Подано математичне моделювання міцності стільникових панелей для тари і упаковки. Запропоновано удосконалення моделі міцності тришарової панелі з урахуванням впливу відхилень стільникового заповнювача для випадку комбінованого навантаження зусиллями стискання та зсуву. Проаналізовано місця виникнення недосконалостей параметрів стільникового заповнювача. Розглянуто особливості впливу недосконалостей заповнювача на фізико-механічні властивості стільникового заповнювача та панелі в цілому

Ключові слова: пакувальні матеріали, тришарова панель, стільниковий заповнювач, недосконалості заповнювача, модель міцності

Представлено математическое моделирование прочности сотовых панелей для тары и упаковки. Предложено усовершенствование модели прочности трехслойной панели с учетом влияния отклонений сотового заполнителя для случая комбинированного нагружения усилиями сжатия и сдвига. Проанализированы места возникновения несовершенств параметров сотового заполнителя. Рассмотрены особенности влияния несовершенств заполнителя на физико-механические характеристики сотового заполнителя и панели в целом

Ключевые слова: упаковочные материалы, трехслойная панель, сотовый заполнитель, несовершенства заполнителя, модель прочности

UDC 621.798.019
DOI: 10.15587/1729-4061.2016.85853

MATHEMATICAL MODELING OF STRENGTH OF HONEYCOMB PANEL FOR PACKING CONTAINERS AND PACKAGING WITH REGARD TO DEVIATIONS IN THE FILLER PARAMETERS

V. Gaydachuk
Doctor of Technical Sciences, Professor
Department of Constructions and Designing of Rocket Engineering*
E-mail: gaydachukvitaly@gmail.com
G. Koloskova
PhD, Associate Professor
Department of Theoretical Mechanics, Engineering Science and Robotics Systems*
E-mail: sceperich@mail.ru
*N. E. Zhukovskiy National Aerospace University «Kharkiv Aviation Institute»
Chkalova str., 17, Kharkiv, Ukraine, 61070

1. Introduction

Honeycomb filler (HF) is the universal construction material, which is applied in different sectors. Structures with HF are used in aviation and cosmonautics, shipbuilding, building, production of doors between rooms and furniture. One of the most promising trends of using HF is packaging.

The use of honeycomb panels in the production of packaging is a reasonable alternative to wood and plastics. Panels with HF possess high specific characteristics of strength, hardness and special properties (soundproofing and thermal insulation, vibration stability, shockproof quality and others) [1]. Such panels are applied both for manufacturing packaging and as the damping gaskets during packing and transportation of goods, for the filling of voids in packagings and transport, as well as for the production of light-weight pallets and boxes, which do not need phytosanitary inspection. The rational application of honeycomb structures in one or another area should be based on the principle of sufficiency in their functional characteristics, in relation to economic aspect [2].

For the honeycomb structures, used in the packaging industry, the basic aspects, which determine their economic profitability, are the manufacturing quality of HF and of panel as a whole, as well as the conditions for its operation. The main factor, which reduces effectiveness and performance resource of using honeycomb panels in the packaging industry, is the non-uniformity of HF structure, which may arise both in the process of manufacturing and during operation under the action of external factors.

The optimization of preparation for the production of HF may essentially influence technological cost, and, therefore, price of the semi-finished products of honeycomb structures. This, in turn, requires a comprehensive analysis of the design and engineering solutions for the honeycomb structures in use, taking into account the technology of HF production at different preparation stages for the production and manufacturing of articles.

Physical wear manifests itself in the process of operating an article, effect of natural conditions and is characterized the loss in the course of time of technical and operational properties (strength, hardness, stability, etc.). It is determined by the technical condition of a honeycomb structure as a whole or its separate structural elements, as well as by the comparison of the normative period of service with the actual age. The main factor, which determines this condition, is a set of the imperfections of filler, attained by it in the process of manufacturing and subsequent operation. Therefore, for maintaining the strength of panels, it is necessary to normalize the acting loads for the real values of HF geometric parameters. In this connection, a study of the influence of deviations in the dimensions of filler on the bearing capacity of a three-layer panel as a whole is undoubtedly relevant.

2. Literature review and problem statement

When studying three-layered structures with HF, applied in the packaging, considerable attention is paid to the examination of amortization properties of filler, which are manifested at static and dynamic loads. In particular, paper [3] examines deformation of HF and its energy absorbing properties in the panels with different thicknesses under the uniform vertical static compression load. A comparison of HF work under static and dynamic loads [4] revealed that the rate of deformation of filler substantially affects its mechanical properties. When studying the performance of honeycomb panels under dynamic loads, the energy absorbing properties of HF were also investigated depending on the impact velocity and defects of the walls of the cells [5]. Authors of [6] experimentally proved that the absorbing properties of multilayer honeycomb packaging materials do not differ considerably from those with a single layer, and the basic parameter, which influences their cushioning properties, is the height of the wall of the cell. Thus, studies demonstrate that the main parameters, which influence resistance of honeycomb structure to compression, is the height of HF wall, thickness of the material that the honeycombs are made of, the presence of defects in it, as well as the intensity of impact loads. A change in type and intensity of load requires correction of the mechanical properties of filler; however, the ways of determining quantitative values of the correcting changes in the parameters of filler were not proposed by authors of the indicated articles.

However, a three-layered structure in packaging can work not only as amortization material under the action of compressive forces, but also as a load bearing surface, which, in this case, is under the action of the combined load by the forces of compression and shear. The problem of calculating the three-layered honeycomb panels under various forms of load was solved in such a science-intensive sector as aviation and rocket design. It is established that the strength of honeycomb panel depends on the precision of its manufacturing, the inaccuracies having accumulative nature, and may appear both in the process of manufacturing the materials and in the process of assembling HF and the panel as a whole. In particular, paper [7] proposed a classification of technological defects, which occur in the process of assembling honeycomb structures from the composite materials. The estimation of technological possibilities for an improvement in quality of honeycomb filler at the stage of its production was conducted based on a study of influence of defects on strength [8]. The methods of estimation and control of the occurrence of defects in HF in the process of stretching the packet are presented in [9]. Next, article [10] proposed a procedure for the goal-directed correction of modulus of rigidity at honeycomb filler shift, as well as described the possibilities of designing honeycomb fillers with the assigned physical-mechanical characteristics. Author in study [11] proposed methods for the setting of admittances for the technological parameters of the HF production process. However, it is necessary to note that such an approach to the design is expedient only in the case when at the minimum mass, it is necessary to attain the maximum strength of panel. When calculating the three-layered panels, applied in the packaging industry, this problem is not posed and, therefore, the above-indicated approaches cannot be used without considerable improvement for the solution of the basic problems, which occur for the honeycomb panels, applied in the packaging industry, namely:

– a justified use of panels of existing size range without enlarging the safety factor, which will lower expenditures for material for its production;

– prolongation of the resource of using packing containers from a three-layered material with HF, which will lower expenditures for its processing and recycling.

3. The aim and tasks of the study

The aim of present work is the development of mathematical model for the strength of a three-layered panel with HF taking into account imperfections of the filler. This will make it possible to determine optimum geometric characteristics of fillers taking into account the planned intensity of loads and multiplicity of use, thus lowering expenditures for their production, operation and recycling.

To achieve the set aim, it was necessary to solve the following tasks:

– to improve the model of strength of a three-layered panel taking into account deviations in HF sizes;

– to analyze the influence of deviations in HF sizes on the bearing capacity of a three-layered panel.

4. Materials and methods for exploring the influence of deviations in sizes of honeycomb filler on the bearing capacity of a three-layered panel

4. 1. Improvement of the model of strength of a three-layered panel taking into account the deviations in sizes of honeycomb filler

When calculating three-layered panels, for the simplification, HF is conditionally evenly distributed by the volume. However, real HF, having discrete structure and imperfections, which occur at its production and operation, may have zones of irregularity, which will influence the bearing capacity of both the three-layered panel and the structure as a whole. Therefore, an important task when designing three-layered panel is taking account of HF imperfections.

The impact of efforts cause compressive and shift stresses in the panel. Consequently, honeycomb structure operates at the specific combinations of the compressive N_{11} and shifting N_{12} efforts, used in the theoretical calculations of geometric parameters of the bearing layers and filler. A solution for the problem on stability of a three-layered plate with HF at its combined loading by the forces of compression and shift by the Bubnov-Galerkin method was represented in paper [12]. The solution of this problem comes down to determining the lowest critical value shear N_{12} at the given values of compression force N_{11}. A scheme of the solution for the problem on stability of a three-layered plate is represented in Fig. 1.

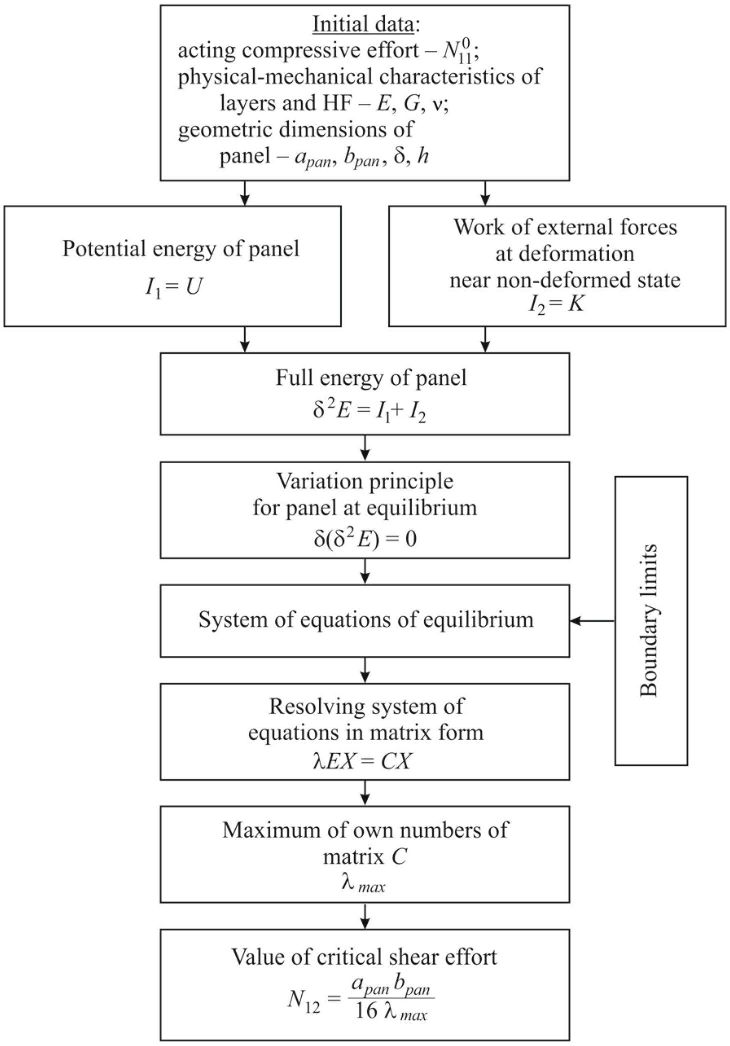

Fig. 1. Scheme of solution of the problem on the stability of a three-layered plate with HF

A honeycomb panel is exposed to normal N_{11}^0 and tangential N_{12}^0 linear efforts, well-defined from the structure scheme. At certain ratio N_{11}^0/N_{12}^0, the loss of stability of panel occurs. Different ratio of these efforts N_{11}/N_{12} differently influences the bearing capacity of panel, which depends both on the physical-mechanical properties of the material of the bearing layers, their thickness and ratio of the sides of the panel, and on the characteristics of HF.

As is known, the physical-mechanical characteristics of HF, which are reflected in the largest degree on the bearing capacity of panel, are its shear moduli. This fact is confirmed by their inclusion into the system of resolving equations of the problem on stability [12]. Theoretical values of the HF shear moduli are determined by dependences [13].

$$G_{xz}^0 = \frac{a_c + b_c \cos\beta}{(a_c + b_c)\sin\beta} \frac{tG_m}{a_c}, \tag{1}$$

$$G_{yz}^0 = \frac{b_c \sin\beta}{a_c + b_c \cos\beta} \frac{tG_m}{a_c}. \tag{2}$$

Formulas (1), (2) are obtained from the condition of ideality of honeycomb cells. Real cells may have deviations from the ideal sizes. Places of the occurrence of imperfections and their corresponding parameters of HF are shown in Fig. 2.

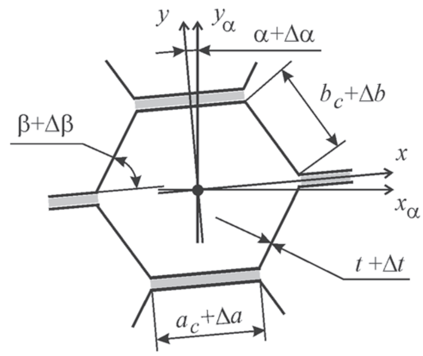

Fig. 2. Places of occurrence of HF imperfections

The faults, which occur at the stages of production of a three-layered panel with HF, can be divided into three types of technological imperfections [8, 14].

Imperfections of the first type occur even at the stage of production of semi-finished products: in the case of most common paper HF – the non-uniformity in the thickness of paper. Thus, as early as at the stage of production of semi-finished products, (1), (2) are introduced with error Δt, which deviates the value of thickness t from the nominal.

Technological imperfections of the second type occur as early as at the stage of HF production. The size of side of gluing a_c of a cell in a cell packet is equal to the width of applied glue strip. Contemporary equipment provides for the implementation of appropriate technological operations with high accuracy. Different authors present different limiting values of the obtained deviation in strip width from ±0,05 mm to ±0,1 mm [13, 15]. That is why the actual side of cell a_c will attain size ($a_c+\Delta a$).

It should also be noted that the value of maximum error in putting the glue strips does not depend practically on the nominal size of a honeycomb cell.

Error Δb in the size of free sides of cells, in addition to the deviation in width of the applied glue zone, is determined by the deviation in the distance between the strips. The maximum values of deviation in the step of strips, according to data from different authors, also take different values from ±0.05 mm to ±0.2 mm [13, 15]. Thus, the errors will occur also in the actual size b_c: ($b_c+\Delta b$).

When aligning the sheets, there are errors connected to their displacement to the right or to the left (equally probably), which causes systematic error in the real packet. These errors will lead subsequently, when stretching HF, to the shift (distortion) of cells. In this case, to predict these displacements by the thickness of packet is impossible.

In the process of pressing, glue strips can be enlarged. Integrally, this operation leads to a certain additional error a_c and b_c in (1), (2).

After the production of a cell packet, it is most often cut into strips of the standard height, or they are delivered to customer by a block and the cutting is carried out by equipment of the honeycomb structure producer.

After cutting, the stretching of strip is achieved; in this case, error occurs and it is registered (β+Δβ).

As a result, formulas (1), (2) take the following form:

$$\begin{cases} G_{xz} = \dfrac{\left[(a_c+\Delta a)+(b_c+\Delta b)\cos(\beta+\Delta\beta)\right]}{\left[(a_c+\Delta a)+(b_c+\Delta b)\right]\sin(\beta+\Delta\beta)} \dfrac{(t+\Delta t)G_m}{(a_c+\Delta a)}; \\ G_{yz} = \dfrac{(b_c+\Delta b)\sin(\beta+\Delta\beta)}{\left[(a_c+\Delta a)+(b_c+\Delta b)\cos(\beta+\Delta\beta)\right]} \dfrac{(t+\Delta t)G_m}{(a_c+\Delta a)}. \end{cases} \quad (3)$$

At the stage of final assembling a three-layered panel, the stretching of cell packet is carried out by magnitude L, equal to [9]:

$$L = 2b_c \sin\beta \cdot Z, \quad (4)$$

where b_c is the nominal value of size of the free side of a honeycomb cell; β is the nominal value of the angle of honeycomb stretching; Z is the number of rows of cells.

Taking into account technological imperfection of HF:

$$L = \sum_{i=1}^{Z} 2(b_c + \Delta b_i)\sin(\beta + \Delta\beta_i), \quad (5)$$

where β_i is the value of angle of stretching of cell in the i-th row of honeycomb.

For a small section of HF, the deviations in dimensions of cell may be considered constant. In this case, formula (5) is converted to the form:

$$L = 2(b_c + \Delta b)\sin(\beta + \Delta\beta)Z. \quad (6)$$

Equating the right sides (4) and (6), we will obtain after transformations:

$$(b_c + \Delta b_c)\sin(\beta + \Delta\beta) = b_c \sin\beta,$$

hence

$$\Delta\beta = \arcsin\left(\frac{b_c \sin\beta}{b_c + \Delta b}\right) - \beta. \quad (7)$$

Thus, deflection in the angle of stretching can be determined through the known deviation in size of the free side of a honeycomb cell. By the considerations of geometry, 0≤β≤ 90°. Taking this into account, it is possible to obtain from ratio (7) a limitation to the angle of stretching

$$\Delta b \geq \frac{b_c \sin\beta - b_c}{\sin 90^\circ} = b_c(\sin\beta - 1) = \Delta b_\beta. \quad (8)$$

At $\Delta b < b_\beta$, it is possible to observe the break of glue or material of honeycomb in the process of stretching.

Technological imperfections of the third type occur in the process of final assembly of panel and include error of stacking angle of honeycomb unit Δα.

In the practice of design of three-layered panels, the structures with nominal value of stacking angle, equal to and different from zero, may be used. When the stacking angle of a cell packet is different from zero, formulas for calculating the shear moduli will take form [13]:

$$\begin{cases} G^*_{xz}(\alpha) = \dfrac{1}{\dfrac{\cos^2\alpha}{G_{xz}} + \dfrac{\sin^2\alpha}{G_{yz}}}; \\ G^*_{yz}(\alpha) = \dfrac{1}{\dfrac{\cos^2(\alpha+\pi/2)}{G_{xz}} + \dfrac{\sin^2(\alpha+\pi/2)}{G_{yz}}}, \end{cases} \quad (9)$$

where G_{xz} and G_{yz} are calculated by formulas (3).

In connection with inaccuracies in the HF and layers stacking, there are imperfections of the third group – stacking angle error Δα – and formulas (9) take the form:

$$\begin{cases} G^*_{xz}(\alpha+\Delta\alpha) = \dfrac{1}{\dfrac{\cos^2(\alpha+\Delta\alpha)}{G_{xz}} + \dfrac{\sin^2(\alpha+\Delta\alpha)}{G_{yz}}}; \\ G^*_{yz}(\alpha+\Delta\alpha) = \dfrac{1}{\dfrac{\cos^2((\alpha+\Delta\alpha)+\pi/2)}{G_{xz}} + \dfrac{\sin^2((\alpha+\Delta\alpha)+\pi/2)}{G_{yz}}}. \end{cases} \quad (10)$$

The given classification of technological imperfections reflects the nature and sequence of their occurrence in the process of panel production. However, while using a panel, to predict the sequence of occurrence and the degree of deviation in the sizes of honeycomb cells is difficult.

Deviations in the HF geometry in the implementation of procedure for determining critical forces N^0_{11}, N^0_{12}, will cause a change in the critical forces to certain magnitude N_{11} and N_{12}, moreover, these values are, as before, interconnected, which makes it possible to examine their deviations from N^0_{11} and N^0_{12} only for one of the critical linear forces. Since HF in a panel works predominantly on the shift, then one should accept N_{12} as the determining magnitude.

Depending on the assigned accuracy of calculation, the permissible deviations in loads can be limited by certain magnitude ξ in the range 0,03<ξ≤0,1, then:

$$\left|\Delta\bar{N}_{12}\right| \leq \xi, \quad (11)$$

where

$$\Delta\bar{N}_{12} = \frac{N_{12} - N^0_{12}}{N^0_{12}}.$$

The problem of quantitative assessment of the influence of HF imperfection on the bearing capacity of a three-layered structure is reduced to finding such values of the HF parameters, at which stability condition of the bearing capacity is satisfied (11), which determines HF quality and honeycomb structure as a whole.

4. 2. Analysis of the influence of imperfections in honeycomb filler on its physical-mechanical characteristics

Deviation in the sizes of HF leads to the change in strength characteristics of panel, which may lead to its malfunctioning in the process of operation. The complexity of determining the working values of HF parameters is due to their large quantity (five). An analysis makes it possible to decrease their number to four, but even in this case the problem has a fairly complicated structure.

A change in each of the parameters influences the bearing capacity of panel differently.

Derivatives from functions (1), (2) by thickness t

$$\frac{dG_{xz}^{0}}{dt}=\frac{a_c+b_c\cos\beta}{(a_c+b_c)\sin\beta}\frac{G_m}{a_c}>0;$$

$$\frac{dG_{yz}^{0}}{dt}=\frac{b_c\sin\beta}{a_c+b_c\cos\beta}\frac{G_m}{a_c}>0,$$

therefore, with an increase in thickness t, the values of shear moduli and panel bearing capacity grow.

Derivatives of shear moduli by the dimensions of side of cell $a=a_c+\Delta a$ and $b=b_c+\Delta b$ with regard to (7) take the following values

$$\frac{dG_{xz}}{da}=-\frac{a^2\sin\beta+2ab\cos\beta\sin\beta+b^2\cos\beta\sin\beta}{a^2(a+b)^2\sin^2\beta}tG_m<0;$$

$$\frac{dG_{yz}}{da}=-\frac{b\sin\beta(2a+b\cos\beta)}{a^2(a+b\cos\beta)^2}tG_m<0;$$

$$\frac{dG_{xz}}{db}=\frac{\dfrac{b}{\sqrt{b^2-b_c^2\sin^2\beta}}\left(\dfrac{a}{b}+1\right)+\sqrt{b^2-b_c^2\sin^2\beta}\,\dfrac{a}{b^2}}{\left(\dfrac{a}{b}+1\right)^2 b_c\sin\beta}\frac{tG_m}{a}>0;$$

$$\frac{dG_{yz}}{db}=-\frac{b_c\sin\beta\dfrac{2b}{\sqrt{b^2-b_c^2\sin^2\beta}}}{\left(a+\sqrt{b^2-b_c^2\sin^2\beta}\right)^2}\frac{tG_m}{a_c}<0.$$

Consequently, with an increase in the size of cell a, the values of HF shear moduli and bearing capacity decrease, and with an increase in size of the free side of cell b, shear modulus G_{xz} grows (HF is strengthened lengthwise), and module G_{yz} decreases (in the transverse direction, the weakening of filler occurs).

A change in shear moduli leads to a change in the limit of compressive strength of panel, moreover, if the shear modulus increases, then the corresponding strength limit grows, and vice versa. Therefore, there is a task of finding such values of angle α, for which $G_{xz}^*(\alpha)$ and $G_{yz}^*(\alpha)$ take extreme values. Derivative from function (9) by angle α

$$\left(G_{xz}^*(\alpha)\right)'=G_{xz}\frac{\sin 2\alpha\cdot(1-G_{xz}/G_{yz})}{\left(\cos^2\alpha+(G_{xz}/G_{yz})\sin^2\alpha\right)^2};$$

$$\left(G_{yz}^*(\alpha)\right)'=G_{xz}\frac{\sin\left[2(\alpha+\pi/2)\right]\cdot(1-G_{xz}/G_{yz})}{\left(\cos^2(\alpha+\pi/2)+(G_{xz}/G_{yz})\sin^2(\alpha+\pi/2)\right)^2}.$$

Consequently, the points of the extremum of functions $G_{xz}^*(\alpha)$ and $G_{yz}^*(\alpha)$:

$$\left(G_{xz}^*(\alpha)\right)'=0 \text{ at } \alpha=\frac{\pi}{2}\cdot n,\ n\in Z;$$

$$\left(G_{yz}^*(\alpha)\right)'=0 \text{ at } \alpha=\frac{\pi}{2}\cdot n,\ n\in Z.$$

The second derivative from function (9)

$$\left(G_{xz}^*(\alpha)\right)''=2G_{xz}\frac{(1-G_{xz}/G_{yz})}{\left(\cos^2\alpha+(G_{xz}/G_{yz})\sin^2\alpha\right)^2}\times$$
$$\times\left[\frac{\sin^2 2\alpha(1-G_{xz}/G_{yz})}{\left(\cos^2\alpha+(G_{xz}/G_{yz})\sin^2\alpha\right)}+\cos 2\alpha\right];$$

$$\left(G_{yz}^*(\alpha)\right)''=2G_{xz}\frac{(1-G_{xz}/G_{yz})}{\left(\cos^2(\alpha+\pi/2)+(G_{xz}/G_{yz})\sin^2(\alpha+\pi/2)\right)^2}\times$$
$$\times\left[\frac{\sin^2\left[2(\alpha+\pi/2)\right]\cdot(1-G_{xz}/G_{yz})}{\left(\cos^2(\alpha+\pi/2)+(G_{xz}/G_{yz})\sin^2(\alpha+\pi/2)\right)}+ +\cos\left[2(\alpha+\pi/2)\right]\right].$$

At $\alpha=\pi n,\ n\in Z$

$$\left(G_{xz}^*(\alpha)\right)''<0,\ \left(G_{yz}^*(\alpha)\right)''>0$$

therefore, $\alpha=\pi n,\ n\in Z$ are the points of maximum of function $G_{xz}^*(\alpha)$ and the points of minimum of function $G_{yz}^*(\alpha)$.

It is analogous at $\alpha=\frac{\pi}{2}\cdot(2n+1),\ n\in Z$

$$\left(G_{xz}^*(\alpha)\right)''>0,\ \left(G_{yz}^*(\alpha)\right)''<0,$$

and it means that $\alpha=\frac{\pi}{2}\cdot(2n+1),\ n\in Z$ are the points of minimum of function $G_{xz}^*(\alpha)$ and the points of maximum of function $G_{yz}^*(\alpha)$.

The obtained points of extremum determine limiting changes in the values of shear moduli of HF in the panel, and, consequently, the bearing capacity of entire three-layered structure.

When solving an engineering problem on designing a three-layered panel, nominal stacking angles of HF are taken in the interval [0; π/2]. Actually, since the functions $G_{xz}^*(\alpha)$ and $G_{yz}^*(\alpha)$ are even, then it is possible not to examine negative values of angle α. On the other hand, for $\alpha\in(\pi/2;\pi]$

$$G_{xz}^*(\alpha)=G_{xz}^*(\pi-\alpha);\ G_{yz}^*(\alpha)=G_{yz}^*(\pi-\alpha),$$

and instead of value $\alpha\in(\pi/2;\pi]$, it is possible to use value $(\pi-\alpha)\in[0;\pi/2]$ in the calculations.

In practice, nominal stacking angle of HF α can take sufficiently large values. When determining the range of maximum permissible deviations in the case when points of extremum of functions (10) happen to be inside the examined interval of values α+Δα, then they must be included in the solution as the control points.

Thus, deviations in the HF parameters from the nominal values lead to a change in the strength of a three-layered panel. The main problem that occurs in this case is the strength instability of the panel, impossibility to predict its behavior under conditions of combined loading with compressive and shear loads. The guarantee of stability of values of critical forces for a three-layered panel in the permissible limits becomes the most important task in this case. Formally, this condition is expressed by formula (11).

Quality of HF under this setting is determined by meeting the condition of stability of the strength properties at the given values of deviations on the geometric parameters of filler Δt, Δa, Δb, $\Delta\beta$, $\Delta\alpha$, which make up a set of deviations. The limiting values of parameters, which compose the set of deviations, are achieved when equality in (11) is fulfilled, and determine the maximum permissible condition of HF quality. There may exist many of such combinations; however, in each particular case it is possible to introduce additional conditions, which refine the solution.

5. Results of study of the influence of deviations in the parameters of honeycomb filler on the bearing capacity of a three-layered panel

It is established in the course of separate examination of the influence of deviations Δt and Δb on the bearing capacity of panel that the larger compression predominates over the shear, the more precise the production of honeycomb material must be. However, since the precision of manufacturing a material is difficult to improve, then the loss of bearing capacity should be compensated for by other parameters – side of the cell, HF stacking angle.

A magnitude of the free side of a cell also affects the bearing capacity of a three-layered panel, an increase in the free side of cell leads to the strengthening of panel $\Delta N_{12}>0$, while the decrease – to the reduction in its strength $\Delta N_{12}<0$. Also, similarly for thickness of the material of honeycomb, with an increase in the predominance of compression over the shear, a deviation in the value of the free side of a honeycomb cell from the nominal must be minimum. In addition, since value β must be lower or equal to 90°, being connected to Δb by dependence (8), then for honeycombs of correct hexahedral form, value Δb must not take values lower than $-0.134 \cdot b_c$.

A deviation in the size of dual cell of honeycomb Δa does not practically affect the maximum available deviations, which occur at the free side of HF cell and thickness of the material. A difference in these parameters at $\Delta a=+0{,}1$mm and $\Delta a=0$ comprises less than 1 %. That is why, when computed, value Δa may be accepted equal to zero, and this deviation, when evaluating the working ability of panel under combined loading, can be disregarded.

The maximum strength of panel is reached at nominal value $\alpha=0$. For the identical compressive force at $\alpha=0$, we obtained a larger value of shear critical force than at α, different from zero.

Thus, the occurrence of deviation by honeycomb unit's stacking angle leads to the reduction in the strength of panel, and an increase in the side of cell and thickness of the HF material increases its strength. Consequently, if the stacking angle of honeycomb filler is different from zero, then, to compensate for the weakening of the panel, values Δb and Δt may (must) be increased for strengthening the panel.

6. Discussion of results of examining the influence of deviations in the parameters of honeycomb filler on the bearing capacity of a three-layered panel

Results of analysis of the influence of deviations in the parameters of HF on the bearing capacity of a three-layered panel demonstrate that, with a regard to stacking angle deviation of cell packet $\Delta\alpha$, it is possible to determine the region of permissible deviations of values Δt and Δb (Fig. 3) by the intersection of regions, obtained for $\alpha\in[\alpha;\alpha+\Delta\alpha]$. This region also determines the limiting value of the ratio of acting loads $(N_{11}/N_{12})_{\lim}$.

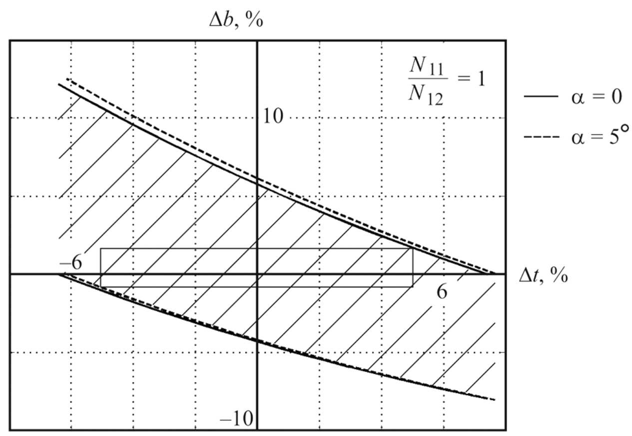

Fig. 3. Determining permissible values Δb and Δt in combination with regard to α

Practical significance of the conducted research consists in the possibility to use constructed model of strength of a three-layered panel with HF for determining the optimum dimensions of fillers, which ensure bearing capacity assigned by operating requirements. For the newly produced packing containers, this will make it possible to select the optimum characteristics of structure of the panels taking into account the planned intensity of loads and multiplicity of use. At the same time, for the worn-out containers, already used, the application of the model gives the possibility to estimate the reduction in strength of the panels and to prolong the period of their operation. Taken together, these will substantially lower expenditures for the production, operation and recycling of similar packaging materials.

The absence of simple and reliable methods for determining the magnitudes of imperfections in filler is an essential obstacle for the wide application of the obtained results. Therefore, further studies will be directed toward the development and practical implementation of methods for the nondestructive control of degree of HF wear in the process of operating the packing container.

7. Conclusions

1. A basic result of the represented study is the improvement of the model of strength of a three-layered panel taking into account the influence of deviations in HF for the case of the combined loads by the forces of compression and shear of the elements of packing and packaging. In this case, we examined a dependence of shear moduli of filler G_{xz} and G_{yz} on its geometric parameters, calculated taking into

account possible deviations in the process of manufacturing and operating the panel.

2. The analysis that we carried out of the influence of deviations in the HF parameters on the bearing capacity of a three-layered panel demonstrated that it is manifested by a substantial change in the value of critical shear in comparison with the estimated one. Therefore, for the provision of the assigned permissible values of critical shear, it is necessary that the deviations in the HF parameters are within certain interconnected ranges.

References

1. Slivinskiy, V. I. Ob'yektivnyye predposylki effektivnogo primeneniya sotovykh konstruktsiy [Text] / V. I. Slivinskiy, G. V. Tkachenko, A. N. Koloskova // Voprosy proyektirovaniya i proizvodstva konstruktsiy letatel'nykh apparatov. – 2001. – Vol. 25, Issue 2. – P. 109–115.
2. Gaydachuk, A. V. O kontseptsii kvalimetrii i upravleniya kachestvom proizvodstva sotovykh zapolniteley i konstruktsiy [Text] / A. V. Gaydachuk, V. I. Slivinskiy // Voprosy proyektirovaniya i proizvodstva konstruktsiy letatel'nykh apparatov. – 2000. – Vol. 22, Issue 5. – P. 56–64.
3. Guo, Y. Vertical static compression performance of honeycomb paperboard [Text] / Y. Guo, W. Becker, W. Xu // International Journal of Materials Research. – 2013. – Vol. 104, Issue 6. – P. 598–602. doi: 10.3139/146.110896
4. Wang, D. Mechanical property of paper honeycomb structure under dynamic compression [Text] / D. Wang, Z. Bai // Materials & Design. – 2015. – Vol. 77. – P. 59–64. doi: 10.1016/j.matdes.2015.03.037
5. Fan, T. Dynamic crushing behavior of functionally graded honeycomb structures with random defects [Text] / T. Fan // International Journal of Materials Research. – 2016. – Vol. 107, Issue 9. – P. 783–789. doi: 10.3139/146.111403
6. Wang, D.-M. Experimental investigation into the cushioning properties of honeycomb paperboard [Text] / D.-M. Wang, Z.-W. Wang // Packaging Technology and Science. – 2008. – Vol. 21, Issue 6. – P. 309–316. doi: 10.1002/pts.808
7. Gaydachuk, A. V. Technological Defects Classification System for Sandwiched Honeycomb Composite Materials Structures [Text] / A. V. Gaydachuk. M. V. Slivinsky, V. A. Golovanevsky // Materials Forum. – 2006. – Vol. 30. – P. 96–102.
8. Slyvyns'kyy, V. Technological possibilities for increasing quality of honeycomb cores used in aerospace engineering [Text] / V. Slyvyns'kyy, M. Slyvyns'kyy, A. Gajdachuk, V. Gajdachuk, S. Melnikow, V. Kirichenko // 58th International Astronautical Congress, 2007. – P. 1–7.
9. Gaydachuk, V. Ye. O vozmozhnosti reglamentatsii defektov, voznikayushchikh v protsesse rastyazhki sotopaketa v sotoblok pri proizvodstve sotovykh zapolniteley [Text] / V. Ye. Gaydachuk, S. M. Mel'nikov // Aviatsionno-kosmicheskaya tekhnika i tekhnologiya. – 2006. – Vol. 5, Issue 31. – P. 5–10.
10. Karpikova, O. A. Sozdaniye sotovykh zapolniteley s zadannymi fiziko-mekhanicheskimi kharakteristikami [Text] / O. A. Karpikova // Vísnik Dnípropetrovs'kogo universitetu. Seríya: Raketno-kosmíchna tekhníka. – 2009. – Vol. 13, Issue 2. – P. 7–10.
11. Gaydachuk, V. Ye. Metody normirovaniya dopuskov na tekhnologicheskiye parametry protsessa proizvodstva sotovykh zapolniteley iz metallicheskoy fol'gi [Text] / V. Ye. Gaydachuk, V. V. Kirichenko, S. M. Mel'nikov // Effektivnost' sotovykh konstruktsiy v izdeliyakh aviatsionno-kosmicheskoy tekhniki, 2007. – P. 87–95.
12. Iyerusalimskiy, K. M. Ustoychivost' trekhsloynykh plastin i tsilindricheskikh paneley iz kompozitsionnykh materialov pri kombinirovannom nagruzhenii [Text] / K. M. Iyerusalimskiy, Ye. N. Sinitsin // Uchenyye zapiski TSAGI. – 1973. – Vol. 4, Issue 4. – P. 65–72.
13. Panin, V. F. Konstruktsii s zapolnitelyami [Text]: spravochik / V. F. Panin, Yu. A. Gladkov. – Moscow: Mashinostroyeniye, 1991. – 272 p.
14. Slivinskiy, M. V. Klassifikatsiya tekhnologicheskikh defektov sotovykh zapolniteley iz polimernykh bumag i puti yeye realizatsii dlya povysheniya ikh kachestva [Text] / M. V. Slivinskiy // Otkrytyye informatsionnyye i komp'yuternyye integrirovannyye tekhnologii. – 2007. – Vol. 36. – P. 56–61.
15. Mel'nikov, S. M. Defekty formy yacheyki sotovogo zapolnitelya, voznikayushchiye v protsesse formoobrazovaniya sotopaketa, i ikh reglamentatsiya [Text] / S. M. Mel'nikov // Otkrytyye informatsionnyye i komp'yuternyye integrirovannyye tekhnologii. – 2006. – Vol. 32. – P. 69–75.