the linear part of the function

$$\operatorname{erf}\left(x/2\sqrt{k^2\xi}\right)$$

and assuming that $x=x_0$, we get:

$$x(t)=k_{H0}\frac{x_0}{2\sqrt{k^2\xi}}F_{B0}\exp(-t/\tau_B)=k_F x_0\exp(-t/\tau_B), \quad (19)$$

where $k_F=k_{H0}F_{B0}/2\sqrt{k^2\xi}.$

Upon termination of EEP action, and, accordingly, EDF, free fall of MU in GM will lead to its oscillatory process. The process is damped with ultrasonic frequency. GM produces appropriate counter-action to the MU motion. Rheological conversion of the MU displacement to mechanical USV can be described by the following equation:

$$\frac{\partial x\left(y_B, \varsigma\right)}{\partial \varsigma}=\operatorname{div}\left(D_{y_B} \nabla^2 x\left(y_B, \varsigma\right)\right)+\gamma_x\left(t_k\right), \tag{20}$$

where y_B is the direction of oscillation motion of MU; D_{y_B} is the effective coefficient of transfer rate of mechanical energy; ς is the transfer time of mechanical energy; $\gamma_x\left(t_k\right)$ is the rate of USV energy flow during the oscillation time t_k.

If the coefficient D_{y_B} is insignificantly dependent on the direction of the transfer and the MU dislacement occurs in one direction, then the equation (20) is simplified to the following:

$$\frac{\partial x\left(y_B, \varsigma\right)}{\partial \varsigma}=-D_{y_B} \frac{\partial^2 x\left(y_B, \varsigma\right)}{\partial y_B^2}+\gamma_x\left(t_k\right). \tag{21}$$

The flow of rheological transformation is free damped mechanical oscillations of MU, which are described by the following equation:

$$\frac{d^2 y}{dt_k^2}+2\delta_\beta \frac{dy}{dt_k}+\omega_{0M}^2 y=0, \tag{22}$$

where y is the current deviation of MU; $\delta_\beta=\beta / 2m$ is the damping coefficient of mechanical oscillations; m is the mass of MU; β is the coefficient of friction of MU in GM; $\omega_{B0}=\sqrt{D_M / m}=2\pi f_0$ is the angular frequency of natural oscillations; D_M is the MU rigidity; m is the mass of MU; f_0 is the frequency, Hz.

As the process is oscillating, so the rate of flow of mechanical motion of MU is described by the following equation:

$$\gamma_x\left(t_k\right)=\frac{d^3 y}{dt_k^3}+2\delta_\beta \frac{d^2 y}{dt_k^2}+\omega_{B0}^2 \frac{dy}{dt_k}. \tag{23}$$

Substituting (23) to (21), we obtain the following nonlinear differential equation of the MU motion:

$$\begin{aligned} & \frac{\partial x\left(y_B, \varsigma\right)}{\partial \varsigma}+D_{y_M} \frac{\partial^2 x\left(y_B, \varsigma\right)}{\partial x^2}= \\ & =k_{y_B}\left[\frac{d^3 y}{dt_k^3}+2\delta_\beta \frac{d^2 y}{dt_k^2}+\omega_{B0}^2 \frac{dy}{dt_k}\right]. \end{aligned} \tag{24}$$

The factor k_{y_B} is the IRT conversion factor. According to the method of zero gradient, the equation (24) is divided into the following system:

$$\frac{\partial x\left(y_B, \varsigma\right)}{\partial \varsigma}+D_{y_B} \frac{\partial^2 x\left(y_B, \varsigma\right)}{\partial x^2}=0; \tag{25}$$

$$\frac{d^2 y}{dt_k^2}+2\delta_\beta \frac{dy}{dt_k}+\omega_{B0}^2 y=k_{y_B} x\left(y_B, \varsigma\right). \tag{26}$$

Under boundary conditions, the solution to the equation (25) will take the form:

$$x(\varsigma)=k_{yB} x_0 \operatorname{erf}\left(y_B / 2\sqrt{D_{y_B} \varsigma}\right). \tag{27}$$

In a practical choice of PCE, it is assumed that the largest deviation of MU will occur when it equals its thickness z_0. Taking into account that the maximum initial deflection of MU is $y_B=z_0$, then under initial conditions the solution to the equation (26) will be a follows:

$$\begin{aligned} & y(t)=k_{x0} x_0 \operatorname{erf}\left(z_0 / 2\sqrt{D_{y_B} \varsigma}\right) \times \\ & \times\left\{1-\exp\left(-\delta_\beta t\right)\left[\cos\left(\omega_{B0} t_k\right)\right]\right\}. \end{aligned} \tag{28}$$

The function $\operatorname{erf}\left(z_0 / 2\sqrt{D_{y_B} \varsigma}\right)$ in (28) we shall approximate by exponential. Then

$$\operatorname{erf}\left(z_0 / 2\sqrt{D_{y_B} \varsigma}\right) \approx \exp\left(-k_z z_0 / 2\sqrt{D_{y_B} \varsigma}\right),$$

where is the scale factor. Taking this into account, a mathematical model of IRT motion of the MU motion takes the following form:

$$\begin{aligned} & y_B(t)=k_{x0} x_0\left\{1-\exp\left(-\delta_\beta t\right)\left[\cos\left(\omega_{B0} t_k\right)\right]\right\} \times \\ & \times \exp\left(-k_z z_0 / 2\sqrt{D_{y_B} \varsigma}\right). \end{aligned} \tag{29}$$

The form of mechanical USV of MU is shown in Fig. 3.

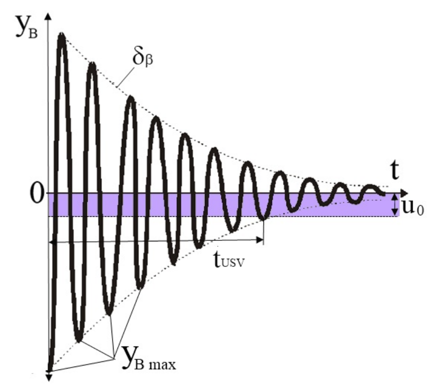

Fig. 3. Free USV of membrane unit

As stated above, PCE refers to electric conductive element and is characterized by active resistance R, capacity C and inductance L. During free motion of MU in the PCE plate an EMF $e_B(t)$. occurs. In this case, electromagnetic field with ultrasonic frequency is created in PCE. This field is insignificant but it impacts the ultrasound parameters. First, it leads to a decrease in the amplitude bypass of ultrasound, and second, to the offset of the initial fluctuations phase $\phi_0=\omega_0 t$. If the timing of passing of ultrasound through GM starts from the moment of EEP supply, then the ultrasound phase shift will lead to occurrence of an error of measuring control (EMC). Observing the work of ultrasonic level gauge MTM-900 [17] showed that with an increase in their operation period, measuring control time for the same level of fluid increases, and it results in the occurrence of EMC. We can assume that this could be due to electrical and mechanical aging of PCE material. Extending the distribution time of ultrasound in measuring the same level of fluids under normal operating conditions may indicate an increase in the time constant of the transition process, and therefore, such electrical parameters of PCE as active resistance R and capacity C. Let us consider the impact of electromechanical parameters of PCE on the formation process of USV. At the set design parameters of PCE, its active resistance is:

$$R=\rho_R\left(z_0/S\right), \quad (30)$$

where ρ_R is the specific resistannce of PCE material; z_0, S are the length and cross sectional area of the element, respectively.

As the PCE material is characterized by the G module of displacement and by the density ρ_M, the time constant of distribution of free USV equals:

$$\tau_p=S\sqrt{z_0\rho_M/G}. \quad (31)$$

After determining the thickness of the PCE plate from the equation (31) and substituting to (30), we get:

$$R=\tau_p^2\rho_R G/S^3\rho_M. \quad (32)$$

PCE is a real capacitor, the equivalent circuit of which can be represented by parallel connection of active electrical $G_E=1/R$ and reactive electrical conductivity $B_C=\omega C$, where ω is the ultrasound frequency. Then for active conductance we have:

$$G_E=S/\rho_R z_0. \quad (33)$$

Taking into account that the capacity of PCE plate can be determined by the approximate formula $C=\varepsilon_a S/z_0$, where ε_a is the absolute dielectric penetration, then the expression for the reactive conductivity takes the form:

$$B_C=\omega_{B0}\varepsilon_a S/z_0. \quad (34)$$

Then the shift angle of instantaneous value of current in PCE equals:

$$\varphi_i=\operatorname{arctg}\left(B_C/G_E\right)=\operatorname{arctg}\left(\omega_{B0}\varepsilon_a\rho_R\right). \quad (35)$$

The equation (35) shows that the secondary electromagnetic actions arising in PCE plate by free mechanical oscillations of MU cause braking effect on its mechanical displacement. Taking into account this action of displacement, the MU will be described by the following equation:

$$y_B(t)=k_{к0}x_0\left\{1-\exp\left(-\delta_\beta t\right)\left[\cos\left(\omega_{B0}t_k+\varphi\right)\right]\right\}\times$$
$$\times\exp\left(-k_z z_0/2\sqrt{D_{y_B}\varsigma}\right). \quad (36)$$

If we assume that the change in frequency of mechanical oscillations ω_{B0} and of absolute dielectric constant ε_a are insignificant, then the main influential factor will be the change in the PCE specific resistance.

5. Discussion of results of the study of a level gauge with circular motion of ultrasonic pulse

Solids, which include the PCE, expand, when heated, in all directions. In addition, the temperature deviation from normal will lead to changes in specific resistance of PCE. The thickness of the PCE plate at the temperature deviation changes according to the formula:

$$z(T)=z_0\left[1+\alpha_z\left(T-T_H\right)\right], \quad (37)$$

where z_0 is the thickness of PCE plate at normal temperature T_H (normal is the temperature, at which calibration of USML is performed); α_z is the linear thermal expansion coefficient of PCE material; T is the current temperature.

The density ρ_M of solids at the deviation of temperature from the nominal one changes by the following formula:

$$\rho_M=\rho_{MH}/\left[1+3\alpha_\rho\left(T-T_H\right)\right], \quad (38)$$

where ρ_{MH} is the density of material at a normal temperature; α_ρ is the volumetric coefficient of thermal expansion.

The equation (35) with regard to (30) takes the following form:

$$\varphi_i=\operatorname{arctg}\left(\omega_{B0}\varepsilon_a R z_0/S\right). \quad (39)$$

Let the degree of damping of mechanical oscillations is $\delta_\beta=1/\tau_p$, where τ_p is the constant of time of distribution of free mechanical USV in GM. Then, with regard to (31), we obtain:

$$\delta_\beta=\left(1/S\right)\sqrt{G/z_0\rho_M}. \quad (40)$$

Substituting equation (38) to (40), the equation (36) takes the following form:

$$y_B(t)=k_{x0}x_0\left\{1-\exp\left(-t\left(z_0/S\right)\sqrt{G/z_0\rho_M}\right)\times\right.$$
$$\times\cos\left[\omega_{B0}t_k+\operatorname{arctg}\left(\omega_{B0}\varepsilon_a R_E z_0/S\right)\right]\Big\}\times$$
$$\times\exp\left(-k_z z_0/2\sqrt{D_{y_B}\varsigma}\right). \quad (41)$$

The analysis of the equation (41) shows that the main influential USE factors include PCE plate thickness, its area, as well as the shift module and active resistance of the element. Increasing the thickness of the plate PCE leads to deterioration of the degree of attenuation, increases the shift of USF by phase, and shifts the maximum of bypassing ultrasound. The shift module G depends on the elasticity modulus E, which, in turn, depends on EDF F_B and the area oS. The shift module can be determined by the formula:

$$G=\frac{1}{2(1+\mu)}\cdot\frac{F_B l_H}{S\left(1-l_H\right)}, \quad (42)$$

where μ is the Poisson coefficient.

Considering the PCE as a flat capacitor, the distance between the plates of which equals z_0, electrodynamic force F_B can be defined by the formula:

$$F_B=\varepsilon_a SU^2/2z_0{}^2, \quad (43)$$

where U is the voltage applied to a PCE plate; S is the PCE plate area; ε_a is the absolute dielectric penetration.

As the main influential factor of USE is the temperature, then with regard to the equations (37), (38), (42) and (43), and ignoring the changes in temperature of higher order, a mathematical model of USE takes the following form:

$$y_B(t)=k_{x0}x_0\left\{\begin{array}{l}1-\exp\left(-t\dfrac{U}{z_0(1+\alpha_z\Delta T)}\sqrt{\dfrac{\varepsilon_a}{4(1+\mu)z_0\rho_{M0}S^2}\left(\dfrac{1+3\alpha_\rho\Delta T}{(1+\alpha_z\Delta T)}\right)}\right)\times\\ \times\cos\left[\omega_{B0}t_k+\operatorname{arctg}\left(\dfrac{\omega_{B0}\varepsilon_a R_{E0}z_0}{S}\left[1+\alpha_R\Delta T\right]\left[1+\alpha_z\Delta T\right]\right)\right]\end{array}\right\}\times$$

$$\times\exp\left(-\frac{k_z z_0}{2\sqrt{D_{y_B}\varsigma}}\left[1+\alpha_z\Delta T\right]\right), \qquad (44)$$

where $\Delta T=T-T_H$ is the temperature deviation from the nominal one; α_R is the temperature coefficient of resistance.

For practical use, the equation (44) can be simplified. With this purpose we shall divide the equation into Teylor series and we will restrict it by the linear part. Then for normal conditions we get:

$$y_B(t)=k_{x0}x_0\left[1-\frac{k_z z_0}{2\sqrt{D_{y_B}\varsigma}}\right]\left\{\begin{array}{l}1-t\dfrac{U}{2Sz_0}\sqrt{\dfrac{\varepsilon_a}{(1+\mu)z_0\rho_{M0}}}\times\\ \times\cos\left[\omega_{B0}t_k+\operatorname{arctg}\left(\dfrac{\omega_{B0}\varepsilon_a R_{E0}z_0}{S}\right)\right]\end{array}\right\}. \qquad (45)$$

According to the equation (9), the maximum deviation of PCE from its initial position is $x_0=F_{B0}/k_{E0}E_{10}$. It was stated above that PCE can be regarded as a capacitor, to one of the conditional plates of which the EEP of the voltage U is connected. Then the force, formed between the plates, is $F_{B0}=U^2C/2z_H$, and EFI is $E_{10}=U/z_H$ [K]. Thus, the maximum deviation of PCE is If we assume that the thickness of the PCE plate is $z_H << 2\sqrt{D_{y_B}\varsigma}$, and the coefficient $k_z<1$, then the first factor in the equation (45) can be neglected. Then, taking into account the above-mentioned, a mathematical model for ultrasound MU takes the following form:

$$y_B(t)=k_{xE}UC\left\{1-t\frac{U}{2Sz_0}\sqrt{\frac{\varepsilon_a}{(1+\mu)z_0\rho_{M0}}}\cos\left[\omega_{B0}t_k+\operatorname{arctg}\left(\frac{\omega_{B0}\varepsilon_a R_{E0}z_0}{S}\right)\right]\right\}, \qquad (46)$$

where $k_{xE}=k_{x0}/2k_{E0}$.

The largest amplitude USV of the membrane unit equals

$$y_{B\max}(t)=k_{xE}UC\left(1-t\frac{U}{2Sz_0}\sqrt{\frac{\varepsilon_a}{(1+\mu)z_0\rho_{M0}}}\right). \qquad (47)$$

The equation (47) can determine the duration of ultrasound:

$$t_{УЗI}=\frac{2Sz}{U}\sqrt{\frac{(\ \ +\mu)}{_a}z_0\rho_{Мз0}}\left[1-\frac{_{B\max}}{k_{xE}UC}\right]. \qquad (48)$$

It was indicated above that ultrasound, which passed GM, is perceived by UST, turns in it into the EMF $e(\Re)$. The latter arrives to BOVI comparator, where it is compared to the reference voltage u_O (Fig. 3). At $e(\Re)=u_O$ the clock counter of pulses stops, by the number of which the time t_B of passing the ultrasound through the GM is determined. When designing USML of fluids, they consider technical capacity of measuring short intervals of time, which makes it possible to reduce the zone of insensitivity and EMC at large values of the fluid level in the tank. On the other hand, the duration of ultrasound $t_{УЗI}$ should be minimal and not exceed a certain minimum time $t_{УЗI\ min}$, set by technical conditions. This time is usually determined experimentally by selecting the voltage U of EEP. As shown in the equation (48), between the time $t_{УЗI}$ and the voltage U there is an extremum, by which one can determine optimal duration of ultrasound by PCE specifications.

Taking into account that the capacity of the capacitor $C=\varepsilon_a S/z_0$, optimal duration of ultrasound can be defined at the formula:

$$t_{УЗI\ опт}=k_{xE}\left(S^2/2y_{B\max}\right)\sqrt{\varepsilon_a(1+\mu)z_0\rho_{M0}}. \qquad (49)$$

The equations (47)–(49) can be used for selecting PCE and for calculation of optimal parameters of ultrasonic transducers, which will enable reducing the zone of insensitivity and increase the accuracy of measuring control of fluid level.

6. Conclusions

1. The formation of the ultrasonic pulse, which then enters the gas medium, consists of the following irreversible rheological transitions (IRT): the energy of electric exciting momentum to the electric field intensity (EFI); EFI to electrodynamic force (EDF); EDF to the displacement of a piezoceramic element (PCE); PCE displacement to electromotive force.

2. Each rheological irreversible transition is a pulse integral Dirac delta function with the core in the form of the function of transfer of energy, mass and momentum, which are described by the nonlinear differential equations with dissipative function in the form of flow rate of the converted magnitude.

3. The method of zero gradient for solving nonlinear differential equations was used that allowed obtaining analytical mathematical models for each rheological irreversible transition.

4. Mathematical models for ultrasonic pulse were obtained and it was shown that the main influential parameter for ultrasound emitter is the temperature, which leads to change not only in the amplitude of the ultrasonic pulse but the phase shift of mechanical ultrasonic oscillations.

5. The effect of secondary electromotive force in the form of ultrasonic pulse was studied, which is emitted in the gas medium. It was shown that the secondary electromotive force leads to a shift of the leading edge of the ultrasonic pulse and a phase shift, which causes additional error of measurement control of a fluid level gauge.

6. Studies of the impact of structural parameters of ultrasonic transducer on the duration of ultrasonic pulse were performed. It was established that the length of the ultrasonic pulse depends on the voltage of electric excitation pulse, capacity of piezoceramic element PCE and its geometrical parameters.

References

1. Patent 2195635 Rossijskaya Federaciya, MPK G01F23/28. Sposob izmereniya urovnya zhidkih i sypuchih sred [Text] / Zhmylev A. B., Titov S. V., Toom K. E., Topunov A. V. – Zayavitel' i patentoobladatel' zakrytoe akcionernoe obshhestvo «Vzlet». – № 2002104724/28; declared: 21.02.02; published: 27.12.02, Byul. № 19. – 3 p.
2. Subhash, N. N. Fluid level sensing using ultrasonic waveguides [Text] / N. N. Subhash, K. Balasubramaniam // Insight – Non-Destructive Testing and Condition Monitoring. – 2014. – Vol. 56, Issue 11. – P. 607–612. doi: 10.1784/insi.2014.56.11.607
3. Moore, P. I. Ultrasonic transit-time flowmeters modeled with theoretical velocity profiles: metrology [Text] / P. I. Moore, G. J. Drown, D. A. Jackson // Measurement Science and Technology. – 2000. – Vol. 11, Issue 12. – P. 1802–1811. doi: 10.1088/0957-0233/11/12/321
4. Zheng, X. M. The Development of an Automatic Ultrasonic Non-Destructive Testing System [Text] / X. M. Zheng, J. Hu, Y. S. Chen // Applied Mechanics and Materials. – 2014. – Vol. 599-601. – P. 1120–1123. doi: 10.4028/www.scientific.net/amm.599-601.1120
5. Ultrasonic level measurement [Text]. – Level: Technical Information/Endress+Hauser GmbH+Co.KG. – Endress+Hauser GmbH+Co.KG, 2002.
6. Patent 82594 Ukrayina, MPK G01S15/00, G01F23/28. Sposib vymiryuvannya rivnya ridkyx seredovyshh i ultrazvukovyj rivnemir [Text] / Lagoda D. P., Myetolkin M. I., Pososhko V. N., Uvarov A. Ya. – zayavnyk i patentovlasnyk tovarystvo z obmezhenoyu vidpovidalnistyu Naukovo-vyrobnyche pidpryyemstvo «Mikroterm». – #a200608554; declared: 31.07.06; published: 25.04.08. Byul. # 8. – 3 p.
7. Piezoelectric ceramic sensors (Piezotite): Catalog №P19E-6 [Text]. – Murata Manufacturing Co., Ltd. – 33 p.
8. International Organization for Standardization [Text]. – ISO 17989-1: Measurement of fluid flow in closed conduits – Ultrasonic meters for gas. Part 1: Meters for custody transfer and allocation measurement. Geneva, Switzerland: ISO, 2010.
9. Ultrasonic level measurement [Text]. – Level: Technical Information/Endress+Hauser GmbH+Co.KG. – Endress+Hauser GmbH+Co.KG, 2002.
10. Froysa, K-E. A ray theory approach to investigate the influence of flow velocity profiles on transit times in ultrasonic flow meters for gas and liquid [Text] / K-E. Froysa, P. Lunde // Paper presented at the 24 Scandinavian Symposium on Physical Acoustics, Ustaoset, 2001.
11. Patent 74227 Ukrayina, MPK G01F 23/28 (2006/01). Ultrazvukovyj prystrij dlya vymiryuvannya rivnya seredovyshh z nerivnomirnoyu poverxneyu [Text] / Stencel J. I., Tomson A. V., Shapovalov O. I., Litvinov K. A., Ryabichenko A. V. – zayavnyk i patentovlasnyk Sxidnoukrayinskyj nacionalnyj universytet im. V. Dalya. – # u201203182; declared: 19.03.2012; published: 25.10.2012. Byul. # 20. – 3 p.
12. Yoder, J. Part II: The Role of Oil & Naturel Gas [Text] / J. Yoder // Flow Control. – 2013. – Vol. 2. – P. 26–31.
13. Stencel, J. I. Elektrodeformacijni procesy v p'yezoelektrychnyx peretvoryuvachax [Text] / J. I. Stencel, A. V. Tomson; V. V. Elyseev (Ed.) // Systemi kontrolya y upravlenyya texnologycheskymy processamy. Sbornyk nauchnix statej. – Lugansk: Svitlycya, 2006. – P. 144–149.
14. Patent 110220 Ukrayina, MPK G01F 23/296 (2006/01). Ultrazvukovyj prystrij dlya kontrolyu rivnya ridynnyx seredovyshh [Text] / Stencel J. I., Litvinov K. A., Ryabichenko A. V. – zayavnyk i patentovlasnyk Sxidnoukrayinskyj nacionalnyj universytet im. V. Dalya. – # a201305151; declared: 22.04.2013; published: 10.12.2015. Byul. # 23. – 3 p.
15. Stencel, Y. Y. Matematycheskye modely ultrazvukovyx datchykov urovnya veshhestv [Text] / Y. Y. Stencel, A. V. Tomson // Voprosy xymyy y xymycheskoj texnologyy. – 2007. – Issue 5. – P. 182–185.
16. Stencel, J. I. Fotokolorymetrychni gazoanalizatory [Text]: mogografiya / J. I. Stencel. – Kyiv: NMK VO, 1992. – 124 p.
17. Urovnemery ul'trazvukovye MTM900. Rukovodstvo po ekspluatacii: AALU.407632.000 [Text]. – Severodonec'k: TOV NVP «Mikroterm», 2007. – 71 p.