Проведено короткий аналіз традиційних методів розв'язання нечітких задач математичного програмування. Виявлені недоліки відомих підходів. Розв'язання задачі досягається з використанням двоетапної процедури. Спочатку розв'язується оптимізаційна задача, яка породжується початковою задачею при заміні нечітких параметрів їх модальними значеннями. Потім відшукується чітке рішення, що забезпечує максимальну компактність нечіткого значення цільової функції і мінімально ухиляється від модального. Виклад теоретичного матеріалу супроводжується прикладами

Ключові слова: задача математичного програмування, нечіткі параметри, метод розв'язання, комплексний критерій, регуляризація

Проведен краткий анализ традиционных методов решения нечетких задач математического программирования. Выявлены недостатки известных подходов. Решение задачи достигается с использованием двухэтапной процедуры. Сначала решается оптимизационная задача, которая порождается исходной задачей при замене нечетких параметров их модальными значениями. Затем отыскивается четкое решение, обеспечивающее максимальную компактность нечеткого значения целевой функции и минимально уклоняющееся от модального. Изложение теоретического материала сопровождается примерами

Ключевые слова: задачи математического программирования, нечеткие параметры, метод решения, комплексный критерий, регуляризация

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METHOD OF SOLVING FUZZY PROBLEMS OF MATHEMATICAL PROGRAMMING

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1. Introduction

The need for solving optimization problems exists everywhere. Numerous tasks of making decisions and those of the synthesis of systems are reduced to the computational scheme, typical for the problems of mathematical programming [1–3]. However, in the problems of analysis, assessment and prediction of the state of technical, economic, military, medical and other systems, specific methods of optimization are used for finding the adequate models of behavior of these systems [4–8]. When solving such tasks, two situations are distinguished. The first one is when parameters of objective function of the problem and the constraints are the determined values. Well-known optimization methods are used for solving these problems [9–12]. In the second situation, initial information contains elements of uncertainty. The problems of optimization, the parameters of which are random variables with the known distribution laws, are combined into a class of problems of stochastic programming [13–15]. In practice it happens very often that obtaining an adequate analytical description of the required density of distribution of random parameters of the problem is impossible due to insufficient volume of the sample of initial data. In this case, when solving the problem, first of all, the mini-max approach may be used, with which the solution is obtained under assumption about "the worst" density of distribution of random parameters, found by the methods of continuous linear programming [16]. Another approach lies in the description of inexact elements of the problem in the terms of fuzzy sets [17–25]. In this case, the problem of fuzzy mathematical programming is obtained. Let us note that with the use of the theory of fuzzy sets there appears a real possibility of obtaining adequate description of optimization problems with uncertainty, much less demanding than the stochastic. This is connected to a fundamental peculiarity of the basic tool of this theory – membership functions of specific fuzzy numbers to the assigned set of these numbers: the absence of the obligatory need to meet the the normalization condition. In this case, the possibilities of constructing analytical models of the problems of acceptable quality are considerably extended. The application of the apparatus of theory of fuzzy sets simplifies setting and describing the problems but makes it impossible to directly use well studied and proven determined methods. Thus, there appears a problem of developing specific methods of mathematical programming for solution of the problems, parameters of which are not clearly defined.

2. Literature review and problem statement

A general task of mathematical programming with parameters that are not clearly defined is formulated as follows: to find a set of variables $X=(x_1, x_2,..., x_n)$, which maximizes objective function

$$f\left(X;a_1,a_2,...,a_q\right) \tag{1}$$

And meets the limitations

$$\Psi_i\left(X;b_{i1},b_{i2},...,b_{ip}\right)\leq 0,\quad i=1,2,...,m, \tag{2}$$

where parameters a_k, k=1, 2,..., q, and b_{il}, i=1, 2,..., m, l==1, 2,..., p, are the fuzzy numbers with the assigned membership functions

$$\mu_k(a_k),\ k=1,2,\dots,q,\ \nu_{il}(b_{il}),$$
$$i=1,2,\dots,m,\ l=1,2,\dots,p, \tag{3}$$

The simplest problem of fuzzy programming is the problem of achieving a not clearly defined purpose, the solution of which is proposed in [20]. In this case, it is assumed that the purpose of decision making and a set of alternatives are the equal fuzzy subsets of a certain universal set of alternatives. The technology of obtaining the solution in this case lies in the following. Assume that a certain alternative X provides for achievement of the objective with degree $\mu_G(X)$ and satisfies the limitation with degree $\mu_C(X)$. Then it is assumed that the degree of belonging of this alternative to the required decision of the problem is equal to the minimal of these magnitudes. In this case, obtaining a solution to the problem lies in the selection of alternative X*, delivering the maximum degree to the required solution, that is,

$$X^* = \arg\max_X \min\{\mu_G(X), \mu_C(X)\}.$$

It is clear that this method is effective in cases when the set of possible alternatives contains not too many elements and the accomplishment of operations of intersection of fuzzy subsets of the objective and the limitations are easy to implement.

A general approach to the solution of fuzzy problem of mathematical programming [21–23] lies in the transformation of the original problem (1), (2) with fuzzy parameters into a distinct problem of mathematical programming. The formulation of this problem takes the form: to find the sets $X=(x_1, x_2, ..., x_n)$, $A=(a_1, a_2,..., a_q)$, $B=(b_{ij})$ that maximize (1), which satisfy the limitations (2) and, besides, additional constraints

$$\mu_k(a_k) \ge \alpha,\ k=1,2,\dots,q, \tag{4}$$

$$\nu_{il}(b_{il}) \ge \alpha,\ i=1,2,\dots,m,\ l=1,2,\dots,p. \tag{5}$$

Here α is the chosen value of membership functions of parameters of the problem.

The set X*, obtained as a result of solving the problem (1)–(5), belongs to the totality of maximizing alternatives with the degree not lower than α, and the value f (X*, A) belongs with the same degree to the fuzzy evaluation of this alternative X^* [21–23]. Let us list the shortcomings of this approach [17]:

– an increase in dimensionality and computational complexity of the obtained problem in comparison with the initial one;

– the indistinctness of transformation of uncertainty levels of initial data into the uncertainty of result;

– the degree of belonging of all not clearly defined parameters of the problem is limited from the bottom by one and the same value α. In this case, it is not clear how to choose this value.

One of the alternative approaches is connected to the proposed in [26] concept of the expected value of fuzzy magnitude. The application of this method makes it possible to obtain a description of distinct function of the varying variables of the problem and in this way it transforms initial fuzzy problem into a conventional problem of mathematical programming.

A fundamentally different approach is based on the operation of constructing membership function of fuzzy value of the objective function of the problem, by using which we may pass from initial fuzzy problem to a distinct problem of mathematical programming. Let us assume that only parameters of objective function of a problem are fuzzy. Repeatedly described, for example, in [27], the procedure of construction of membership function of fuzzy value of objective function of the problem lies in the following. Their values, for which the level of belonging is assigned (for example, α), are calculated with the use of membership functions of fuzzy parameters of objective function. These values of fuzzy parameters determine the value of membership function of objective function with the same level α. Then, with the variation on α, the corresponding values of membership function of objective function are found with the variation on α, the set of which is then approximated by the appropriate curve. Obtained in this way, analytical description of membership function of objective function can be constructively used for finding a distinct solution.

Another variant of direct use of membership function of objective function lies on the surface and is based on the possibility of evaluating the level of preference of one fuzzy value of the function to another. Taking into account this circumstance, we can easily construct the procedure of finding the sequence of solutions to the problem, in which the sequential solution will be more preferable than the previous one. In this case, any method of zero-order can be used, for example, the method of branches and limits, as it was done in [28]. It is clear that the effectiveness of methods of solving an original problem based on this variant is limited by the possibilities of the applied optimization procedures and, therefore, they can actually be used only in the problems of low dimensionality.

Let us examine another additional possible approach [7, 17]. Assume that with the use of (1), (3) according to rules [17] (or in any other way) we constructed the membership function $\mu(f(X,A))$ of fuzzy value of objective function of the problem, corresponding to set X. Let us select a certain fixed value α<1 of the level of belonging $\mu(f(X,A))$ and solve the equation

$$\mu(f(X,A)) = \mu(y) = \alpha. \tag{6}$$

Since any membership function is a convex upward function, this equation has two roots

$$y_{1,2} = \left(\mu_1^{-1}(\alpha), \mu_2^{-1}(\alpha)\right).$$

Assume, for example, the problem of linear programming is being solved: to find the set $X=(x_1, x_2, ..., x_q)$, maximizing L(X)=AX and satisfying limitations BX–C=0, in this case, parameters of the objective function $A=(a_1, a_2, ..., a_q)$ are fuzzy numbers, for example, with Gaussian membership functions

$$\mu_k(a_k) = \exp\left\{\frac{(a_k - m_k)^2}{2\sigma_k^2}\right\},\ k=1,2,\dots,q.$$

In this case, in accordance with rules [7, 17], membership function of the objective function of the problem will take the form:

$$\mu\left(L(x)\right)=\exp\left\{\frac{\left(L-m_\Sigma(x)\right)^2}{2\sigma_\Sigma^2(x)}\right\},$$

where

$$m_\Sigma(X)=\sum_{k=1}^{q}m_k x_k,\quad \sigma_\Sigma^2(X)=\sum_{k=1}^{q}\sigma_k^2 x_k^2.$$

Now for $\alpha<1$, chosen in any way, we will solve the equation

$$\exp\left\{\frac{\left(L-m_\Sigma(X)\right)^2}{2\sigma_\Sigma^2(X)}\right\}=\alpha. \tag{7}$$

Hence

$$L_{1,2}(X)=m_\Sigma(X)\pm\sigma_\Sigma(X)\left(2\ln\frac{1}{\alpha}\right)^{\frac{1}{2}}. \tag{8}$$

Let us select the smaller of these roots – L_1 and set the problem of finding the set X*, maximizing $L_1(X)$ and satisfying limitations (2). It is clear that as a result of performing this procedure, the body of uncertainty, which corresponds to the membership function of the obtained fuzzy value of the objective function of the problem, will maximally shift to the area of larger values of the objective function.

The shortcomings of this approach are obvious. First, the obtained solution of problem X* depends on which of the obtained roots L_1 or L_2 of the equation (6) is used for solving the problem of maximization. In this case it is clear that the solutions will be different. Second, it is still not clear how to select value α, on which, obviously, the desired result also depends. Let us consider an example.

Example. Let us solve the problem of maximization of linear form

$$L(X,A)=a_1x_1+a_2x_2 \tag{9}$$

with constraints

$$x_1+x_2=2,\quad x_1\ge 0,\quad x_2\ge 0. \tag{10}$$

Coefficients of the optimized function are fuzzy numbers with the membership functions

$$\mu_1(a_1)=\exp\left\{\frac{(a_1-2)^2}{2}\right\},\quad \mu_2(a_2)=\exp\left\{\frac{(a_2-5)^2}{32}\right\}.$$

Let us solve the problem in a traditional way [19, 20]. Let us assign α=0.1 and solve inequalities

$$\mu_1(a_1)=\exp\left\{\frac{(a_1-2)^2}{2}\right\}\ge 0.1,$$

$$\mu_2(a_2)=\exp\left\{\frac{(a_2-5)^2}{32}\right\}\ge 0.1.$$

Solution of these inequalities leads to permissible intervals of values a_1 and a_2:

$$a_1\in\left[2-\left(2\ln\frac{1}{0.1}\right)^{0.5};\ 2+\left(2\ln\frac{1}{0.1}\right)^{0.5}\right]=[-0.146;4.146],$$

$$a_2\in\left[5-\left(32\ln\frac{1}{0.1}\right)^{0.5};\ 5+\left(32\ln\frac{1}{0.1}\right)^{0.5}\right]=[-3.58;13.58].$$

Taking into account the obtained ranges for a_1 and a_2, it is obvious that the maximum (9) is reached if: $x_1^*=0,\ x_2^*=2,\ a_2=13.58.$

Now let us use technology [7, 17] for the solution of the problem.

Let us determine membership function of the objective function of the problem, which takes the form

$$\mu\left(L(X,A)\right)=\exp\left\{\frac{\left(L-m_\Sigma\right)^2}{2\sigma_\Sigma^2}\right\},$$

where

$$m_\Sigma=m_1x_1+m_2x_2=2x_1+5x_2,\ \sigma_\Sigma^2=\sigma_1^2x_1+\sigma_2^2x_2=x_1^2+16x_2^2.$$

In this case

$$\mu\left(L(X,A)\right)=\exp\left\{\frac{\left(L-(2x_1+5x_2)\right)^2}{2\left(x_1^2+16x_2^2\right)}\right\}. \tag{11}$$

From equation (7)

$$\exp\left\{\frac{\left(L-(2x_1+5x_2)\right)^2}{2\left(x_1^2+16x_2^2\right)}\right\}=\alpha$$

we have the roots

$$L_1(X)=2x_1+5x_2-\left(2\ln\frac{1}{\alpha}\right)^{0.5}\left(x_1^2+16x_2^2\right)^{0.5}, \tag{12}$$

$$L_2(X)=2x_1+5x_2+\left(2\ln\frac{1}{\alpha}\right)^{0.5}\left(x_1^2+16x_2^2\right)^{0.5}. \tag{13}$$

For the purpose of maximization of functions (12) or (13) taking into account (10), we will reduce them to one-dimensional.

$$\begin{aligned}L_1(x_1)&=2x_1+5(2-x_1)-\\&-\left(2\ln\frac{1}{\alpha}\right)^{0.5}\left(x_1^2+16(2-x_1)^2\right)^{0.5}=\\&=10-3x_1-\left(2\ln\frac{1}{\alpha}\right)^{0.5}\left(17x_1^2-64x_1+64\right)^{0.5},\end{aligned} \tag{14}$$

$$L_2(x_1)=10-3x_1+\left(2\ln\frac{1}{\alpha}\right)^{0.5}\left(17x_1^2-64x_1+64\right)^{0.5}. \tag{15}$$

Maximization $L_1(x_1)$ by x_1 for α=0.1 leads to the solution $x_1^*=1.75,\ \ x_2^*=0.25.$ The following values of parameters of membership function of fuzzy objective function of the problem correspond to this solution:

$$m_\Sigma = 2\cdot 1.75 + 5\cdot 0.25 = 4.75;$$

$$\sigma_\Sigma^2 = 1.75^2 + 16\cdot 0.5^2 = 4.0625.$$

Maximization $L_2(x_1)$ leads to another solution $x_1^*=0,$ $x_2^*=2,$ to which other values of parameters of membership function of the objective function correspond:

$$m_\Sigma = 5\cdot 2 = 10,\ \ \sigma_\Sigma^2 = 16\cdot 2^2 = 64.$$

Thus, as a result of maximization by the left root, we obtain compact membership function of fuzzy values of the objective function, shifted to the left. On the contrary, during maximization by the right root, we obtain the blurred membership function, shifted to the right and covering the area of larger values of the objective function. Therefore, the question about which of the roots of equation (8) is expedient to use for maximization does not have the definite answer. Furthermore, it is still not clear which values of the level α should be chosen.

The noted drawbacks of the known methods of solving fuzzy problems of mathematical programming stimulate the search for an alternative technology.

3. The aim and tasks of the study

The purpose of the work is to develop a method for solving a fuzzy problem of mathematical programming, which must satisfy the following requirements:

– maximally compact membership function of fuzzy values of objective function must correspond to the solution of the problem;

– the resulting solution must take into account a modal solution of the problem, obtained if fuzzy parameters of the problem are assigned to be equal to their modal values;

– technology of the solution must be free from the binding ambiguous selection of value of the level α.

One of the possible variants of construction of the method for solution of fuzzy problems of mathematical programming, which satisfies the assigned requirements, is the use of the following two-stage procedure.

4. A method of solving a problem of mathematical programming with parameters that are not clearly defined and discussion of the obtained result

Two-stage procedure is proposed for the solution of the problem.

The first stage. Using membership functions of fuzzy values of parameters, let us assign their values as equal to the modal ones. Let us solve a distinct problem of mathematical programming generated by this: to find the set X=(x_1, x_2, ..., x_n), maximizing the objective function

$$f\left(X, a_1^{(0)}, a_2^{(0)}, \ldots, a_q^{(0)}\right) \tag{16}$$

and satisfying the limitations

$$\Psi_i\left(X, b_{i1}, b_{i2}, \ldots, b_{ip}\right) \le 0,\ \ i = 1, 2, \ldots, m. \tag{17}$$

Here parameters a_k, k=1, 2,..., q are fuzzy numbers with membership functions $\mu_k(a_k)$, which have modal values a_k^0. Assume $X^{(0)}$ is the solution for the obtained problem.

At the second stage, another distinct problem of mathematical programming is solved: to find the set X=(x_1, x_2, ..., x_n), minimizing complex criterion, formed as follows.

Using membership functions of fuzzy parameters of the problem, according to the rules of performing operations with fuzzy numbers [17], let us determine membership function of fuzzy value of the objective function of the problem $\mu(f(X,A))$. Now let us introduce criterion

$$\Phi\left[\mu(f(X,A)), A^{(0)}\right] = \lambda F_1\left(A^{(0)}\right) + F_2\left(\mu(f(X,A))\right). \tag{18}$$

In this case, the first component of criterion (18) determines the deviation of solution X from the modal $X^{(0)}$, and the second one assigns the measure of uncertainty (compactness) of the obtained membership function of fuzzy value of the objective function of the problem.

The measure of compactness of membership function can be evaluated differently. First, through entropy of distribution of fuzzy values of this function according to formula

$$H = -\int_0^\infty \mu(f(X,A)) \log\left(\mu(f(X,A))\right) df(X,A). \tag{19}$$

Second, it may be defined as the ratio of area under the curve, corresponding to the membership function of fuzzy value of the objective function of the problem to the value of this area, calculated for the modal set of variables. For the convenience of calculations, it is expedient to square the obtained ratios.

In accordance with this, we have the following two expressions for criterion (18).

$$\Phi_1\left[\mu(f(X,A)), A^{(0)}\right] = \lambda \frac{\left(X - X^{(0)}\right)^T \left(X - X^{(0)}\right)}{\left(X^{(0)}\right)^T X^{(0)}} + \frac{\left(\int_{-\infty}^{\infty} \mu(f(X,A))\, df(X,A)\right)^2}{\left(\int_{-\infty}^{\infty} \mu\left(f\left(X,A^{(0)}\right)\right) df\left(X,A^{(0)}\right)\right)^2}; \tag{20}$$

$$\Phi_2\left[\mu(f(X,A)), A^{(0)}\right] = \lambda \frac{\left(X - X^{(0)}\right)^T \left(X - X^{(0)}\right)}{\left(X^{(0)}\right)^T X^{(0)}} + \frac{\left(\int_{-\infty}^{\infty} \mu(f(X,A)) \log\left(\mu(f(X,A))\right) df(X,A)\right)^2}{\left(\int_{-\infty}^{\infty} \mu\left(f\left(X,A^{(0)}\right)\right) \log\left(\mu\left(f\left(X,A^{(0)}\right)\right)\right) df\left(X,A^{(0)}\right)\right)^2}. \tag{21}$$

Example. Let us return to problem (9), (10).

The first stage. We will assign the values of fuzzy parameters of the problem on the level of their modal values: $a_1^{(0)}=2,\ \ a_2^{(0)}=5.$ The obtained problem takes the form: to find the set (x_1, x_2), maximizing

$$L\left(X, A^{(0)}\right) = 2x_1 + 5x_2$$

and satisfying (10).

Solution to the problem is obvious: $x_1^{(0)}=0,\ \ x_2^{(0)}=2.$

At the second stage we form functional (18).

Since

$$\mu\left(L(X,A)\right)=\exp\left\{\frac{\left(L-\bar{L}(X)\right)^2}{2D(X)}\right\},$$

$$\bar{L}(X)=2x_1+5x_2,\ \ D(x)=x_1^2+16x_2^2,$$

then

$$\int_{-\infty}^{\infty}\mu\left(L(X,A)\right)dL=$$

$$=\int_{-\infty}^{\infty}\exp\left\{\frac{\left(L-\bar{L}(X)\right)^2}{2D(X)}\right\}dL\underset{\frac{L-\bar{L}(X)}{D^{\frac{1}{2}}(X)}=u}{=}D^{\frac{1}{2}}(X)\int_{-\infty}^{\infty}e^{-\frac{u^2}{2}}du=$$

$$=\sqrt{2\pi}D^{\frac{1}{2}}(X)\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-\frac{u^2}{2}}du=\sqrt{2\pi}\left(x_1^2+16x_2^2\right)^{\frac{1}{2}}.$$

Then

$$J(X,A^{(0)})=\lambda\frac{\left(x_1-x_1^{(0)}\right)^2+\left(x_2-x_2^{(0)}\right)^2}{\left(x_1^{(0)}\right)^2+\left(x_2^{(0)}\right)^2}+$$

$$+\frac{2\pi\left(x_1^2+16x_2^2\right)}{2\pi\left[\left(x_1^{(0)}\right)^2+16\left(x_2^{(0)}\right)^2\right]}=$$

$$=\frac{x_1^2+16x_2^2}{64}+\lambda\frac{x_1^2+\left(x_2-2\right)^2}{4}.$$

Now, taking into account (10), we transform the obtained expression to the function of one variable x_1. In this case, we obtain

$$J(x_1)=\frac{1}{64}\left(17x_1^2-64x_1+64\right)+\lambda\frac{x_1^2}{2}.$$

Hence

$$\frac{dJ(x_1)}{dx_1}=\frac{34}{64}x_1-1+\lambda x_1=0,$$

$$x_1(\lambda)=\frac{32}{32\lambda+17}.$$

Then

$$x_2(\lambda)=2-x_1(\lambda)=\frac{64\lambda+2}{32\lambda+17}.$$

In this case

$$J_1\left(x_1(\lambda)\right)=\frac{1}{64}\left[17\left(\frac{32}{32\lambda+17}\right)^2-64\left(\frac{32}{32\lambda+17}\right)+64\right],$$

$$J_2\left(x_1(\lambda)\right)=\frac{1}{2}\left(\frac{32}{32\lambda+17}\right)^2.$$

An analysis of behavior of components of solution of the problem depending on λ reveals that with an increase in λ (increase in the weight of the second component), the value $J_2(\lambda)$ decreases to zero while the value of $J_1(\lambda)$ grows. It is clear that the chosen value λ, assigning the result of the solution, is defined subjectively by the level of preference of one particular criterion over the other one. A possible compromise is reached at the point, in which the values $J_1(\lambda)$ and $J_2(\lambda)$ are equal. Let us find value λ from equation

$$\frac{1}{64}\left[17\left(\frac{32}{32\lambda+17}\right)^2-64\left(\frac{32}{32\lambda+17}\right)+64\right]=\frac{1}{2}\left(\frac{32}{32\lambda+17}\right)^2.$$

We will introduce

$$u=\frac{32}{32\lambda+17}.$$

Then the equation takes the form

$$17u^2-64u+64=32u^2$$

or

$$15u^2+64u-64=0.$$

$$u_{1,2}=\frac{-64\pm\sqrt{4096+3840}}{30}=\frac{-64\pm 89.08}{30};\ \ u_1=0.836.$$

Then

$$\frac{32}{32\lambda+17}=0.836,$$

hence

$$\lambda=0.56.$$

Realization of the proposed approach for the solution of fuzzy problem of mathematical programming gets more complicated if parameters of not only of objective function, but also of limitations are fuzzy. A natural way of overcoming the difficulties appearing here is the application of the method of penalty functions. In this case, with the use of rules [7, 17], membership function of fuzzy value of penalty function is formed, after which the problem comes down to that described above.

The proposed method of solving the problem of mathematical programming with not clearly defined parameters has a number of advantages. The obtained solution of the problem takes into account the modal solution, which, in the situation when the level of uncertainty of fuzzy parameters of the problem is not high, may be used as the approximated. On the other hand, this solution provides minimum uncertainty of fuzzy value of the objective function. The computational procedure of obtaining the result is simple and comes down to solving two distinct problems of mathematical programming.

5. Conclusions

1. We proposed a method for solving a fuzzy problem of mathematical programming, implementing correct transition to distinct problem of mathematical programming. Its essence lies in step-by-step solution of the original problem.

First, we search for the set of variables, which is the solution to the problem, if fuzzy values of initial data are assumed to be equal to modal. Then we formulate and solve the problem of determining the set, which, minimally deviating from the modal, maximally decreases uncertainty of the result.

2. The obtained solution of the problem ensures the required interrelation between the level of uncertainty of the value of objective function and the proximity of solution to the modal by the introduction of regularizing coefficient to the complex criterion.

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