Розроблено принципи побудови математичної моделі системи передачі тепла, найпростішими елементами якої вибрано елементарний теплообмінник, дільники та змішувачі потоків теплоносія, для яких характерна найпростіша схема руху теплоносіїв. Це дозволяє реалізувати математичний опис різних систем передачі тепла без наступного виділення дискретних складових частин теплообмінних апаратів, що дасть змогу підвищити ефективність аналізу роботи теплообмінників

Ключові слова: математична модель, система передачі тепла, теплоенергетична установка, теплоносій, теплообмінник, теплоенергетичні параметри

Разработаны принципы построения математической модели системы передачи тепла, простейшими элементами которой выбраны элементарный теплообменник, делители и смесители потоков теплоносителя, для которых характерна самая простая схема движения теплоносителей. Это позволяет реализовать математическое описание различных систем передачи тепла без последующего выделения дискретных составляющих частей теплообменных аппаратов, что позволит повысить эффективность анализа работы теплообменников

Ключевые слова: математическая модель, система передачи тепла, теплоэнергетическая установка, теплоноситель, теплообменник, теплоэнергетические параметры

UDC 621.184.4/.5
DOI: 10.15587/1729-4061.2016.74826

DEVELOPMENT OF MATHEMATICAL MODELS AND THE CALCULATIONS OF ELEMENTS OF CONVECTIVE HEAT TRANSFER SYSTEMS

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1. Introduction

Efficient functioning and stable development of almost all sectors of industry requires a rational use of the existing and a search for new reliable and environmentally friendly sources of energy. During design and operation of thermal energy units, usual is the application of technological schemes, in which the units' elements are defined by their constructive or technological features: superheater, capacitor, deaerator, separator, compressor, engine, etc. In accordance with these schemes, the methods of design (constructive) and testing calculations of energy units are elaborated.

For solving the problems of diagnosis and optimization of these units, the actual state of which, as a result of prolonged operation, differs significantly from the project, the mentioned procedures and methods of calculation proved not efficient enough. The main difficulties are caused by the fact that under the operating conditions, practically inaccessible is the numerical information about the actual state of the equipment, needed to check the calculations. For solving such practical problems, mathematical methods are applied that imply consistent formulation of the task, development of a mathematical model, selection of the research methods on the obtained mathematical model and analysis of the obtained mathematical result.

2. Scientific literature analysis and the problem statement

Improvement of the existing technologies of energy production, as well as developing and designing of the new ones is impossible without the use of modern methods of numerical modeling [1]. These methods make it possible to compare the various modes of operation of the equipment and its design features much faster and more economically efficient compared to traditional approaches.

Mathematical modeling has been used successfully to conduct numerical analysis of energy balances when comparing different variants of combining solar thermal systems with photo electrical converters [2] for simultaneous production of thermal and electrical energy. Increase in the efficiency of hybrid systems that simultaneously use different renewable energy sources [3] can also be achieved by the mathematical modeling methods, which allow making the most optimal selection of the system by its performance, controlling the amount of available energy from each source of renewable energy and the amount of energy used from each source. Design of the systems for thermal energy storage that use a hidden heat of phase transitions [4], without the use of methods of mathematical modeling, is a very labor-intensive and time-consuming process, especially when choosing material for a thermal accumulator. Methods of mathematical modeling are also used to increase the efficiency of capturing the greenhouse gases. A dynamic model was proposed for intellectual control of the capture process of carbon emissions while burning pulverized coal [5].

During development of numerical methods of simulation that can be most efficiently used to address technological, engineering and operational tasks in the field of thermal power systems, it is necessary to use maximally adequate mathematical models. For example, a boiler model with circulating fluidized layer of coal [6], which takes into account hydrodynamics, heat transfer and peculiarities of fuel combustion, is consistent with experimental data, which were obtained at an industrial boiler, in a part of predicting the temperature of smoke gases, their composition and concentration of components of the gas mixture at various points of the boiler. The use of optimal models, which take into account maximum number of variables [7], allows determining the effect of various factors on the work of thermal power equipment of thermal power stations and carrying out calculations of normative and actual figures of its work. In particular, by using a regression analysis [8], one may, based on different initial information, receive single or multifactor equations. The simulation results of a turbine [9] are essential for correct operation of the system of dynamic analysis and control of its power. A model of a spiral heat exchanger was developed, which takes into account different modes of heat transfer [10] and allows designing a heat exchanger, which will ensure the best heat transfer at minimal metal consumption. In order to study the impact of the location of deaerator on the efficiency of heat recovery of a steam generator, the analysis was conducted [11], which revealed that the location of a deaerator between the heaters of low and medium pressure provides greater performance efficiency as compared to the location of deaerator in the condenser of a turbine, which is consistent with experimental data.

By using the methods of mathematical modeling, one may develop the models and analyze their performance not only relative to the individual elements of thermal power equipment, but also to implement modeling of complex multicomponent objects. One example is the simulation of a coal-fired power plant with the capacity 500 MW [12], the thermodynamic models of which are based on the first law of thermodynamics using the equations of balance for each component. The studied component, in turn, is modeled by a separate object of control under stationary conditions. Also based on parametric design of real power station, a mathematical model of the cycle of a gas turbine of combined generation of heat and electricity was developed [13]. The adequacy of this model is confirmed by the results of testing the work of the unit, which were sufficiently close to the real results.

The results, obtained in the process of developing mathematical models, should be the foundation for improving their constructions and structural schemes [14, 15], for optimization of the modes of their operation, for reducing harmful impact [16] and finding new approaches and solutions [17].

3. The purpose and objectives of the study

The purpose of this work is development of mathematical models of the elements of heat transfer systems (HTS), which can be used to create programs of automated analysis of the work of thermal power equipment.

To achieve the set goal, the following tasks were to be solved:

– to depict a power unit in the form of a system whose elements are divided by thermodynamic features: compression and expansion, generation and absorption of heat, separation and mixing of heat carriers, heat transfer, atmosphere;

– to develop mathematical models of the elements of heat transfer systems for analysis of the work of thermal power equipment.

4. Principles of division of heat transfer systems into discrete components

Any power unit for normal functioning must consist of three compulsory parts: a source of high potential thermal energy, a thermal engine and the environment, as well as a working body (heat carrier), which connects these elements into a single system. In the simplest case, one heat carrier may be a working body of a power unit, which circulates in a closed circuit through the hot source, the engine and the environment (Fig. 1, *a, d*), but in real power units, several autonomous circuits are used as a rule, for the transfer of heat among which the heat exchangers are required (Fig. 1, *b, e, f*).

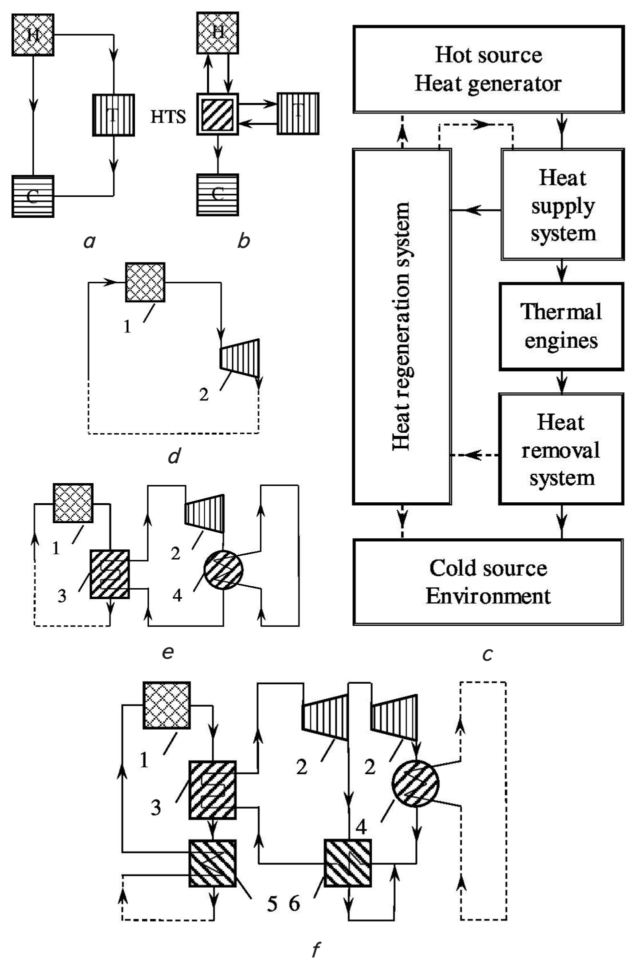

Fig. 1. HTS in the structure of a power unit: *a, b* is the structure of power units without HTS selection and with HTS selection; *c* is the scheme of heat flows in a power unit; *d, e, f* are the energy units schemes with different levels of development of HTS; H, C is the hot and cold source; D is the thermal engine; 1 is the combustion chamber; 2 is the engine; 3, 4 are the HTS on the supply and the removal of heat of the engine; 5, 6 are the HTS of heat recovery

During thermodynamic analysis of the influence of heat transfer systems (HTS) on the indicators of work of a power unit, it is expedient to divide HTS into three subsystems, which differ in their functions relative to the engine. These are subsystems that ensure heat supply to the engine, heat removal from the engine and heat regeneration (Fig. 1, *c, f*).

The limit of HTS division into the simplest discrete components is the selection of HTS elements, which clearly reflect the essence of processes in HTS and are suitable in terms of their mathematical modeling. Suitable for HTS are the elements that perform the transfer of heat between movable heat carriers and the elements that distribute flows of these heat carriers. Thus, the simplest elements of HTS are elementary heat exchangers, as well as the dividers and mixers of the flows of heat carriers. By the elementary ones will be assumed such heat exchangers, in which the simplest scheme of motion of heat carriers is in place and a mathematical description of which is performed without consequent separation of discrete components of the heat exchanger.

5. Building a mathematical model of a heat transfer system

The use of the specified elementary heat exchangers, mixers and dividers of flows can represent any subsystems and the systems of heat transfer of any energy plants. The most common elements that reflect thermodynamic processes in a power unit are: elementary heat generator (hot source), elementary thermal engine, the environment (cold source), elementary compressor, technological heat consumer. The specified set of elements is sufficient for depicting any HTS and is suitable for mathematical modeling. Their conditional images are shown in Fig. 2, *a–h*.

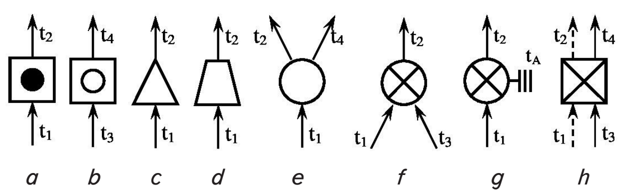

Fig. 2. Elements of thermodynamic systems: *a* is the heat generator (hot source); *b* is the technological consumer of heat; *c* is the compressor (pump); *d* is the thermal engine; *e, f* are the divider and mixer of flows of heat carriers; *g* is the environment (cold source); *h* is the heat exchanger; t_1, t_2 is the temperature of the heated heat carrier at the input and output, respectively; t_3, t_4 is the temperature of the heating heat carrier at the input and output, respectively; t_A is the ambient temperature

Such system scheme provides rigid and complete image of the unit, which is important for its mathematical modeling. The advantages of special designation of the elements of HTS are especially vivid during the analysis and comparison of various subsystems, as well as during building up complex schemes of energy units (Fig. 3).

Peculiarities of the structure of HTS influence appropriately the methods of their research. During mathematical modeling and the study of properties of HTS we determined the following main types of HTS (Fig. 4):

a) open HTS, two-flow at the input and output (inside HTS, the flows of heat carriers may divaricate and mix);

b) open HTS, two-flow only at the input and multi-flow at the output;

c) open HTS, multi-flow at the input and output;

d) semi-open HTS with a closed circuit of heat carrier that passes through a heat generator;

e) semi-open HTS with a closed circuit of heat carrier that passes through the environment;

f) closed HTS that do not have connections with other systems.

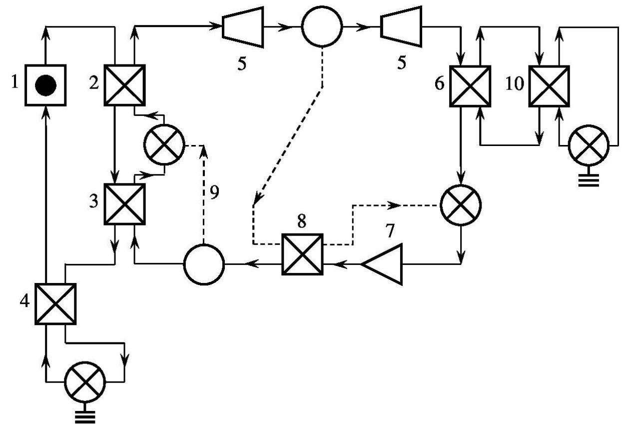

Fig. 3. System scheme of a steam turbine block: 1 is the combustion chamber; 2, 3 are the surfaces of heating a steam-water tract; 4 is the air heater; 5 is the steam turbine; 6 is the capacitor; 7 is the pump; 8 is the regenerative feed water heater; 9 is the water supply to a steam cooler; 10 is the circulating water cooler

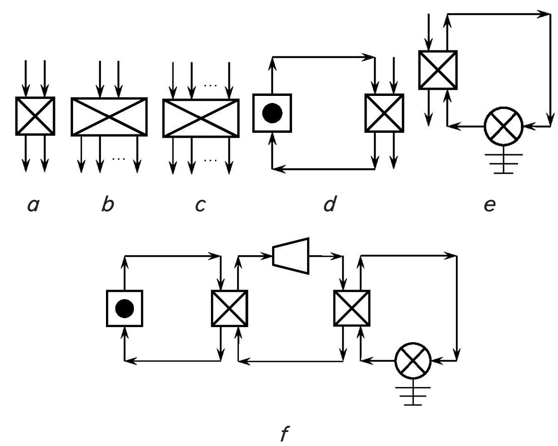

Fig. 4. Main types of HTS: *a—c* are the open HTS (two-flow, two-flow only at the input, multi-flow); *d, e* are the semi-open HTS; *f* is the closed HTS

The set of magnitudes that are used to describe both the actual HTS of energy units and the conditions of their operation is divided into the following groups in this work:

a) object parameters – parameters of the object itself (element, subsystem, HTS as a whole);

b) mode parameters – parameters of the object's external relations with other objects;

c) power.

Object parameters characterize, first of all, the capacity of the object to perceive, release and transfer heat. To ensure certainty and ease of formation of mathematical models of the objects, it is accepted that the processes of supply, removal and transfer of heat, as well as the processes of mixing and separation of the flows of heat carriers, proceed in an isobaric manner. Accordingly, the following parameters are adopted as the HTS object parameters:

a) Heat capacity of the flow of heat carrier:

$$\dot{C} = G \cdot c, \tag{1}$$

where G is the heat carrier consumption; c is the specific isobaric heat capacity of heat carrier.

b) Thermal bypass of the heat transfer surface:

$$\dot{K} = k \cdot F, \tag{2}$$

where k is the coefficient of heat transfer; F is the area of the heat transfer surface.

These parameters are sufficient as the object parameters for heat exchangers, generators and heat transmitters, mixers and dividers of the flows of heat carriers.

It is accepted for compressors and heat engines of HTS that the processes that occur in them (compression and expansion of the working body, respectively) proceed in an adiabatic manner. Thus, the object parameters are P_2/P_1 – the ratio of pressures of a working body, at the output and the input of the element, respectively, to n – the specified indicator for adiabatic polytropic process with friction.

The environment can conditionally be regarded as the limit case of a heat exchanger with infinitely large value of the thermal bypass of the surface of heat transfer and heat capacity of the flow of one of the heat carriers.

For system-wide flows of heat carriers (contours, recirculation, overflows, leaks, adulteration), heat capacity of the flow of a heat carrier is used as the object parameter.

If necessary, each of the parameters of the right part of the expressions (1) and (2) – G, c, k, F – can be used as a standalone object parameter.

The connection between the elements of HTS is made with the flow of heat carriers, the hydraulic regime of which is specified, and accordingly the consumption of heat carriers in each line of connection between elements is the pre-set value. Under this condition, the only means of information transfer between the elements when changing in HTS is the temperature change of heat carriers in the lines of connections of the elements.

Thus, there is only one mode parameter of HTS – the temperature of heat carriers at the input and output of elements. In this case, the temperature at the input reflects external influence on the element while the temperature at the output is the result of changes at the input and inside the element.

Similar is the role of temperature of heat carriers at the input and output of subsystems of HTS.

Thermal power supplied to a heat carrier in the heat generator (Q_{HG}) and removed from the heat carrier in the heat consumer (Q_{HC}), relates to the group of those magnitudes that reflect the primary changes in HTS. The power spent on compression of a working body in the compressor (N_C) and obtained by expanding the working body in the thermal engine (N_E) is assigned to a group of magnitudes that reflect the effects of the primary changes in HTS.

The mode parameters, as well as power, which reflect the primary influence on HTS, will further be called the input parameters. The parameters that reflect the effects of the primary influences are the output parameters.

For the open HTS, the input parameters are the temperature of heat carriers only at the general inputs in HTS. The temperature of heat carriers at the general outputs of flows from HTS, as well as in the lines of connections of the elements, refers to the output parameters. In the closed HTS, there is only one input temperature (t_A) – ambient temperature.

Dependency of the output parameters on the input and object parameters is called a characteristic of HTS. In this case, main attention is paid to the determining and analysis of temperature characteristics of HTS, their subsystems and elements. These characteristics reflect the dependency of temperature of heat carriers at the output and inside the object on the values of input and object parameters. Presentation of mathematical models of each of the elements of HTS in the form of temperature characteristic provides unification of models of all elements, subsystems and HTS in general, as well as the clarity and convenience of the description of the elements' connections.

In the lines of connections, the temperature of a heat carrier at the output from one element equals the temperature of the heat carrier at the input to another element.

In the end, the original mathematical description of any HTS will take the form of the system of equations of temperature characteristics of the elements and connections of the elements in the form of equity of temperatures at the beginning and at the end of the connection line. The obtained form of a mathematical model of HTS, suitable for specific tasks, may serve as the solution to the specified system of equation.

Diversity of flows of heat carriers in HTS and the desire to ensure visibility of the calculated expressions requires the use of additional variants to mark the parameters:

t_1, t_2 is the temperature of the heated heat carrier at the input and output;

t_3, t_4 is the temperature of the heating carrier at the input and output;

G_1 is the consumption of the heated heat carrier;

G_3 is the consumption of the heating heat carrier;

c_1 is the heat isobaric capacity of the heated heat carrier;

c_3 is the specific isobaric heat capacity of the heating heat carrier.

The power that is spent on heating the heat carrier in the heat generator is described by the expression

$$Q_{HG} = G_1 \cdot c_1 \cdot (t_2 - t_1). \tag{3}$$

Similar is for the power obtained by cooling a heat carrier in a heat consumer

$$Q_{HC} = G_3 \cdot c_3 \cdot (t_3 - t_4). \tag{4}$$

During the study of temperature characteristics of HTS, it is expedient to use the specific value of thermal power:

– for a heat generator

$$q_{HG} = \frac{Q_{HG}}{G_1 \cdot c_1} = t_2 - t_1; \tag{5}$$

– for a heat consumer

$$q_{HC} = \frac{Q_{HC}}{G_3 \cdot c_3} = t_4 - t_3. \tag{6}$$

Temperature characteristics of generators and heat consumers will be presented in the following uniform form:

$$t_2 = t_1 + q_{HG}; \tag{7}$$

$$t_4 = t_3 + q_{HC}. \tag{8}$$

As can be seen, ideal generators and heat consumers are linear elements of HTS.

Unification of the recordings of the equations of temperature characteristics of generators and heat consumers is implemented to ensure uniformity in the following formation of mathematical models of HTS.

The HTS parameters described above are dimensional and suitable for the display of the final result of solution to a particular applied problem. The use of generalized dimensionless parameters makes it possible to visually represent the characteristics of HTS for all theoretically possible range of the system performance in a compact form. This increases efficiency of the analysis of general properties of HTS, provides a possibility to assess a mode similarity, significantly expanding the possibilities of HTS calculation based on different variants of the original information.

The essence of formation of generalized heat carrier temperature lies in the fact that a basic temperature difference is chosen in the composition of HTS, as a rule, this is the difference of the highest and the lowest temperatures of heat carriers at the HTS input, and the values of the temperature in other points of HTS that are expressed relative to this basis.

We have for heat exchangers:

– generalized temperature of the heated heat carrier

$$P=\frac{t_2-t_1}{t_3-t_1}; \qquad (9)$$

– generalized temperature of the heating heat carrier

$$S=\frac{t_4-t_1}{t_3-t_1}. \qquad (10)$$

Similarly generalized flow heat capacity is a dimensionless ratio of the heat capacity of the given flow to heat capacity of the basic flow:

$$R=\frac{G_1\cdot c_1}{G_3\cdot c_3} \qquad (11)$$

and generalized thermal bypass of the surface of heat transfer is the ratio of thermal bypass of the surface to the heat capacity of the flow of heat carrier:

$$H=H_1=\frac{k\cdot F}{G_1\cdot c_1}; \qquad (12)$$

$$L=H_3=\frac{k\cdot F}{G_3\cdot c_3}, \qquad (13)$$

while

$$L=H\cdot R. \qquad (14)$$

It is necessary to pay attention to the fact that the expressions (9)–(14) are, in fact, the identity, equality by definition.

P, S are the generalized mode parameters, and R, H, L are the generalized object parameters of the heat exchanger [18]. It follows from (14) that independent are just two object parameters (R and H or R and L).

Temperature characteristics of the heat exchanger may be represented by dimensional or generalized form:

$$\{t_2,\ t_4\}=f\left[t_1, t_3, k,\ F,\ G_1,\ c_1,\ G_3,\ c_3\right]; \qquad (15)$$

$$\{P,\ S\}=\varphi(R,\ H). \qquad (16)$$

Defining a reverse dependency is also possible

$$\{R,\ H\}=\psi(P,\ S). \qquad (17)$$

As we can see, in order to clearly record the condition of the heat exchanger, there must be eight dimensional parameters while only two generalized parameters are sufficient. The dependency (16) is more compact in the record and graphic image; the values P and S are dimensionless and are in a convenient range 0÷1. Of important practical significance is the fact that each generalized parameter can be determined based on different variants of the initial information: dimensional values of mode parameters only or dimensional values of object parameters only. This property of generalized parameters manifests itself while comparing the expressions (9)–(14) and (16), (17).

The same properties are possessed by generalized parameters of other HTS elements, and subsystems of HTS and HTS as whole.

For ideal dividers of heat carriers flows in each flow divider, the temperature of heat carriers at the output (after the separation of the flow) is the same as at the input (before the division of the flow)

$$t_2=t_4=t_1, \qquad (18)$$

where t_2 is temperature of the heat carrier, which in the subsequent element becomes the heated one; t_4 is the original temperature of the heat carrier, which in the subsequent element becomes the heating one.

We receive in a general case, when dividing one input flow into n of output flows,

$$t_{2i}=t_1, \qquad (19)$$

where $i\in n$.

The equalities (18) and (19) meet the conditions of reversibility of isobaric mixing and the separation of ideal gases with the same temperature.

It follows from the expressions (18) and (19) that all elementary dividers of heat carriers flows are linear elements of HTS.

Interrelation of changes in input and output parameters is described by the following expression:

$$\Delta t_{2i}=\Delta t_1. \qquad (20)$$

Thus, the temperature signal, supplied to the input of divider, is transmitted to the outputs of the divider without changes.

For ideal mixers of heat carriers flows at isobaric mixing of two heat carriers that have the properties of ideal gases, we receive:

$$t_2=(1-Z)\cdot t_1+Z\cdot t_3, \qquad (21)$$

where

$$Z=\frac{G_3\cdot c_3}{G_3\cdot c_3+G_1\cdot c_1}; \qquad (22)$$

t_2 is the temperature of the heat carrier at the output of the mixer;

t_1, t_3 is the temperature of heat carriers before mixing;

$G_1 \cdot c_1$ is the heat capacity of a heat carrier flow at the temperature t_1;

$G_3 \cdot c_3$ is the heat capacity of a heat carrier flow at the temperature t_3.

Based on (21), we receive:

$$Z \equiv \frac{t_2 - t_1}{t_3 - t_1}. \quad (23)$$

As can be seen, the structure of the equation of temperature characteristic of the mixer of two flows is the same as that of elementary heat exchanger. Similarly as for the heat exchanger, the value of the generalized object parameter of the mixer can be determined based on both the object parameters only and the temperature only.

If only the input temperatures change

$$\Delta t_2 = (1 - Z) \cdot \Delta t_1 + Z \cdot \Delta t_3. \quad (24)$$

If only the object parameters change

$$\Delta t_2 = (t_3 - t_1) \cdot \Delta Z = (t_3 - t_1) \cdot \left(\frac{1}{1 + R_x} - \frac{1}{1 + R} \right). \quad (25)$$

For ideal compressors and thermal engines, the properties of a working body in ideal compressors and thermal engines are the same as the properties of ideal gas. In this connection, the temperature characteristic of the specified elements of HTS may be depicted by the expression that is received based on the known equation of adiabatic compression of ideal gas

$$T_2 = K \cdot T_1, \quad (26)$$

where

$$K = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}, \quad (27)$$

T_1, T_2 is the absolute temperature of a working body at the input and output of the element, respectively;

P_1, P_2 is the absolute pressure of a working body at the input and output of the element, respectively;

n is the adjusted indicator of polytropic process with friction [19].

Let us pay attention to the fact that only in the expression (26) the unit of temperature, the starting point of temperature, is essential.

In the formulas of other HTS elements, described above, the temperature values can be expressed both by Kelvin and Celsius degrees. Only because in the calculations of energy units the application of Celsius degrees is predominant, the use of this unit (°C) is implied in the above-mentioned formulas for t_1, t_2, t_3, t_4.

To ensure uniformity of application of the units of temperature in the formulas of temperature characteristics of all elements of HTS, we will perform appropriate transformation of the formula (26):

$$t_2 = (1 - K) \cdot (-273) + K \cdot t_1. \quad (28)$$

As can be seen, the formula (28) is unified by the structure with the formulas of temperature characteristics of heat exchangers mixers.

It follows from the equations (26)–(28) that ideal compressors and thermal engines also refer to linear elements of HTS. The value of their generalized parameter can be defined based on the known object parameters only (formula (27) and also based on known temperatures only:

$$K = \frac{T_2}{T_1} = \frac{t_2 - (-273)}{t_1 - (-273)}. \quad (29)$$

The environment as an element of HTS can be conditionally represented in the form of a heat exchanger with infinitely large surface of heat transfer, in which heat capacity of the flow of a heat carrier has also infinitely large values.

Accordingly, the external heat carrier as a result of passing through the environment has, at the output, the temperature equal to ambient temperature

$$t_2 = t_A. \quad (30)$$

The environment finalizes considered complex of ideal elements of HTS, in which there is a linear dependency of the output temperatures on the input temperature.

For the "j" element of the open HTS with two input flows of heat carriers [19, 21], we receive:

$$U_{ij} = \frac{t_{ij} - t_{10}}{t_{30} - t_{10}}, \quad (31)$$

where t_{30}, t_{10} is the temperature of the heating and heated heat carriers at the HTS input.

If an open HTS has more than two input flows of heat carriers, then

$$U_{ij} = \frac{t_{ij} - t_{100}}{t_{300} - t_{100}}, \quad (32)$$

$$U_{0ij} = \frac{t_{0ij} - t_{100}}{t_{300} - t_{100}}, \quad (33)$$

where t_{300}, t_{100} is the temperature of heating and heated heat carriers, accepted conditionally as the base for reference temperatures in HTS; t_{0ij} is the temperature of a heat carrier in the input flow "i" of HTS.

Generalized power of "j" element is described by the following expression:

$$\Omega = \frac{q_j}{t_{300} - t_{100}}. \quad (34)$$

The expressions (31)–(34) can be used to describe any element of an open HTS.

6. Discussion of results of the research into HTS of energy units

To enhance the efficiency of work on mathematical modeling and examination of HTS, it is necessary to carry out classification of the set of all possible HTS with regard to their functions, structure and properties. A classification of HTS for the calculations of HTS energy units, which is proposed in this work, is given in Table 1.

The total estimated expression of a temperature characteristic for any HTS has the form

$$t_{2j}=\sum_{i=1}^{m} a_{ij}\cdot\left(t_{10i}\right)^{\alpha_i}+\sum_{k=1}^{n} b_{jk}\cdot q_k^{\beta_k}, \qquad (35)$$

where t_{2j} is the heat carrier temperature at the output of element "j"; t_{10i} is the temperature of a heat carrier in the flow "i" at its input in HTS; q_k is the specific thermal power of generator or heat consumer; a_{ij}, b_{jk} are the coefficients, the calculated expressions of which are formed only from the object parameters; further on they will be named object coefficients; α_i, β_k are the indicators of degree; m is the total number of incoming "k" flows of HTS; n is the total number of generators and heat consumers in HTS.

We receive in a general case:

$\alpha_i \neq 1$; $a_{ij}=f_a(t)$;

$\beta_k \neq 1$; $b_{jk}=f_b(t)$,

that is, the values of indexes of degree do not equal one and the values of object coefficients depend on the temperature. This means that the shape of temperature characteristic of HTS that represents the dependency of the output temperature on the input temperature and the input power is nonlinear.

Table 1

Classification of HTS of energy units

Feature	Class
HTS functions	Systems of heat supply
	Systems of heat removal
	Systems of heat recovery
Shape of temperature characteristic	Linear HTS
	Non linear HTS
Nature of external relations	Open HTS multi-flow
	Open HTS two-flow
	Semi-open HTS
	Closed HTS
Availability of generators or heat consumers	HTS with generation and heat consumption
	HTS without generation and heat consumption
Changes inside HTS	Active HTS
	Passive HTS

All real HTS of energy units are non-linear, but in many cases, in the interval of practically possible changes in temperature, the nonlinearity is so small that this nonlinearity can be neglected when solving practical tasks.

Thus, the expression (35) is reduced to the following form:

$$t_{2j}=\sum_{i=1}^{m} a_{ij}\cdot t_{10i}+\sum_{k=1}^{n} b_{jk}\cdot q_k, \qquad (36)$$

where $a_{ij}\neq f(t)$; $b_{jk}=f(t)$.

The elements of HTS, in which the value of the object parameters does not depend on temperature, are referred to in this paper as the ideal elements and the systems that consist only of such elements – the ideal systems.

Due to the fact that the calculations of HTS essentially depend on the linearity or nonlinearity of temperature characteristics, the shape of these characteristics is selected as one of the features of the classification of HTS.

The expression (36) describes HTS with an arbitrary number of input flows of heat carriers. In reality, quite common are the HTS, in which there are only two incoming flows of heat carriers. Calculation expressions of such HTS have specific form:

$$t_{2j}=\left(1-a_j\right)\cdot t_{10}+a_j\cdot t_{30}+\sum_{k=1}^{n} b_{jk}\cdot q_k. \qquad (37)$$

The semi-open HTS are possible, in which there is only one input flow. For these HTS we obtain:

$$t_{2j}=t_{10}+\sum_{k=1}^{n} b_{jk}\cdot q_k. \qquad (38)$$

In the limiting case, HTS may have no external connections at all. For such a closed HTS, the equation of temperature characteristic has the following form:

$$t_{2j}=t_A+\sum_{k=1}^{n} b_{jk}\cdot q_k. \qquad (39)$$

If the considered linear HTS does not contain generators and heat consumers, then it is described by this expression:

$$t_{2j}=t_A+\sum_{i=1}^{m} a_{ij}\cdot t_{10i}. \qquad (40)$$

The given dependency of calculated expressions on the number of input flows, as well as on the presence of generators and heat consumers, is reflected accordingly in the calculations and simulation of HTS. In this regard, the classification of HTS is also needed by the number of input parameters.

Formation of a mathematical model of a heat transfer subsystem significantly depends on the functions of this subsystem in the composition of HTS. If a linear subsystem performs the role of a transmitter of signals only that are supplied to its input in the form of changes in the temperature of heat carriers, then it, as a rule, can be replaced by a simpler subsystem and, therefore, may be quite simply described mathematically. This subsystem is called passive. Active are those subsystems, inside which at least one object parameter changes or power of a generator and a heat consumer. HTS as a whole can also be passive or active.

The above attached calculated expressions of temperature characteristics (35)–(40) have the form that corresponds to the logic of performing of checking calculations, that is, the values of the output parameters are determined based on the known values of the input and the object parameters.

Certain peculiarities of calculations when using generalized and dimensional parameters can be shown on the example of determining a dependency of the changes in the output temperature of the heated heat carrier on the changes in the input temperatures of heat carriers in a counter-flow convective heat exchanger.

The interrelation of the parameters in ideal counter-flow heat exchanger (when the values of the object parameters do not depend on temperature changes) is described by the following expression:

$$t_2 = t_1 + \left(t_3 - t_1\right) \frac{1-\exp\left[\frac{k\cdot F}{G_1\cdot c_1}\left(1-\frac{G_1\cdot c_1}{G_3\cdot c_3}\right)\right]}{\frac{G_1\cdot c_1}{G_3\cdot c_3}-\exp\left[\frac{k\cdot F}{G_1\cdot c_1}\left(1-\frac{G_1\cdot c_1}{G_3\cdot c_3}\right)\right]}. \quad (41)$$

A required dependency can be determined by using the method of verifying calculations but in this case the values of all object parameters should be set.

In a case when, in the output state of the heat exchanger, the value of the output temperature "t_2" is already known, then by the usual constructive calculation only one object parameter can be defined, for example, F. The values for other object parameters must be specified.

Qualitatively new capacities of calculations occur in the case when, with the known value "t_2", the generalized parameters are used in the calculations.

Let us transform the above-attached expression (41), with regard to the fact that the multiplier in the form of a fraction contains only objective parameters:

$$P = \frac{t_2 - t_1}{t_3 - t_1} = \frac{1-\exp\left[\frac{k\cdot F}{G_1\cdot c_1}\left(1-\frac{G_1\cdot c_1}{G_3\cdot c_3}\right)\right]}{\frac{G_1\cdot c_1}{G_3\cdot c_3}-\exp\left[\frac{k\ F}{G_1\cdot c_1}\left(1-\frac{G_1\cdot c_1}{G_3\cdot c_3}\right)\right]}. \quad (42)$$

It follows from the expression (42) that the value of the parameter P can be defined based on both the known temperatures only and object parameters only. If the values of object parameters remain unchanged at the change in input temperatures, then the value of the parameter P also maintains a constant value.

Let us write down the equation of temperature characteristic in another form:

– for output state

$$t_2 = \left(1-P\right)\cdot t_1 + P\cdot t_3 = \left(1-P_2\right)\cdot t_1 + P_2\cdot t_3, \quad (43)$$

– for the state with changed values of the input temperatures

$$t_{2x} = \left(1-P\right)\cdot t_{1x} + P\cdot t_{3x} = \left(1-P_2\right)\cdot t_{1x} + P_2\cdot t_{3x}. \quad (44)$$

Perform, as an example, a particular calculation. Assume that in the output state of a heat exchanger, the temperature values are as follows:

$$t_3 = 530\ °C;\ \ t_1 = 30\ °C;\ \ t_2 = 330\ °C.$$

Define the value of the parameter P:

$$P = \frac{330-30}{530-30} = 0{,}6.$$

Then the required dependency of the output temperature on the input temperatures will take the form:

$$t_2 = 0{,}4\cdot t_1 + 0{,}6\cdot t_3.$$

Thus, it can be argued that to determine the required dependency, the information about the values of object parameters is not needed.

7. Conclusions

1. A division of heat transfer systems into elementary elements was developed (heat generator (hot source), thermal engine, the environment (cold source), compressor, technological heat consumer, heat exchanger, mixer and divider), which may be used to depicting any subsystems and the systems of heat transfer of energy units.

2. Mathematical models of elements of HTS were developed in the form of thermal characteristic and schemes of heat transfer systems, which reflect thermodynamic processes in an energy unit. Models of all elements are unified, generalized dimensionless object parameters are applied in them. The use of generalized dimensionless parameters makes it possible to perform calculations based on different variants of the initial information and to improve efficiency of the analysis of work of thermal power equipment.

References

1. Rychkov, A. D. Modelyrovanye v teploenerhetyke [Text] / A. D. Rychkov // Vychyslytelnye tekhnolohyy. – 2002. – Vol. 7, Issue 2. – P. 94–105.
2. Baskar, P. A review of mathematical models for performance analysis of hybrid solar photovoltaic – thermal (PV/T) air heating systems [Text] / P. Baskar, G. Edison // Advanced Materials Research. – 2013. – Vol. 768. – P. 29–39. doi: 10.4028/www.scientific.net/amr.768.29
3. Bhandari, B. Mathematical modeling of hybrid renewable energy system: a review on small hydro-solar-wind power generation [Text] / B. Bhandari, S. R. Poudel, K.-T. Lee, S.-H. Ahn // International journal of precision engineering and manufacturing-green technology. – 2014. – Vol. 1, Issue 2. – P. 157–173. doi: 10.1007/s40684-014-0021-4
4. Verma, P. Review of mathematical modeling on latent heat thermal energy storage systems using phase-change material [Text] / P. Verma, S. K. Singal // Renewable and Sustainable Energy Reviews. – 2008. – Vol. 12, Issue 4. – P. 999–1031. doi: 10.1016/j.rser.2006.11.002
5. Zhang, Q. Development of model and model-predictive control of an MEA-based postcombustion CO_2 capture process [Text] / Q. Zhang, R. Turton, D. Bhattacharyya // Industrial & Engineering Chemistry Research. – 2016. – Vol. 55, Issue 5. – P. 1292–1308. doi: 10.1021/acs.iecr.5b02243
6. Huilin, L. A coal ombustion model for circulating fluidized bed boilers [Text] / L. Huilin, Z. Guangbo, B. Rushan, C. Yongjin, D. Gidaspow // Fuel. – 2000. – Vol. 79, Issue 2. – P. 165–172. doi: 10.1016/s0016-2361(99)00139-8

7. Wang, J. Eulerian simulation of heterogeneous gas–solidflowsin CFB risers: EMMS-based sub-grid scale model with a revised cluster description [Text] / J. Wang, W. Ge, J. Li // Chemical Engineering Science. – 2008. – Vol. 63, Issue 6. – P. 553–571. doi: 10.1016/j.ces.2007.11.023
8. Halashov, N. N. Avtomatyzyrovannyi raschet normatyvnykh y faktycheskykh pokazatelei TETs [Text] / N. N. Halashov, S. V. Metnev // Elektrycheskye stantsyy. – 2008. – Vol. 11. – P. 26–28.
9. Dulau, M. Mathematical modelling and simulation of the behaviour of the steam turbine [Text] / M. Dulau, D. Bica // Procedia Technology. – 2014. – Vol. 12. – P. 723–729. doi: 10.1016/j.protcy.2013.12.555
10. Guha, P. Mathematical modeling of spiral heat exchanger [Text] / P. Guha, V. Unde // International journal of engineering research. – 2014. – Vol. 3, Issue 4. – P. 226–229. doi: 10.17950/ijer/v3s4/409
11. Srinivas, T. Study of a deaerator location in triple-pressure reheat combined power cycle [Text] / T. Srinivas // Energy. – 2009. – Vol. 34, Issue 9. – P. 1364–1371. doi: 10.1016/j.energy.2009.05.034
12. Mathematical modelling and simulation of steam power plant [Text]. – International Conference on Electrical, Electronics, Signals, Communication and Optimization (EESCO), 2015. – P. 1–5.
13. Kowalczyk, C. Mathematical model of combined heat and power plant using GateCycle™ software [Text] / C. Kowalczyk, R. M. Rolf, B. Kowalczyk, K. Badyda // Journal of Power Technologies. – 2015. – Vol. 95, Issue 3. – P. 183–191.
14. Popyryn, L. S. Matematycheskoe modelyrovanye y optymyzatsyia teploenerhetycheskykh ustanovok [Text] / L. S. Popyryn. – Moscow: Enerhyia, 1978. – 416 p.
15. Hyl, A. V. Prymenenye chyslennoho modelyrovanyia topochnykh protsessov dlia praktyky perevoda kotlov na neproektnoe toplyvo [Text] / A. V. Hyl, A. V. Starchenko, A. S. Zavoryn. – Tomsk: STT, 2011. – 184 p.
16. Zeng, Y. A review on optimization modeling of energy systems planning and ghg emission mitigation under uncertainty [Text] / Y. Zeng, Y. Cai, G. Huang, J. Dai // Energies. – 2011. – Vol. 4, Issue 12. – P. 1624–1656. doi: 10.3390/en4101624
17. Grabner, M. Numerical simulation of coal gasification at circulating fluidised bed conditions [Text] / M. Grabner, S. Ogriseck, B. Meyer // Fuel Processing Technology. – 2007. – Vol. 88, Issue 10. – P. 948–958. doi: 10.1016/j.fuproc.2007.05.006
18. Chaban, O. Y. Modeli i rozrakhunky elementarnykh konvektyvnykh teploobminnykiv [Text] / O. Y. Chaban, I. R. Halianchuk // Visnyk DU "Lvivska politekhnika". "Teploenerhetyka. Inzheneriia dovkillia. Avtomatyzatsiia". – 1999. – Vol. 365. – P. 32–40.
19. Andriushchenko, A. Y. Osnovy tekhnycheskoi termodynamyky realnykh protsessov [Text] / A. Y. Andriushchenko. – Moscow: Vysshaia shkola, 1975. – 264 p.
20. Halianchuk, I. R. Mathematical models of heat transfer system for two- and three-way heat exchangers [Text] / I. R. Halianchuk, M. Ya. Kuznetsova // Eastern-European Journal of Enterprise Technologies. – 2013. – Vol. 2, Issue 8 (62). – P. 29–32. – Available at: http://journals.uran.ua/eejet/article/view/12428/10325
21. Halianchuk, I. R. Matematychne modeliuvannia ta doslidzhennia vlastyvostei povitropidihrivnyka kotla [Text] / I. R. Halianchuk, M. Ya. Kuznetsova // Visnyk NU "Lvivska politekhnika". Teploenerhetyka. Inzheneriia dovkillia. Avtomatyzatsiia". – 2014. – Vol. 795. – P. 40–50.