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Розроблено метод корекції за надлишковою інформацією параметрів орієнтації об'єкту, що швидко обертається навколо вздовжної осі. Специфіка об'єкту, що швидко обертається, в тому, що стрімко накопичується похибка визначення орієнтації, пов'язана із похибкою масштабного коефіцієнту для гіроскопу та пропорційна куту повороту відносно його вимірювальної осі. Для вирішення цієї проблеми був розроблений метод корекції, вільний від зазначених недоліків. Проведене моделювання та продемонстрована працездатність та ефективність запропонованого методу. Цей метод може бути використаний для побудови високоточних систем керування

Ключові слова: високодинамічний об'єкт, акселерометр, корекція, кут крену, похибка орієнтації, інерціальна навігаційна система, керування рухом

Разработан метод коррекции по избыточной информации параметров ориентации быстровращающегося вокруг продольной оси объекта. Специфика объекта, который быстро вращается, в том, что стремительно накапливается погрешность определения ориентации. Она обусловлена погрешностью масштабного коэффициента для гироскопа и пропорциональна углу поворота относительно его измерительной оси. Разработан метод коррекции, свободный от указанных недостатков. Проведено моделирование и продемонстрирована работоспособность и эффективность предложенного метода. Этот метод может быть использован для построения высокоточных систем управления

Ключевые слова: высокодинамичный объект, акселерометр, коррекция, угол крена, погрешность ориентации, инерциальная навигационная система, управление движением

#### 1. Introduction

The traditional field of using navigation equipment is air and sea transport, rocket and space technology. With regard to the importance of fulfilled functions, high-precision platform or platform-free inertial navigation system (INS) have been used at these objects until now. The high cost of such systems, which is determined by the price of precision gyroscopic meters of angular velocity, limits the scope of application of INS.

An important alternative to INS in the field of navigation provision is the satellite-navigation systems (SRNS) GPS, GLONASS. However, though surpassing INS by parameters of long-term accuracy, SRNS do not ensure receiving the entire set of parameters required for the use in a contour of automatic motion control. In particular, the required frequency of information update, continuity of operation under conditions of possible interruptions in satellite signals. It is important that the listed disadvantages of SRNS are missing in INS. Thus, it is impossible to completely eliminate inertial equipment from the contour of automatic motion control, replacing it with a SRNS signals receiver, neither today nor in the near future.

Based on these prerequisites, as well as utilizing opportunities provided by the mass production of low-cost inertial sensors of low accuracy and generally accessible information from SRNS, the first integrated navigation systems apUDC 531.383

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# IMPROVING THE ACCURACY OF DETERMINING ORIENTATION OF A RAPIDLY ROTATING OBJECT

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peared and began to be used more than 20 years ago. They performed complexation of inertial and satellite information to form so-called hybrid data that are characterized by high accuracy and all useful attributes (continuity of formation, comprehensiveness, etc.) of inertial parameters [1].

Further development of this type of equipment proceeded in the direction of easing the requirements for inertial sensors while maintaining high accuracy of hybrid navigation information. This tendency is economically justified because the cost of systems based on inertial sensors of low and medium accuracy is the order of magnitude less that the cost of high-precision INS. This fact contributed to the comprehensive expansion of the scope of application of such systems: from control systems of unmanned aerial vehicles and guided projectiles to applications for mobile phones.

The success of integrated inertial-satellite navigation systems inspire hope in the possibility of further simplifying inertial subsystem with maintaining useful functional properties of the system as a whole. One of such simplifications is full or partial replacement of gyroscopic measuring module of a platform-free inertial navigation system (PINS) by the excessive number of accelerometers.

Navigation systems, built on the elements that measure linear acceleration, are called accelerometer PINS. At present, this direction is being developed, although the area of application of such systems is extremely specific and, therefore, limited [2, 3]. However, there are a number of objects that are characterized by high linear and angular dynamics at a relatively small time of motion. The issue of on-board navigation provision may be specifically relevant for them. The described motion conditions create a possibility of efficient usage of accelerometer PINS, and MEMS technology that is now actively developing, gives a developer in the field of motion meters necessary equipment in the required dynamic range at low cost.

Thus, the listed factors, in particular, expansion of the scope of application of low-cost navigation systems to a specific area of highly dynamic objects, rather high characteristics of advanced MEMS-accelerometers and their low price, provide relevance to the studies in the field of creation of accelerometer PINS on their basis.

#### 2. Literature review and problem statement

Let us consider a few of the known ways of correcting and improving the accuracy of determining orientation of an object.

It should be noted that to control the motion of an object that rapidly rotates around the longitudinal axis, it is necessary, with high frequency, to determine and use information on its triaxial orientation in space. In a general case, this is achieved by using a platform-free inertial navigation system. It contains three gyroscopes and three accelerometers, located along the orthogonal system of measuring axes. The specificity of the object that rapidly rotates is that in such PINS an error of determining orientation swiftly accumulates. It is associated with error of the so-called large-scale coefficient for gyroscope and is proportional to the angle of rotation relative to its measuring axis. This order of things leads to the fact that it becomes impossible to use information regarding orientation for determining control by the factor of its unreliability. Thus, improving the accuracy of determining the orientation of an object under these conditions is the necessary condition for creating highly accurate systems of motion control.

Of all the ways of increasing the accuracy of the systems of orientation, the most widely used are those based on the improvement in the instruments of systems, techniques of preliminary calibration of sensors, algorithmic provision of measurements processing during operation and periodic correction of vector of state with the use of excess information.

One of the obvious ways to improve accuracy of the systems of orientation is to improve the hardware base, which implies the use of more precise gyroscopes [4]. It has significant limitations because today the most precise gyroscopes have stability of relative scale factor at the level of  $10^{-5}$  [5]. At the speed of rotation at 6000 rpm, in only 3 s, it leads to error in determining the angle of rotation, larger than 1°. Less accurate and more affordable gyroscopic sensors degrade this value by 2–3 orders of magnitude. Thus, the specified method is not efficient for the examined conditions.

There is a known way to improve accuracy of the system of orientation based on preliminary calibration of sensors [6]. This method is based on determining systematic component of error, description of its mathematical dependency on certain factors, identification of parameters of this dependency and use of the specified model as a compensating one during operation of the sensor. It has disadvantages linked to the high cost of special equipment, long period of experiment, as well as the inability to achieve sustainable compensation quality during the entire period of life cycle of the product. It should be noted that this method, similar to the previous one, significantly increases the final price of navigation system, which is not always economically justified.

One more known way is to increase accuracy of the systems of orientation by improving algorithmic base of processing of measurements during operation [7]. It implies the use of more efficient algorithms of processing information from gyroscopic sensors. The essence of it is that it provides the ability to use cheap and less in size sensors (gyroscopes and accelerometers, and sometimes only accelerometers), compensating algorithmically for their low accuracy and processing information from these sensors so that it is possible to obtain all the necessary navigation parameters with sufficient accuracy. The disadvantage of this method is, first, significant restrictions on permissible speed, which is measured with regard to the desired accuracy of determining orientation, second, sensitivity to instrumental errors in gyroscope. In addition, a necessary condition for efficient application of this method is the use of gyroscopes with fairly high accuracy in the system.

Finally, as the most similar in the technical sense, there is a way to improve accuracy of the systems of orientation in PINS by a periodic correction of vector of state (including angles of orientation) through the use of information that is in excess for the system as a whole, which is given by a receiver of signals from satellite navigation system [1, 8, 9]. This correction is carried out using the Kalman filter algorithm and requires rather sophisticated program and mathematical provision.

Schematic diagram of the method is depicted in Fig. 1. The essence of the method consists in the following: signals  $\omega_X, \omega_Y, \omega_Z$  from three gyroscopes, proportional to the value of projection of angular velocity onto the measuring axes, and the signals  $a_X$ ,  $a_Y$ ,  $a_Z$  from three accelerometers, proportional to the values of projection of imaginary acceleration onto the measuring axes, enter the block of calculation of navigation parameters. From the output of this block, components, calculated by the PINS algorithm, of the coordinate vector  $\overline{R}_{I}$ , components of the velocity vector  $\overline{V}_{I}$  and the angles of orientation  $\psi_I$  (angle of course),  $\theta_I$  (pitch angle),  $\gamma_{I}$  (angle of heel) of the object enter the block of filtration and correction. The information from the receiver of radio-navigation signals in the composition of the coordinates vector  $\overline{R}_{s}$  and components of the velocity vector  $\overline{V}_{s}$  of the object is also received here. The outgoing variables of the block of filtration and correction  $\hat{R}, \hat{V}, \hat{\psi}, \hat{\theta}, \hat{\gamma},$  the so-called hybrid parameters, are proportional to the corrected values of the same-name navigation parameters and are delivered to the output of the system, ensuring at the same time sustainability of the system performance through feedback.

As a result of implementation of the method, the system's diagram will take the form, displayed in Fig. 2.

The advantages of the given method are less strict requirements for inertial sensors while maintaining long-term high accuracy of final navigation definitions. The disadvantages of the specified method are, first, dependency on the availability of radio signals from the satellite systems and on external obstacles along the way of radio signals. Second, low frequency of correction implementation, due to low frequency of satellite information update. Third, the difficulties in accurate synchronization of inertial and satellite information under conditions of highly dynamic objects. All this makes it problematic to use this method under conditions of rapid rotation of the object.



Fig. 1. Diagram of the system that is corrected by the satellite information



Fig. 2. Diagram of the system with additional unleashed correction of angle of heel

Thus, the ways to increase accuracy of determining orientation, existing today, cannot be used for the objects that rotate rapidly.

### 3. Aim and objectives of the study

The aim of the work is to develop additional method of correction of orientation of an object that rotates rapidly. It is necessary to put the idea of a high-frequency correction of angle of heel at the core of design, based on the use of measurements of vector of imaginary acceleration, the projections of which onto the axes of sensitivity of accelerometers have modulated character as a result of rotation.

To achieve the set aim, the following tasks were to be solved:

- in the method, which uses satellite information, to determine extreme value by the signals of accelerometers yaw channels and pitch (in the appropriate block (Fig. 2); - to perform formation of correcting signal, which is used in the block of unleashed correction of angle of heel

(BUCAH) to override current value of the angle of heel that was obtained in computation of navigation parameters;

- to carry out filtration and correction of angle of heel instead of the signal, proportional to the angle of heel from the block of calculation of navigation parameters.

### 4. Materials and methods of research

Before defining the essence of the method, let us consider some of the general provisions.

Assume that  $O_1XYZ$  is a motionless topocentric Cartesian coordinate system (TCCS), in which the  $O_1Y$  axis is directed upwards in parallel to the vector of acceleration of free fall  $\overline{g}$ . Let us consider an object that moves along some trajectory at the speed  $\overline{V}$ , and associate with it the connected coordinate system (CCS) Oxyz (Fig. 3). In a separate case that does not confine the next consideration we accept that longitudinal axis of the object Ox coincides with the velocity vector  $\overline{V}$ , and the axis  $O_1X$  – with direction to the North. Under these conditions, the angle of heel of the trajectory is equivalent to the pitch angle  $\theta$  and the angle of path is equivalent to the course angle  $\psi$ .

In addition to the trajectory motion, the object performs rotation around the Ox axis with a significant angular speed  $\omega$  that may change, but slowly. Let us denote the angle of rotation around the Ox axis as the angle of heel  $\gamma$  and note that when  $\theta=0$  and  $\gamma=0$ , then the plane xOz is parallel to the horizontal plane XO<sub>1</sub>Z.

There are two accelerometers Ay and Az on the object among other meters, at distance  $\rho$  from the axis of rotation and with the axes of sensitivity, collinear to the corresponding axes of CCS (Fig. 4).



Fig. 3. Topocentric and connected coordinate systems

The essence of the proposed method of correction is in the following. The method described above, using three gyroscopes, three accelerometers and a receiver of satellite signals, performs determining the angles, speed and coordinates of the object. But under conditions of rapid rotation of the object, the error of scale factor of the x-gyroscope leads to constant sharp increase in the error of determining angle of heel in this way. To increase accuracy of determining the angle of heel, we propose, with the frequency of rotation of the object increased by four times, to perform its value correction by the readings of y-and z-accelerometers, which in a general case have the form of the curve, modulated with rotation frequency, with variable amplitude of modulation and slow trend of the center line (Fig. 5).







Fig. 5. Qualitative character of the y-accelerometer measurements

That is, the designed method is as follows. By fixing in the block of selection of extreme value and formation of signal of correction of passing by the sequence of measurements of y-accelerometer its local maximum or local minimum, the signal of correction "1" or "2" is formed, which corresponds to the true value of the angle of heel  $\gamma_e=0$  ("1") or  $\gamma_e=\pi$  ("2"). For the maximum and minimum values of the z-accelerometer measurements, we form a signal of correction "-1" or "-2", which corresponds to the true value of the angle of heel  $\gamma_e=3/2\pi$  ("-1") and  $\gamma_e=1/2\pi$  ("-2"). The signal of correction, formed in this way, enters the block of correction of BUCAH, where override of the current value of the angle of heel is performed by the rule  $\gamma_I=\gamma_e$ .

To substantiate the described method, let us proceed to appropriate mathematical models.

Since under considered conditions the projection of vector of total angular velocity of rotation of the object to the longitudinal axis is much larger than all other projections, we will accordingly ignore the speed of change in the angles of pitch and course. Under these conditions, let us consider a model of current values of the measurements, which implements y-accelerometer, assuming that the initial value of the angle of heel is zero. The projection of imaginary vector of acceleration of the object onto the axis of sensitivity of y-accelerometer has the form

$$a_{v}(t) = W_{Ov}(t) - \rho \cdot \omega^{2}(t) + g \cdot \cos\theta(t) \cdot \cos\gamma(t), \qquad (1)$$

where  $a_y(t)$  are the measurements of y-accelerometer in current time t;  $W_{Oy}(t)$  is the current projection of actual acceleration of the point O onto the Oy axis of CCS;  $\rho$  is the distance of placement of the sensitive element of y-accelerometer from the axis of rotation Ox;  $\omega(t)$  is the current speed of rotation of the object around longitudinal axis; g is the value of free fall acceleration;  $\theta(t)$  is the current value of the angle of pitch;  $\gamma(t){=}\omega(t){\cdot}t$  is the current value of the angle of heel.

Typically  $\omega(t)$  and  $\theta(t)$  change in time at the speed far lower than other variables.

The real acceleration of the point O, the projection of which is being considered, is the result of actions of all external forces that are brought to the point O, and consists, in particular, of the force of gravity  $Q_w=gm$ , resistance strength of environment  $\overline{Q}_R(t)$ , lifting force  $Q_U(t)=q_U(t)m$ , and probably reactive force  $\overline{Q}_T(t)$  of the engines (Fig. 6).



Fig. 6. Diagram of forces that are applied to the object

Since in the case of collinearity of velocity vector and longitudinal axis of the object the resistance force and reactive force are orthogonal to the axes of sensitivity of y-accelerometer, the projection  $W_{\rm Oy}(t)$  with regard to rotation of the object around the Ox axis takes the form

$$W_{Ov}(t) = (q_{U}(t) - g \cdot \cos\theta(t)) \cdot \cos\gamma(t).$$
(2)

Substitution of expression (2) in equation (1) gives

$$a_{v}(t) = q_{U}(t) \cdot \cos\gamma(t) - \rho \cdot \omega^{2}(t).$$
(3)

Thus, the measurements of y-accelerometer have a low variable component, associated with the change in the speed of rotation  $\omega(t)$ , and a high frequency component, modulated by the low variable, compared to the speed of rotation, acceleration  $q_U(t)$  from the lifting force.

Qualitative character of the measurements (3) is displayed in Fig. 5.

It follows from model (3) that at quasi-stationary character of  $q_U(t)$ , a local maximum in the period of rotation of the

object corresponds to  $\gamma=0$ , and a local minimum is achieved at  $\gamma=\pi$ , which confirms the developed method of correction of the angle of heel in the system of determining orientation.

Analogous substantiation and conclusions are true for z-accelerometer as well.

# 5. Results of research into the developed method of correction of angle of heel

Let us consider example to demonstrate the efficiency and effectiveness of the proposed method. For this purpose we will model motion of the object that rotates around longitudinal axis at angular velocity  $\Omega=100$  rev/s. Mathematical model of the object motion has the form:

$$\begin{cases} \frac{dV}{dt} = g(n_{x1} - \sin\theta); \\ \frac{d\theta}{dt} = \frac{g}{V} \left( n_{y1} \cos \gamma - \cos \theta \right) \\ \frac{d\psi}{dt} = \frac{g}{V \cos \theta} n_{y1} \sin \gamma; \\ \frac{dx}{dt} = V \cos \theta \cos \psi; \\ \frac{dy}{dt} = V \sin \theta; \\ \frac{dz}{dt} = V \cos \theta \sin \psi; \\ \gamma = -\Omega \cdot t. \end{cases}$$

Here V is the module of linear speed;

$$n_{x1} = \frac{-kV}{mg}$$

is the overload along longitudinal axis;  $n_{y1}=0$  is the overload along the y axis (missing); k is the coefficient of environment resistance; g is the Earth's gravitational constant; m is the mass of the object;

We will assume that error of scale factor of gyroscope  $\delta k=0,1$  %, frequency of information update is 10 kHz. Then error of the angle of heel can be calculated by formula:

$$\delta \gamma(t) = \int_{0}^{t} \Omega \cdot \delta k dt.$$

Due to high-frequency correction in accordance with the developed algorithm described above, the signal of correction enters the block of correction BUCAH, where override of current value of the angle of heel is performed by the rule  $\gamma_I = \gamma_e$ .

For clarity, we will build dependency of error of determining the angle of heel without the proposed correction and with it (Fig. 6).

Explanations to Fig. 7 are the following: line 1 is the error of determining the angle of heel in one second by the algorithm that does not require correction. Line 2, almost parallel to the horizontal axis, is the error of determining the angle of heel by the algorithm with correction, which is performed by the extrema of measurements of y-and z-accelerometers four times per a period of rotation. This error in

a larger scale on the shortened time should take the form, shown in Fig. 8.



Fig. 7. Behavior of error of determining the angle of heel: 1 - without correction; 2 - with correction



Fig. 8. Fragment of error in determining the angle of heel by the algorithm with correction

Instantaneous decline of the error is associated with the fact of correction, and slow growth – with the accumulation of error between the moments of correction.

# 6. Discussion of results of modeling of operation of the system with correction

The conducted analysis establishes, in particular, necessary conditions of efficiency of the proposed method – this is the existence of lifting force. Creation and regulation of such force is usually performed by the onboard motion control system using the diving rudders that rotate relative to the hull. The presented method is exactly designed to provide such a system of control with highly accurate angular information.

It follows from the presented graphs that the proposed method significantly limits error of determining the angle of heel (compared with the known algorithms). The level of last error is connected to the speed of rotation and the period of updating the measurements and forms the angle by which the object rotates in a quarter of the period of updating information.

Disadvantages of the already known algorithms of correction are, first, dependency on the availability of radio signals from satellite systems and on external obstacles along the way of radio signals, second, a low frequency of correction execution, due to the low frequency of updating satellite information. All this makes their use problematic under conditions of rapid rotation of the object. The developed method is devoid of these shortcomings through additional high-frequency correction of the angle of heel based on the use of measurements of vector of imaginary acceleration by accelerometers.

Thus, at a relatively simplified implementation, presented method provides for a high accuracy of determining parameters of orientation. This makes it possible to construct advanced motion control systems, albeit for a limited class of moving objects.

#### 7. Conclusions

1. We implemented the idea of rather simple and efficient method of correction of angle of heel, in particular: determining extreme value of angle of heel according to the readings of separate accelerometers of the system. It was confirmed that these readings in a general case have a modulated form. That is, one may separate their local extrema.

2. In the moments of reaching the local maxima and minima, we performed appropriate correction of the angle of heel. This corrected signal was used to override the current value of the angle of heel.

3. Practical modeling using the design confirmed significant increase in accuracy of determining parameters of orientation. It is important that last error of determining the angle of heel has limited character and does not grow over time. There is the accumulation of error only between the moments of correction, but it is insignificant.

That is, the use of the method enables us to significantly improve accuracy of determining parameters of orientation of the objects that rotate rapidly. This, together with the correction by the satellite data, provides for the ability to construct advanced control systems of motion of highly dynamic objects.

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