UDC 536.21
DOI: 10.15587/1729-4061.2016.79990

ANALYTICAL STUDY OF THE PROCESSES OF THERMAL CONDUCTIVITY AT HIGH INTENSITY HEATING

N. Kolesnychenko
Postgraduate Student*
E-mail: kolesnychenko.natalia@yandex.ua
N. Volgusheva
PhD, Associate Professor*
E-mail: n-volgusheva@mail.ru
I. Boshkova
Doctor of Technical Sciences, Associate Professor*
E-mail: ira_boshkova@mail.ru
*Department of Heat-and-Power Engineering and Fuel Pipeline Transportation
Educational & Scientific Institute of Cold, Cryotechnologies and Environmental Energy
Odesa National Academy of Refrigeration
Dvoryanska str., 1/3, Odesa, Ukraine, 65082

Представлено аналіз моделей теплопровідності для високоінтенсивних процесів нагрівання щільних тіл. Визначено, що для існуючих технологічних умов високотемпературної обробки матеріалів швидкість нагрівання значно нижче критичної, вище якої слід ураховувати швидкість поширення теплоти. На основі рівняння теплопровідності із внутрішніми джерелами теплоти при урахуванні теплообміну з навколишнім середовищем отримано аналітичний розв'язок, що дозволяє розрахувати температуру в довільній точці напівобмеженого масиву в умовах нагрівання в мікрохвильовому полі

Ключові слова: теплопровідність, параболічний тип, гіперболічний тип, швидкість поширення теплоти, мікрохвильове нагрівання

Представлен анализ моделей теплопроводности для высокоинтенсивных процессов нагрева плотных тел. Определено, что для существующих технологических условий высокотемпературной обработки материалов скорость нагрева значительно ниже граничной, выше которой следует учитывать конечность скорости распространения теплоты. На основе уравнения теплопроводности с внутренними источниками теплоты при учете теплообмена с окружающей средой получено аналитическое решение, позволяющее рассчитать температуру в произвольной точке полуограниченного массива в условиях нагрева в микроволновом поле

Ключевые слова: теплопроводность, параболический тип, гиперболический тип, скорость распространения теплоты, микроволновой нагрев

1. Introduction

The application of methods of high-energy influence and their combining with the traditional technologies of thermal treatment of clay and plastic materials sets the aim of obtaining the assigned working properties and characteristics: the degree of resistance to wear and cracking, resistance to the influence of high temperatures, and the required mechanical and physical internal structure. Such materials find their use in machine building, microelectronics, biomechanics, power engineering, as well as in the aerospace and transport industry [1–3]. Heating in the microwave (MW) field is also referred to intensive heating technologies [4]. The application of microwave heating offers possibilities of developing fundamentally new technologies of creation of promising ceramic, compositional and semiconductor materials [5]. The efficiency of obtaining such materials depends on the special features of forming the temperature field in a body, and obtaining data about it requires reliable mathematical models. However, at present there is a problem, connected with the uncertainty of approaches to the simulation of high intensity processes, in the first place, as a result of the uncertainty of assumptions, made with the formulation of the differential equation of thermal conductivity. The concept of high intensity heating is used widely enough; in the course of construction of models, the equations of both hyperbolic and parabolic type are permissible, depending on the specific character of the heat propagation in certain material.

Thus, the relevance of the subject matter of the study is predetermined by the need of determining correct mathematical models of thermal conductivity for the conditions of high intensity heating of material. Analytical solutions give the opportunity to carry out computational experiments and, as a result, to obtain new knowledge about the influence of a wide spectrum of parameters of the process on the thermal state of the body.

2. Literature review and problem statement

The simulation of high intensity processes of heat propagation, in the course of which the disturbance of linear connection between the heat flux and the temperatures gradient is possible, presents special complexity [6–8]. Usually, while solving the problems of thermal conductivity, differential equation is usually used, in which a temporal and spatial change in temperature is described by the equation of parabolic form, however, with the description of high intensity processes, its application can lead to obtaining incorrect results.

As was noted in [9], a certain type of isothermal surface corresponds to a specific differential operator of thermal conductivity, among which the operator of the parabolic type is a special case. It is argued [9] that the attempt to obtain temperature fields from the parabolic operator through "imposing" different initial and boundary conditions led to the problem of paradoxes and incorrect problems.

The differential equation of thermal conductivity of parabolic type, connecting temporal and spatial change in temperature for the medium with variable physical characteristics and the internal heat sources on the assumption that the velocity of heat propagation is infinitely high, takes the following form:

$$\rho c\frac{\partial t}{\partial \tau}=\mathrm{div}(\lambda \mathrm{grad} t)+q_v. \qquad (1)$$

In [10], the hypothesis of finite velocities of heat and mass propagation was proposed. In the cases when a linear relationship between the heat flux and the temperatures gradient is disrupted, density of the heat flux is determined by the generalized Fourier 's Law (on assumption that thermo-physical characteristics do not depend on temperature and there are no internal heat sources):

$$\vec{q}=-\lambda\nabla t-\tau_r\frac{\partial \vec{q}}{\partial \tau}, \qquad (2)$$

where τ_r is the constant of time (relaxation time).

With an abrupt change of q, the reconstruction of the temperature field and the gradient of temperature occurs with the displacement in time (τ_r). The velocity of heat propagation is determined by expression:

$$w_r=\sqrt{\frac{a}{\tau_r}}. \qquad (3)$$

For example, for nitrogen τ_r=10^{-9}s, w_r=150 m/s; for aluminum τ_r=10^{-11} s, w_r=1500 m/s.

The differential equation of thermal conductivity with regard to the relaxation processes was obtained using the equation of heat balance and generalized Fourier 's Law:

$$\frac{\partial t}{\partial \tau}+\tau_r\frac{\partial^2 t}{\partial \tau^2}=a\nabla^2 t. \qquad (4)$$

It is called the hyperbolic equation of thermal conductivity. The transfer to the hyperbolic operator removes some incorrect solutions of classical theory of thermal conductivity [11].

At present, considerable attention is paid to the construction of mathematical models of thermal conductivity on the basis of hyperbolic equation. In [12], the results of studying solutions to boundary-value transferring problems for hyperbolic equations were presented, in which the correctness of setting the task with HE boundary condition of I and III kinds was considered. In [13], the accurate analytical solution of the hyperbolic equation of thermal conductivity for the infinite plate with the boundary conditions of the first order was obtained. It was shown that warming (cooling) of a body is determined by the motion of the front of shock thermal wave, at which the temperature jump occurs. In this case, two sub–areas are obtained: in one of them, temperature varies from the temperature on the wall to the temperature on the wave front, in the other one, the unperturbated, the temperature is equal to the initial temperature. In [14], the wave heat transfer in the linear and nonlinear media on the basis of the law of thermal conductivity is explored, which considers not only the first and second derivatives of time of heat flux, but also derivatives of higher orders. This made it possible to pass over to the problem on the basis of parabolic equation with the argument delayed in time.

In [15], the solution to the nonlinear equation of thermal conductivity, based on the relaxation model of the heat transfer for quasi-stationary heating regime was obtained, which made it possible to receive dependency for the maximum heating rate, above which it is necessary to consider that the velocity of heat propagation is finite:

$$\left.\frac{dT}{d\tau}\right|_{max}=\frac{V_r^2}{a_r-\tau_r V_r^2}\left(T_w-T_0\right), \qquad (5)$$

where V_r is the linear rate of the surface (isotherm) motion, a_r is the diffusivity the, T_w, T_0 are the temperatures of the surface (isotherm) and the environment, respectively.

The thermal diffusivity may be determined from the ratio for the relaxation time, given in [16]:

$$\tau_r=3a_r/\upsilon_r^2, \qquad (6)$$

where υ_r is the speed of sound.

For evaluating the boundary rate of heating the material on condition that $\tau_r V_w^2\le 0{,}1\cdot a_r$, the following dependency was proposed [15]:

$$\left.\frac{dT}{d\tau}\right|_{max}=\frac{T_w-T_0}{9\tau_r}. \qquad (7)$$

The relaxation time was evaluated by different authors for different types of materials, and it was established that its value lies within the limits from 10^{-9} s for gases to 10^{-14} s for metals. The heating rate above 100 K/s is referred to as high.

With the essential dependency of thermo-physical and electro-physical properties on temperature, the analytical methods of solving prove to be ineffective. In this case, the solutions are obtained with the help of numerical methods. The drawbacks of numerical methods are connected to the fact that, underlying them, there is an error, connected with the replacement of the initial equations with approximating equations, that is, an error of computational algorithm appears, furthermore, their use is labor-intensive [17].

A separate problem of mathematical simulation of the processes of thermal conductivity is heating in the microwave field since here it is necessary to consider the conversion of electromagnetic energy to the internal energy in the volume of a body. At the same time, the technologies of obtaining materials, based on microwave heating, are becoming increasingly common [18–20]. Based on the application of theory of generalized functions through reducing the problem of thermal conductivity for a multilayer construction to the single-layer one with the variable (discontinued) physical properties of the medium in the closed form, the accurate analytical solution of the problem of non-stationary thermal conductivity with the internal heat sources varying in time, was obtained [21]. However, the proposed dependencies have their constraints and may be used exclusively for the multilayer constructions at the assigned values of internal heat sources.

As a result of analysis of literature data, it was revealed that there is no certainty in the selection of differential operator for the simulation of high intensity processes of heating bodies and there is no explicit model, which describes the non-stationary thermal conductivity of heating a semi-infinite array under the action of the microwave field.

3. The aim and the tasks of the study

The purpose of the work is to build up mathematical models of high intensity heating of dense bodies, the properties of which are similar to clay materials and plastics, used in the production of technical ceramics and different compositional articles.

To achieve the aim, the following range of problems was to be solved:

– to analyze existing solutions of hyperbolic equations of thermal conductivity and to estimate their applicability for calculating the high intensity processes of production, in particular ceramics;

– to estimate the contribution of relaxation phenomena to the processes, the intensity of which is limited by requirements for the production of ceramics;

– to obtain dependencies for calculating temperature during microwave heating of a semi-restricted array for the boundary conditions of the III kind;

– to conduct computational experiment regarding the obtained dependencies for the purpose of their verification.

4. Mathematical methods for the analysis of thermal state of a body

4. 1. Analysis of the non-stationary thermal conductivity of a body based on the model with hyperbolic equation

The study of the thermal state of a body was carried out with the use of the dependencies [13]. Fig. 1 presents the graph of the change in the excess body temperature at different values Fo. The following designations were accepted: relaxation Fourier number $Fo_r = \frac{a \cdot \tau_r}{\delta^2}$, relative coordinate $\xi = \frac{x}{\delta}$, dimensionless excess temperature

$$\theta = \frac{t - t_a}{t_0 - t_a},$$

where t_0, t are the initial and current temperatures of material, t_a is the ambient temperature.

The calculations were performed under the following conditions: Fo_r=2,78×10^{-11}, δ=0,06 m. At Fo of the order of 10^{-3}–10^{-4}, a substantial change in temperature at the border of a body is observed, then the temperature curve begins to smooth out: in these cases, the form of curves corresponds to the results, obtained by the known dependencies, based on the differential equation of thermal conductivity of parabolic type. The heat front at Fo=2,78×10^{-4} is limited by dimensionless coordinate ξ=0,97, at Fo=2,78×10^{-3}, the coordinate was displaced to the value ξ=0,92.

It appears that the application of the equation of thermal conductivity of hyperbolic type makes it possible to solve the problem of the Fourier small numbers. At the Fourier small numbers, warming (cooling) of a body is determined by motion of the front of the shock thermal wave, at which the temperature jump occurs.

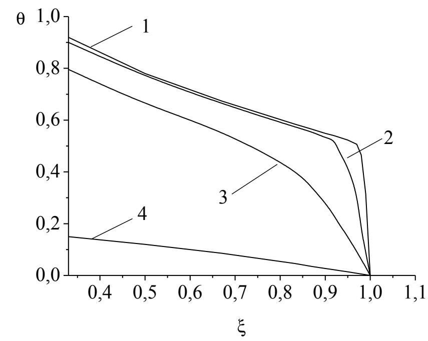


Fig. 1. Change in excess temperature of the body Θ by dimensionless coordinate ξ at different values of the Fourier number: 1 – Fo=2,78×10^{-4}, 2 – Fo=2,78×10^{-3}, 3 – Fo=2,78×10^{-2}, 4 – Fo=2,78×10^{-1}

Analysis of the need to apply the hyperbolic equation of thermal conductivity was conducted based on the calculation data of temperature when assigning extremely high values of temperatures of the heating surface: from 3000 °C to 4000 °C (for example, during sintering of ceramics, the temperature does not exceed 1420 °C). It was found that the heating rate under these conditions is 2,6 K/s –3,8 K/s. At the same time, in accordance with dependency (7) for clay materials, the properties of which were determined according to [22], the boundary heating rate was 13185 K/s (the melting point of kaolin (1800 °C) was accepted as T_w, the starting temperature T_0=20 °C). Thus, in the course of simulation of temperature field in the processes of ceramics sintering, there is a possibility in principle to apply the thermal conductivity equations of parabolic type.

4. 2. Mathematical model of non-stationary thermal conductivity based on parabolic equation with internal heat sources

When formulating the model, the following conditions were accepted. The layer of material is considered as a semi-restricted array with the thermal insulation of lateral surface at the initial temperature t_0. Inside the rod, there acts a positive heat source, caused by the action of the microwave field, specific power of which is q_v, W/m^3. The heat exchange with the environment takes place according to the law of Newton-Richman (boundary condition of the third kind). It is necessary to find the distribution of temperature by the length of the array at an arbitrary moment of time. Mathematical record of this problem is represented as follows:

$$\frac{\partial t(x,\tau)}{\partial \tau} = a \cdot \frac{\partial^2 t(x,\tau)}{\partial x^2} + \frac{q_v}{c \cdot \rho}, \quad (8)$$

$$t(x.0) = t_0, \quad (9)$$

$$\frac{\partial t(\infty,\tau)}{\partial x} = 0, \quad (10)$$

$$\alpha(t(0,\tau) - t_a) = -\lambda \frac{\partial t(0,\tau)}{\partial x}. \quad (11)$$

Here t is the temperature, x is the current coordinate, τ is the time, a, λ, c, ρ are, respectively, the coefficients of temperature conductivity and thermal conductivity, specific heat capacity and density of layer of material; α is the heat emission coefficient, q_v is the positive heat source.

It is accepted, that internal heat sources are the exponential function of the coordinate: $q_v=q_{v0}.e^{-\beta \cdot x}$. Here q_{v0} is the maximum specific power of the positive source, γ is the coefficient of attenuation of electromagnetic energy in the layer, m^{-1}.

By using the Laplace transform, solution (8) was obtained with the conditions of unambiguity (9)–(11):

$$
\begin{aligned}
&T(x,\tau)=\left(\frac{T_a-T_0}{\lambda/\gamma}+\frac{q_{v0}}{c\cdot\rho\cdot a}\right)\cdot\left(-\frac{\lambda}{\alpha}\cdot \mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}\right)+\right.\\
&+\frac{\lambda}{\alpha}\cdot e^{-\frac{\gamma}{\lambda}\cdot\sqrt{a}\cdot(-\frac{\alpha}{\lambda}\cdot\sqrt{a}\cdot\tau+\frac{x}{\sqrt{a}})}\times\\
&\times \mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}-\frac{\alpha}{\lambda}\cdot\sqrt{a\cdot\tau}\right)\Bigg)+\frac{q_{v0}\cdot(\alpha-\lambda\cdot\gamma)}{c\cdot\rho\cdot a}\times\\
&\times\frac{1}{2\cdot\gamma^2\cdot\lambda}\cdot\left\{\frac{1}{\gamma-\frac{\alpha}{\lambda}}\cdot\left(-\mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}\right)+e^{-\gamma\cdot\sqrt{a}\cdot\left(-\gamma\cdot\sqrt{a}\cdot\tau+\frac{x}{\sqrt{a}}\right)}\times\right.\right.\\
&\times \mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}-\gamma\cdot\sqrt{a\cdot\tau}\right)\Bigg)+\frac{1}{\gamma\cdot\left(\gamma+\frac{\alpha}{\lambda}\right)}\cdot\left(\mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}\right)+\right.\\
&+e^{\gamma\cdot\sqrt{a}\cdot\left(\gamma\cdot\sqrt{a}\cdot\tau+\frac{x}{\sqrt{a}}\right)}\cdot \mathrm{Erfc}\left(\frac{x}{2\cdot\sqrt{a\cdot\tau}}+\gamma\cdot\sqrt{a\cdot\tau}\right)\Bigg)\Bigg\}+\\
&+\frac{q_{v0}}{c\cdot\rho\cdot a\cdot\gamma^2}\cdot\left(e^{a\cdot\gamma^2\cdot\tau}-1\right)\cdot e^{-\gamma\cdot x}+T_0. \qquad (12)
\end{aligned}
$$

The obtained dependency enables us to calculate the temperature of a semi-infinite array at different geometric and physical conditions, media and material.

5. Results of analytical study of the temperature field of a semi-restricted array and their discussion

The analytical study of the temperature field of a semi-restricted array was conducted with the variation in the following determining characteristics: the the heat transfer coefficient α, the coefficient of absorption of electromagnetic energy γ, the layer thickness x, initial temperature of material t_0 and the ambient temperature t_a.

Fig. 2 demonstrates the change in temperature by the depth of material.

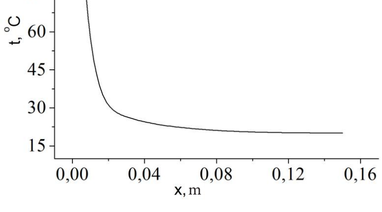


Fig. 2. Change in the temperature of material t by the depth x: τ=100 s

The data were obtained for the following conditions: α=20 W/(m^{2}K), x=0,001 m, $q_{v0}=3\times10^5$ W/m^3, γ=35 m^{-1}, t_0=20 °C, t_a=20 °C. It is evident that at a certain distance from the surface, the intensity of increase in temperature is considerably lower than in the layers, close to the surface, which is connected to the attenuation of electromagnetic energy by depth.

Effect of the heat transfer coefficient on the distribution of temperatures in the material is demonstrated by Fig. 3. It can be seen that the heat emission coefficient exerts a substantial influence on the layers, close to the surface. Low values of α lead to the fact that in the layers, close to the surface, in which the attenuation of electromagnetic energy is insignificant, the temperature grows substantially.

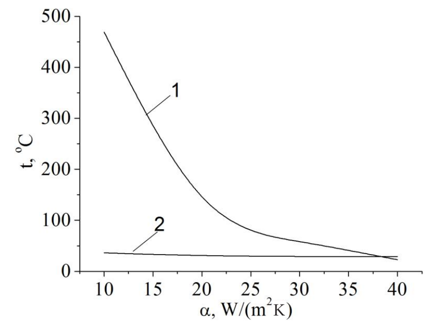


Fig. 3. Effect of the heat transfer coefficient α on the temperature of material t: τ=100 s, 1 – x=0,01 m, 2 – x=0,021 m

The absorption coefficient γ produces considerable effect on the temperature of material and on the uniformity of its heating (Fig. 4). The calculation shows that in immediate proximity from the surface at values $\gamma\leq50$ m^{-1}, temperature of the material grows with an increase in γ, and in this case, this increase slows down. At $\gamma>50$ m^{-1}, a drop in temperature is observed. At a larger depth of the array, an increase in γ leads to the monotonous decrease in temperature. This can be explained by the fact that at the low values of γ, the influence of internal heat sources is more substantial than the heat emission from the surface.

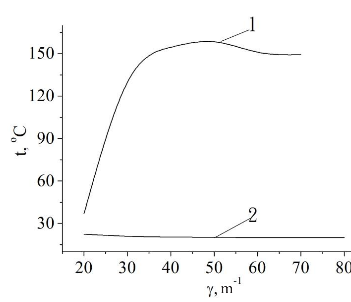
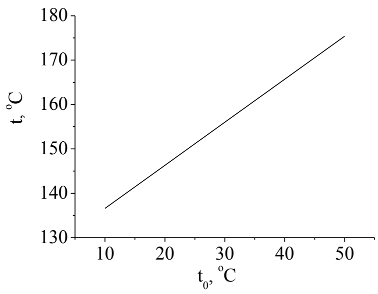


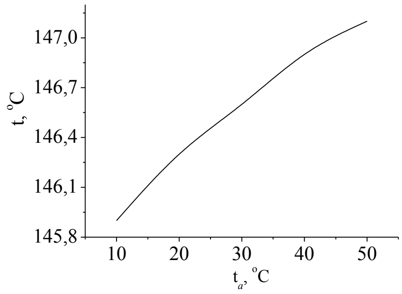
Fig. 4. Influence of the absorption coefficient γ on the temperature of material t at τ=100 s: 1 – x=0,01 m, 2 – x=0,021 m

The effect of initial temperature of the material and temperature of air on the change in the temperature of array is shown in Fig. 5. The nature of change in the body temperature correctly reflects the influence of the varied parameters.

Analysis of results of the calculations allows us to conclude that the obtained analytical dependency (12) may be recommended for calculating the temperature field of dielectric material, in particular, for the production of technical ceramics, under the effect of positive internal heat sources, for example, from the microwave electromagnetic field.



a



b

Fig. 5. Change in the temperature t of array depending on determining temperatures: *a* – effect of the initial temperature of array t_0, *b* – effect of temperature of air t_a, τ=100 s, x=0,001 m

6. Discussion of results of analytical study of the processes of thermal conductivity at high intensity heating

The calculation study of change in the temperature field in the material, the thermo-physical properties of which corresponded to clay, carried out on the dependencies, obtained on the basis of the hyperbolic equation of thermal conductivity, demonstrated influence of the Fourier numbers. It was revealed that at the values Fo, commensurate with the relaxation Fo_r, two areas are formed, which correspond to a thermal layer and the layer, in which relaxation processes were not completed. It is evident from Fig. 1 that the closer the value Fo to Fo_r, the more vividly these areas are expressed. At a considerable disagreement, the nature of change in the temperature curve becomes monotonous (Fig. 1, line 4).

Thus, the solution of the hyperbolic equation of thermal conductivity, given in [13], makes it possible to obtain data for the small Fourier numbers and to determine thickness of a thermal layer at different moments of time.

By the proposed dependencies [13], obtained with with boundary conditions of the first kind, the rates of heating clay semi-array for different surface temperatures were calculated. The results show that the heating rates are considerably below the boundary rate, at which the influence of relaxation processes should be considered. Thus, when assigning the surface temperature t_0=400 °C, the heating rate does not exceed 4 K/s while the boundary rate for the examined material is 13185 K/s. Therefore, all existing technological heating processes may be simulated on the basis of the thermal conductivity equations of parabolic type.

The microwave technologies of the production of technical ceramics and composite materials are of special interest. Due to a specific character of heating in the microwave field, the possibility of obtaining materials with the improved operational and functional properties emerges. However, there are difficulties of obtaining analytical solutions of the equations of thermal conductivity due to the need for considering the internal heat sources and special features of conversion of electromagnetic energy to internal energy. Solutions [10] are limited by the condition, at which ambient temperature must be higher than the temperature of material, which is rarely possible at microwave heating. There is a solution for the unrestricted plate with boundary conditions of the third kind [18], however, similar analytical dependencies were not obtained for the bodies of other form. This led to the need of obtaining analytical solution in the explicit form for a semi-restricted array with boundary conditions of the third kind. In the process of the model composition, we made assumptions that ambient temperature, thermo-physical and electro-physical properties (which are expressed in the value of the absorption coefficient γ) remained constant, and the initial temperature of array in all its points was identical. The solution, obtained with the aid of the operating method, demonstrates its workability when conducting computational experiment. This allowed us to recommend it for the calculation of temperature field of dielectric material when heating under conditions of action of microwave field.

To refine the calculation results, the dependency of the absorption coefficient on the temperature of material and its structural characteristics, changing in the process of heating, should be established. Furthermore, the data on the coefficients of absorption of electromagnetic energy are very limited, which does not allow making accurate calculations for a wide range of materials. Obtaining these data requires conducting separate experimental studies. Nevertheless, the main result of the performed work lies in the fact that we obtained the dependency, which makes it possible to receive information about the thermal state of a body at its heating in the microwave field and to define influence of the determining regime parameters on the heat exchange process.

7. Conclusions

1. The analytical study of change in the temperature field in material demonstrated that at values of the Fourier numbers, commensurate with the Fourier's relaxation numbers, two areas are formed, which correspond to a thermal layer and the layer, in which relaxation processes were not completed.

2. The contribution of relaxation phenomena to the processes, the intensity of which is limited by requirements for the production, for example, of ceramics, can be disregarded. It was shown that the possible heating rates based on the example of the production of technical ceramics are considerably lower than the boundary rates, above which it is not possible to accept the hypothesis about the infinite velocity of heat propagation. In the process of construction of mathematical models, it is expedient to take the equation of thermal conductivity of parabolic type as the basis.

3. Authors proposed a mathematical model of thermal conductivity of a semi-restricted array under the effect of internal heat sources for the boundary conditions of the III kind in the differential form. As the result of its solution, we obtained analytical dependencies for the calculation of temperature of the array at its heating under conditions of action of the internal heat sources, in particular, in the microwave field.

4. The proposed dependency for calculating the dimensionless excess temperature makes it possible to obtain information about a thermal state of a body at its heating in the microwave field and to determine influence of the determining characteristics – the heat emission coefficient, the coefficient of absorption of electromagnetic energy, thickness of the layer, initial temperature of material and ambient temperature – on the heat exchange process.

References

1. Wetherhold, R. C. The use of functionally graded materials to eliminated or control thermal deformation [Text] / R. C. Wetherhold, S. Seelman, J. Wang // Composites Science and Technology. – 1996. – Vol. 56, Issue 9. – P. 1099–1104. doi: 10.1016/0266-3538(96)00075-9
2. Lukasievicz, S. A. Detection of material properties in a layered body by means of thermal effects [Text] / S. A. Lukasievicz, R. Babaei, R. E. Qian // Journal of Thermal Stresses. – 2003. – Vol. 26, Issue 1. – P. 13–23. doi: 10.1080/713855763
3. Ayzikovich, S. M. Analiticheskie resheniya smeshannyih osesimmetrichnyih za-dach dlya funktsionalno-gradientnyih sred [Text] / S. M. Ayzikovich, V. M. Aleksandrov, A. S. Vasilev, L. I. Krenev, I. S. Trubchik. – Moscow: Fizmatlit, 2011. – 192 p.
4. Sheppard, L. M. Manufacturing ceramics with microwave: the potential for economical production [Text] / L. M. Sheppard // Am. Ceram. Soc. Bull. – 1988. – Vol. 67. – P. 3041–3086.
5. Bykov, Yu. V. Fabrication of Metal-Ceramic Functionally Graded Materials by Microwave Sintering [Text] / Yu. V. Bykov, S. V. Egorov, A. G. Eremeev, I. V. Plotnikov, K. I. Rybakov, V. E. Semenov et. al. // Inorganic Materials: Applied Research. – 2012. – Vol. 3, Issue 3. – P. 261–269. doi: 10.1134/s2075113312030057
6. Luo, Zh. A reduced-order extrapolation central difference scheme based on POD for two-dimensional fourth-order hyperbolic equations [Text] / Zh. Luo, Sh. Jin, J. Chen // Applied Mathematics and Computation. – 2016. – Vol. 289. – P. 396–408. doi: 10.1016/j.amc.2016.05.032
7. Shen, W. Leigh Little, Hu, L. Anti-diffusive methods for hyperbolic heat transfer [Text] / W. Shen, Leigh Little, L. Hu // Computer Methods in Applied Mechanics and Engineering. – 2010. – Vol. 199, Issue 17-20. – P. 1231–1239. doi: 10.1016/j.cma.2009.12.013
8. Ordonez-Miranda, J. Determination of thermal properties for hyperbolic heat transport using a frequency-modulated excitation source [Text] / J. Ordonez-Miranda, J. J. Alvarado-Gil // International Journal of Engineering Science. – 2012. – Vol. 50, Issue 1. – P. 101–112. doi: 10.1016/j.ijengsci.2011.08.012
9. Shashkov, A. G. Volnovyie yavleniya teploprovodnosti [Text] / A. G. Shashkov, V. A. Bubnov, S. Yu. Yanovskiy. – Moscow: Editorial USSR, 2010. – 296 p.
10. Lyikov, A. V. Teoriya teploprovodnosti [Text] / A. V. Lyikov. – Moscow, 1967. – 600 p.
11. Maurer, M. J. Non-Fourier Effects at High Heat Flux [Text] / M. J. Maurer, H. A. Thompson // Journal of Heat Transfer. – 1973. – Vol. 95, Issue 2. – P. 284–286. doi: 10.1115/1.3450051
12. Isaev, K. B. K voprosu ob uchete konechnoy skorosti rasprostraneniya tepla v tverdom tele [Text] / K. B. Isaev // Tr. V Minskogo mezhd. foruma MMF-2004, 2004. – P. 1–6.
13. Kudinov, V. A. Ob odnom metode polucheniya tochnogo analiticheskogo resheniya giperbolicheskogo uravneniya teploprovodnosti na osnove ispolzovaniya ortogonalnyih metodov [Text] / V. A. Kudinov, I. V. Kudinov // Vestnik Samarskogo Tehnicheskogo universiteta. Seriya Fiziko-matematicheskie nauki. – 2010. – Vol. 5, Issue 21. – P. 159–169.
14. Antimonov, M. S. Chislenno-analiticheskie metodyi resheniya zadach teploprovodnosti na osnove ortogonalnyih metodov vzveshennyih nevyazok [Text] / M. S. Antimonov. – Ulyanovskiy gosudarstevenyi tehnicheskiy universitet, 2008. – 24 p.
15. Isayev, K. Application of solutions of inverse heat conduction problems for research of materials thermal conductivity in wide range of its values and temperatures [Text] / K. Isayev // Proceed. 4-th Baltic Heat Transfer Conference. "Advances in Heat Transfer Engineering", 2003. – P. 201–208.
16. Chester, M. Second sound in solids [Text] / M. Chester // Physical Review. – 1963. – Vol. 131, Issue 5. – P. 2013–2015. doi: 10.1103/physrev.131.2013
17. Samarskiy, A. A. Metodyi resheniya setochnyih uravneniy [Text] / A. A. Samarskiy, E. S. Nikolaev. – Moscow: Nauka, 1978. – 592 p.
18. Lukas, R. Mikrowelleunterstütze Wärmt- und Stoffübertragung beim Trocknen und Entbindern Technischer Keramik [Text] / R. Lukas // Dissertation zur Erlagung des akademischen Grades Doktor-Ingenieur, 2005. – 125 p.
19. Willert-Porada, M. A. Microwave sintering of metal-ceramic FGM [Text] / M. A. Willert-Porada, R. Borchert // Functionally Graded Materials, 1996. – P. 349–354. doi: 10.1016/b978-044482548-3/50058-5
20. Chatterjee, A. Analysis of microwave sintering of ceramics [Text] / A. Chatterjee, T. Basak, K. J. Ayappa // AIChE Journal. – 1998. – Vol. 44, Issue 10. – P. 2301–2311. doi: 10.1002/aic.690441019
21. Averin, B. V. Obschaya shema resheniya kraevoy zadachi nestatsionarnoy teploprovodnosti s vnutrennimi istochnikami teplotyi dlya mnogosloynyih konstruktsiy [Text] / B. V. Averin // Vestnik Samarskogo Tehnicheskogo universiteta. Seriya Fiziko-matematicheskie nauki. – 2009. – Vol. 2, Issue 19. – P. 274–277.
22. Shumskayte, M. Y. Analiz vliyaniya ob'emnogo soderzhaniya i tipa glinistyih mineralov na relaksatsionnyie harakteristiki peschano-alevritovyih obraztsov kerna. Geologiya, geofizika i razrabotka neftyanyih mestorozhdeniy [Text] / M. Y. Shumskayte, V. N. Glinskih // Vsesoyuzn. NII organizatsii, upravleniya i ekonomiki neftegazovoy promyishlennosti. – 2015. – Vol. 7. – P. 35–38.