Виробничий процес розглядається в системі 4D, в якій фактори виробництва, що використовуються в часі, є об'єктом взаємопов'язаного $i$ взаємозалежного процесу. Встановлено, що кожному збільшенню обсягу робіт $\Delta Y i$ відповідає приріст часу $\Delta T i$ і нескінченному числу збільшень обсягу робіт $\triangle Y i$ відповідає нескінченне число збільшень часу $\triangle$ Ті. Встановлено, що сутність часу в конкретному виробничому процесі проявляється в тому, що система векторів часу колінеарна сонаправлена відповідної системі векторів обсягів робіт

Ключові слова: виробничий прощес, час у просторі 4D, виробничі фактори, коллінеарність векторів, приріст часу

Производственный процесс рассматривается в системе $4 D$, в которой факторы производства, используемые во времени, являются объектом взаимосвязанного и взаимозависимого процесса. Установлено, что каждому приращению объема работ $\Delta Y$ соответствует приращение времени $\Delta$ Ті и бесконечному числу приращений объема работ $\Delta \boldsymbol{V}$ соответствует бесконечное число приращений времени $\Delta$ Ті. Установлено, ито сущность времени в конкретном производственном процессе проявляется в том, что система векторов времени коллинеарна и сонаправлена соответствующей системе векторов объемов работ

Ключевые слова: производственный процесс, время в пространстве 4D, производственные факторы, коллинеарность векторов, приращение времени

# ANALYSIS OF ROLE OF TIME IN THE PRODUCTION PROCESS IN A 4D SPACE 

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## 1. Introduction

An intention to explain a special role of time in the development of processes, taking place in production systems (PS) under conditions of introducing advanced technologies, leads to a number of assumptions, which deserve serious attention [1, 2].

In contemporary knowledge, the concept of "time" as the initial and the undefined, in practice rests on the intuition of a researcher, on his non-reflected professional experience, on the elements of frequently subconscious ideas. It is necessary that time in the production process ( PP ) should become the object of a comprehensive study. Time, which has the properties of vector, such as direction and magnitude, can be accepted as the initial premise of this study [3].

The problems, connected with an increase in the effectiveness of using production resources over time, are of undoubted interest. The need for the solution of this problem was noted in work [4]. In particular, relative to the introduction of time parameter into the production function (PF): "If we want to base our own theory of production on the theory of "roundabout" process of Jevons-Bohm-Bawerk-Taussig, we may introduce time directly into production function, after writing down: $x=\psi\left(v_{1}, v_{2} \ldots v_{n} ; t\right) . .$. [4]. Subsequent studies are directed to giving dynamism to PF, first of all, by introduction of time factor into it [5].

Nonlinear dynamic PF, which considers the fluctuations of production factors (resources volume) over time, more adequately describes the actual production process. In the nonlinear dynamic model $\mathrm{y}=\mathrm{f}\left(\mathrm{t}, \mathrm{x}_{\mathrm{i}}(\mathrm{t}) \ldots \mathrm{x}_{\mathrm{n}}(\mathrm{t})\right.$ ), where $\mathrm{x}_{\mathrm{i}}(\mathrm{t})$ reflects the dynamics of a change in the specific production
factor depending on time. Parameter t is a time independent variable, which implicitly reflects the influence of all disregarded factors on the result of indicator $y$.

However, in light of contemporary concept of space and time, its consideration only as an independent variable implicitly would mean substantial narrowing the possibilities of prognostic calculations when introducing advanced technologies.

There are sufficiently clear tendencies to examine production process in dependence on the joint influence of different factors, which creates prerequisites for more accurate predicting calculations of the production development. According to the specialists of the USA: " A market of 3D and 4D technologies is expected to be worth USD 127.84 billion in 2016 and to grow on average by $16.17 \%$ in the period between 2016 and 2022. The year 2015 was taken as basic for analysis and forecasted period was between 2016 and 2022. The market is segmented based on technology, end users, industry and geography" [6].

## 2. Literature review and problem statement

The most widespread research is conducted in the area of optimization of the time role in the cost of an object (creation of product). For example, paper [7] gives technical and economic substantiation of the technology-designing stages of a complex of works, connected with building a motor road for the purpose of determining the influence of time on the costs of works. The project calculation included correction of risk costs, expected costs, current risk costs and final
actual costs of construction completion. It was established that depending on the combination of different factors and conditions, an increase in the cost of the project implementation may amount up to $18.5 \%$, and the term of the project completion delays by $56 \%$ [7]. The calculation of influence of totality of factors, which depend on the time of works completion, with the technical and economic substantiation is extremely promising.

Control of construction projects, as a rule, taking into account uncertainty and risk, may affect the implementation of time compromise - costs. Paper [8] proposes the model of multi-criterion optimization, which makes it possible to reach a compromise in costs and time with sufficient reliability. The results show that the person, who makes a decision, with the aid of regulating time and costs on the basis of the Pareto principle, establishes a preferable version of construction with a different acceptable risk level.

When implementing construction projects, as a rule, there appear some situations, connected with the need for performing unplanned types of work, which are necessary to complete within a rather tight time range. Solution of these problems inevitably involves an increase in the costs of work. Algorithm LLY, using linear and net graphs of solving the problem of time-costs at the assigned time range of putting an object into operation, is proposed [9]. However, in the work under consideration, an attempt to connect the production process and time in one model is not traced.

There is an experience [10] of solving the problem of optimization of selecting the material of a building structure - steel or reinforced concrete - on the basis of the Microsoft Project software. The results show that application of steel constructions makes it possible to save $55.3 \%$ of construction time, with an increase in costs by $23.1 \%$ [10]. In spite of the established connection between costs and time, the problem of the process of optimization is not set in the work.

In work [11], the optimization of using time and means is proposed to perform applying a mathematical model, based on fuzzy sets. The graph of construction and performing of unforeseen works, as well as delays in material supply, is developed. It is important that the model on the basis of verbal information includes the optimization of parameters of time, costs and supply with building materials. Under these conditions, the priorities of activity, which have minimum values with floating point, are accurately formulated. The function of belonging, including the costs as the objective of mathematical model, is developed, which makes it possible to operationally find the critical path of decreasing long delays of time and costs [11].

Paper [12] examines solution of the problem of effective combination of construction objects by means of achieving a compromise between time and costs as a promising strategy. Under conditions of limited resources the problem is formulated as linear integral program. Effectiveness and usefulness of the proposal is proved by the application of thirty objects. However, the authors emphasize that combination is an exclusively risky measure and consider it necessary to expand the model and to introduce the random estimation relative to several parameters [12]. However, the analyzed work did not establish any dependences, which allow the implementation of prognostic calculations when combining the construction objects.

Some attempts have been made to solve the production problems as a process in multidimensional space, in particu-
lar, 4D space with involvement of time. For example, according to a number of authors [13, 14], the application of a 4D model is a useful alternative to the project of tools planning, such as the CPM of networks and histograms in technical and economic substantiation. There appears an opportunity for a larger number of specialists to understand the process and rapidly identify potential problems, as well as to foresee possible time-and-space conflicts and problems. The need for improving 4D tools, which should include histograms, lists of components and annotation in their graphic user interface, is underlined. However, the authors do not give the examples of problem visualization in 4D [13].

The visualizations of transport projects as an effective method of exchanging the information between the project stakeholders are examined in [14]. In the examined applications, the visualization is used primarily for transmission of information about a geometric construction, or a photo-realistic image of transport projects. The use of 4D visualization is also effective in the implementation of road projects processes for facilitating joint decision making on planning the construction and the traffic of works. However, 4D visualization is limited to highlighting some kinds of works or operations in various colors. This approach was used for the construction of a section of a large-scale highway design in Dallas, Texas.

With an increase in the complexity of contemporary construction projects, there is an imperative need for a higher degree of computer utilization for the purpose of reaching effective planning and management [15]. The authors demarcate the previous developments and the implementation of the prototypes of the four-dimensional site managing model (4DSMM). It is proposed to perform planning on the three-dimensional computer building model (axonometric model), and specialists are offered an opportunity to plan viewing the graphs of the construction process simulation in color for the assumed period of time. Color comes out as the fourth parameter in 4D, where a specific period of time, connected with a specific stage or kind of works, corresponds to a definite color. However, time is not connected into the united model, which makes it possible to optimize resources consumption.

The 4D simulation method was developed for helping designers and better understanding the consequences of implementation of motor roads construction projects. In particular, the most important signs of influencing the environment, namely the spatial areas with continuous information and progression in the course of time, are visualized. The method was used for support of the Dutch project of highway extension [16]. In comparison with the 2D method, the proposed simulation method provides for the integral prospect of evaluating three-dimensional changes and the influence of the project on the environment in the course of time.

It should be noted that in the analyzed works, the 4D use is accomplished by providing the visual image of an object (in perspective and axonometry - 3D) with highlighting in color the sections characterizing stages and kinds of works, performed within a definite period of time. The authors of research in this field pay considerable attention to developing software of the 4 D designing process. As an example we may take Fig. 1, which reflects the sequence of changes, caused by the project influence on the environment [17]. It is noted that the shortcoming of such approach is that the detailing of separate elements is impossible (slopes, trenches, canals, etc., Fig. 1).


Fig. 1. Missing slopes of surfaces

## 4. Essence of time in the production process

For explaining the interrelation between time and a production process, we will turn to PF , for example, of the form [18]:

$$
\begin{equation*}
\mathrm{Y}=\mathrm{C}_{0} \prod_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{x}_{\mathrm{i}}^{\alpha_{i}} \tag{1}
\end{equation*}
$$

where Y is the calculated indicator (for example, the volume of work, etc., in real material or value terms); $x_{i}, i=\overline{i, n}$ are the factors (resources, for example, fixed funds, materials, labour), influencing Y (in real material or value terms); $\alpha_{i}$, $\mathrm{i}=\mathrm{i}, \mathrm{n}$ are the «weights», characterizing contribution of $\mathrm{x}_{\mathrm{i}}$ into $\mathrm{Y} ; \mathrm{C}_{0}$ is the coefficient, characterizing the joint influence of the factors, not taken into account by the model.

The given analysis shows that calculations "costs - time" and attempts to consider them in space and time (4D) do not have systems character. 4D visualization by highlighting in color of stages or kinds of works, carried out at specific time, serves as an addition to an increase in clarity of a project's graphic information. The problem is that PP is not examined in the system, in which the result of resource using over time is not an object of an interconnected and interdependent process. The examination of the problem in 4D space, in which time is reflected only in color, does not include entering the metric space - geometric interpretation of space-and-time.

In connection with this, the sense of hypothesis is in the fact that, probably, obtaining the geometric interpretation of space-and-time with examining the problems of using resources and time as an interconnected and interdependent process will make it possible to solve the problems of forecasting results of the production activity at a qualitatively higher level.


Fig. 2. 3D space model on the basis of PF, reflecting the surface with control when introducing advanced technology, characterized by different combinations of resources consumption at moving from level 1 U to level 2 U

## 3. The purpose and objectives of the study

The purpose of the study is an increase in accuracy of the prognostic calculations of production activity results when introducing advanced technologies. The goal is supposed to be achieved due to including into 4D model of the basic production resources, consumed over time and incorporated into a united model of interconnected and interdependent factors.

To achieve the set goal, the following tasks were to be solved:

- to determine the initial model for including the parameter of time in the production process, developing in a 4D space;
- to establish the essence of time in production process in interrelation and interdependence with the results of using resources;
- to prove, using specific example, the effectiveness of a 4 D model during the calculations, connected with estimating the results of resource using over time in the course of creating a product.

Let us examine a three-dimensional graphic model (Fig. 2) of the PF form (1). The function is most accessible for understanding in view of the possibility of its visual image in three-dimensional space [19, 20]. Curves 1Y and 2 Y connect the points with the identical numerical values of volume of work. Their projections $1 \mathrm{Y}_{1}, 2 \mathrm{Y}_{1}, 1 \mathrm{Y}_{2}, 2 \mathrm{Y}_{2}, 1 \mathrm{Y}_{3}$, $2 \mathrm{Y}_{3}$ are isoquants.

As it is evident in Fig. 1, from point A, characterized, for example, by volume of works 1 Y , volume 2 Y may be achieved in the specific PS via the implementation, for example, of new technology by the infinite set of combinations of labor and other resources. For example, point B, C and D, show three variants of reaching the volume of works, characterized as 2Y. Depending on possibilities of the PS, qualification of personnel, quality control, social conditions, resources quality, purpose of production system, tasks, etc., one or another variant of resources combination is selected.

Let us examine three versions of transition form 1 Y to a higher level 2Y, provided by the implementation of the new technology. Let us examine vectors $\mathbf{A B}, \mathbf{A C}$ and $\mathbf{A D}$. Each of them has its own correspondent combination of resources
use: $\mathrm{Ax}_{1}, \mathrm{Ax}_{2}, \mathrm{Bx}_{1}, \mathrm{Bx}_{2}, \mathrm{Cx}_{1}, \mathrm{Cx}_{2}, \mathrm{Dx}_{1}, \mathrm{Dx}_{2}$. We will consider the variant of $\mathbf{A B}$ vector optimum from mathematical positions, as $\mathbf{A B}{ }^{\wedge} 2 \mathrm{~V}$ and it is the shortest distance between two curves $\mathrm{Y}_{1}$ and $\mathrm{Y}_{2}$. The curve - isoquantum 2 U - connects points with numerical marks, which characterize the identical volume of works $(\mathrm{Y})$ at different combinations of production resources $\left(X_{1}\right.$ и $\left.X_{2}\right)$. From Fig. 2 it is seen that $A B<A C$ and $A B<A D$, and the variants of entering level $\mathrm{Y}_{2}$ are not equivalent by volumes of consumed resources. For example, entering point C is accompanied by a decrease in resource consumption $X_{1},\left(B_{x 1}>\mathrm{Cx}_{1}\right)$, and by an increase in the volume of resources $\mathrm{X}_{2}$, as $\mathrm{Cx}_{2}>\mathrm{B}_{\mathrm{x} 2}$. Analogous situation is connected with entering point D. However, in this situation there is an increase in consumption of resource $\mathrm{X}_{1}$ and a decrease in $\mathrm{X}_{2}$.

When simulating a process in 4 D space, it is necessary to use a symbolic record of certain transformations, which stand, as a rule, for an actual technological process. If transformations of coordinates from the fixed system to the moving one, and vice versa, are performed, this means that the transmission of saved information about an actual motion and an actual method of transformation of parameters takes place. In other words, each form of mathematical record in this case has its own correspondent method of motion and its own method of information transmission [21].

Let's assume that the start of a certain PP was fixed. Taking into account that time is connected with resources and their motion in space, the "past" of a process coincides with its beginning and time "flows" together with the process of converting resources in space. Hence it is possible to draw a conclusion that time conditionally flows in the same direction with the changes in an observed object. It is accepted to characterize motion as a generalized concept
using vectors magnitudes, and it may be assumed that the "own" time of a process will be directed collinearly with the vector of "displacement". Let us show it graphically, using the geometric interpretation of space-and-time, proposed by G. Minkowski (Fig. 3) [22].

Under the actual conditions of production, control in space and time begins from point C (from achieved result) in the direction of point D (planned result - for example, creation of a product). The actual geometric development of the process takes place in the course of time from C to $\mathrm{D}^{\prime}$. Let us note that specific time, the planned time of creating a product, corresponds to this process. Depending on the processes, connected with control and developing over time, point D of vector $\mathbf{C D}$ describes a fairly complicated trajectory (Fig. 3).

This is explained by the fact that the system of mobile equilibrium tends to change so as to reduce the effect of external influence to minimum [23]. Under the influence of a totality of factors, developing over time and not taken into account by the project, (changing conditions, delay in reaction of the managing system to changes in the production process, including overregulation [24], and other reasons) the displacement trajectory of the vector of control over time in general form can be presented, for example, in the form of the undulating surface (Fig. 3). The surface is formed on one side by straight line CC', which coincides with the planned direction of the displacement vector in space, and is characterized by $\mathrm{Y}_{1}$, previously achieved volume of product manufacturing. On the other side it is formed by the curve, characterizing the conditionally actual trajectory, described by point D , the end of vector $\mathbf{C D}$, directed toward reaching the production volume of product $\mathrm{Y}_{2}$ at point $\mathrm{D}^{\prime}$.


Fig. 3. Spatial model, illustrating temporary cross-sections of Minkowski space for vector of control CD (transformation of $\mathbf{C D}$ into $\mathbf{C}^{\prime} \mathbf{D}^{\prime}$ ) taking into account the reaction of vector of control to influence of external and internal factors over time in PS

Then for further consideration of the process, there appears the need for introduction time for completion of works, not taken into account by the project. Each executor needs his own time to perform each operation or a totality of operations.

Let us unite the axes, used for quantitative characteristic of resources, into one axis $\mathbf{O X}$. We will obtain Fig. 4, convenient for examination.


Fig. 4. Illustration of vectors of using resources over time

$$
\begin{aligned}
& \Delta \mathrm{R}=\mathrm{R}_{2}-\mathrm{R}_{1} \\
& \Delta \mathrm{Y}=\Delta \mathrm{R}-\left.\nabla \mathrm{Y}\right|_{\mathrm{x}=x_{1}}=\mathrm{R}_{2}-\mathrm{R}_{1}-\left.\nabla \mathrm{Y}\right|_{\mathrm{x}=\mathrm{x}_{1}}
\end{aligned}
$$

In this case,

$$
\nabla=\frac{\partial}{\partial \mathrm{x}_{1}} \mathrm{e}_{1}+\frac{\partial}{\partial \mathrm{x}_{2}} \mathrm{e}_{2}+\ldots \frac{\partial}{\partial \mathrm{x}_{\mathrm{n}}} \mathrm{e}_{\mathrm{n}},
$$

which proves the vector character of motion, connected with PP.

Specific scalar $\mathrm{Q}=\mathrm{f}(\mathrm{x}, \mathrm{y}, \mathrm{z})$ corresponds to each point of space $\mathrm{x}, \mathrm{y}, \mathrm{z}$, and specific vector $\mathrm{X}=\varphi(\mathrm{x}, \mathrm{y}, \mathrm{z}), \mathrm{Y}=\psi(\mathrm{x}, \mathrm{y}, \mathrm{z})$ and $\mathrm{Z}=\chi(\mathrm{x}, \mathrm{y}, \mathrm{z})$ is applied to each point of space. Scalar energy is converted into vector force by the course of time [26].

Let us turn to dependence:

$$
\begin{equation*}
\Delta \mathrm{Y}_{\mathrm{i}}=\mathrm{Q}(\mathrm{t}) \Delta \mathrm{T}_{\mathrm{i}}, \tag{2}
\end{equation*}
$$

where $\Delta \mathrm{Y}$ is the increment in volume of works; $\mathrm{Q}(\mathrm{t})$ is the productivity; $\Delta \mathrm{T}$ is the period of time in question. The increment in volume of works is equal to the numerical value, created within the specific time interval. However, it is necessary to draw a fundamental conclusion. Taking into account that $\Delta \mathbf{Y}$ is a vector, Q is a scalar, formula (2) should be represented in the form:

$$
\begin{equation*}
\Delta \mathbf{Y}_{\mathrm{i}}=\mathrm{Q}(\mathrm{t}) \Delta \mathbf{T}_{\mathrm{i}} \tag{3}
\end{equation*}
$$

since for retaining the equality in (2), it is necessary that $\Delta T_{i}$ should be a vector (tensor of the 1st rank). Hence follows $\Delta \mathbf{T}_{i} \uparrow \uparrow \Delta \mathbf{Y}_{\mathrm{i}}$, i. e. the vectors of the $\Delta \mathbf{T}_{i}$ and of an increase in gains of works volume $\Delta \mathbf{Y}_{\mathrm{i}}$, ideally are collinear and are co-directed. Consequently, the time, connected with PP, will be as uneven as the volume of production and productivity.

It also follows from formula (3) that the higher the labor productivity, the lower time consumption and the higher the gains of works volume. This is a manifestation of properties of time to be compressed and extended in PP (together with $\Delta \mathrm{Y}$ and $\mathrm{Q}(\mathrm{t})$ ). Taking into account that time is examined
in work in indissoluble connection with production factors, the question of examination it as a resource in models of PP planning is obvious.

It must be noted that reaching $\Delta \mathrm{Y}$ is always accompanied by reaching the infinite number of other $\Delta \mathrm{Y}_{\mathrm{i}}, \mathrm{i}=1,2, \ldots \infty$. For example, the completion of volume of works $\Delta \mathrm{Y}$ is accompanied by the wear of machines, a change in natural conditions, volumes of building materials, changing soil temperature, etc. $\Delta \mathrm{T}_{\mathrm{i}}$ corresponds to each $\Delta \mathrm{Y}_{\mathrm{i}}$. Consequently, the infinite number of $\Delta T_{i}$ corresponds to the infinite number of $\Delta Y_{i}$.

Hence, we can draw a conclusion. In expression (3) time, connected to specific PP, is a system of interdependent and interconnected vectors $\Delta \mathbf{T}_{\mathrm{i}}$, and $\Delta \mathrm{Y}_{\mathrm{i}}$. Then, for example, it is necessary that the vector of assigned directive time should be collinear and co-directed to the vector of performing this volume of works. Or, correspondingly, the vector of performing this volume of works corresponds to the vector of time of performing this volume. If this condition is not satisfied, it will lead to disagreement of PP and an increase in the costs for creating this product. Undulating surface, formed by the displacement of vector $\mathbf{C D}$ over time (Fig. 3) can serve as an illustration of an increase in costs for creating this product. This increase is characterized by the difference between the area of the plane, illustrating the designed PP , and the area of undulating surface, which illustrates its implementation over time.

Let us examine formula (3) in more detail.
Matrix $\mathrm{Q}(\mathrm{t})$ in general case with taking into account non-uniformity of its elements over time $\Delta \mathbf{T} \equiv\left|\Delta \mathrm{t}_{1}, \Delta \mathrm{t}_{2} \ldots \Delta \mathrm{t}_{\mathrm{n}}\right|$ may be written down in the form:

$$
\mathrm{Q}=\left|\mathrm{x}_{\mathrm{t}_{4}}^{\prime}\left(\mathrm{t}_{1}\right) ; \mathrm{x}_{\mathrm{t}_{2}}^{\prime}\left(\mathrm{t}_{2}\right) \ldots \mathrm{x}_{\mathrm{n}_{\mathrm{t}_{\mathrm{n}}}^{\prime}}^{\prime}\left(\mathrm{t}_{\mathrm{n}}\right)\right| .
$$

Then formula (3) in scalar writing will take the form:

$$
\begin{align*}
& \Delta \mathrm{Y}=\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{~K}_{\mathrm{q}} \beta_{\mathrm{i}} \Delta \mathrm{Y}_{\mathrm{i}}=\mathrm{K}_{\mathrm{q}} \beta_{1} \int_{0}^{\mathrm{t}_{1}} \frac{\mathrm{dx}_{1}}{\mathrm{dt}_{1}} \mathrm{dt}_{1}+ \\
& +\mathrm{K}_{\mathrm{q}} \beta_{2} \int_{0}^{\mathrm{t}_{2}} \int_{0}^{\mathrm{dx}_{2}} \mathrm{dt}_{2} \mathrm{dt}_{2}+\ldots+\mathrm{K}_{\mathrm{q}} \beta_{\mathrm{n}} \int_{0}^{\mathrm{t}_{2}} \frac{\mathrm{dx}_{\mathrm{n}}}{\mathrm{dt}_{\mathrm{n}}} \mathrm{dt}_{\mathrm{n}} . \tag{4}
\end{align*}
$$

$\mathrm{K}_{\mathrm{q}}$ is the coefficient of system (enterprise) functioning, determined by the formula:

$$
\mathrm{K}_{\mathrm{qj}}=\frac{\mathrm{Y}_{\mathrm{f}}}{\sum_{1}^{\mathrm{n}} \mathrm{X}_{\mathrm{i}}}
$$

where $\mathrm{Y}_{\mathrm{f}}$ is the actually completed volume of works; $\mathrm{X}_{\mathrm{i}}$ are the resources, which participate in the product creation; j is the size of sample $j=1 \ldots n$

$$
\beta_{1 \mathrm{i}}=\frac{\mathrm{X}_{1 \mathrm{i}} \cdot \mathrm{n}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{4 \mathrm{i}}} ; \beta_{2 \mathrm{i}}=\frac{\mathrm{X}_{2 \mathrm{i}} \cdot \mathrm{n}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{2 \mathrm{i}}} ; \beta_{3 \mathrm{i}}=\frac{\mathrm{X}_{3 \mathrm{i}} \cdot \mathrm{n}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{X}_{3 \mathrm{i}}}
$$

$\beta_{\mathrm{i}}$ is the coefficient, characterizing the use of resources; $n$ is the size of sample.

According to actual production conditions in formula (4) $t_{1}, t_{2} \ldots t_{n}$ may coincide or not coincide with the planned values. In they do not coincide, all upper limits of integration are selected, obviously, from the relationship: $\mathrm{t}_{\mathrm{h}}=$ $=\min \left\{\mathrm{t}_{1}, \mathrm{t}_{2}, \ldots, \mathrm{t}_{\mathrm{n}}\right\}$, as the system works only while its elements
function. Then the values of integrals, as of function $t_{h}$, will change depending on specific production conditions and, correspondingly, $|\Delta \mathbf{Y}|$ will change, which is not considered in traditional calculations with the help of PF.

## 5. Consideration of results: an example of calculation

Let us examine, for example, the calculation of parameters of PF of the form (1) of an enterprise. Where $\mathrm{X}_{1}, \mathrm{X}_{2}, \mathrm{X}_{3}$ are resources: fixed capital of an enterprise, materials and labor costs, respectively. Y is the volume of works, carried out by accepted resources (all parameters are assigned in terms of value). First, calculations of parameters PF according to traditional diagram were performed, the results of which are given in Table 1, columns 2-6. The values of $\mathrm{y}_{\mathrm{r} \mathrm{PF}}$, the actual volume of work, were obtained. Then they were compared with actual $\mathrm{Y}_{\mathrm{f}}$.

All subsequent calculations of $\mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}$ were performed according to the diagram, proposed below, which considers the
development of production process over time. The differences $\Delta_{1}$ and $\Delta_{2}$, characterizing the deviation of calculated volumes of work by the 4 D model ( $\mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}$ ) and calculated volumes of works by the PF model $\left(\mathrm{Y}_{\mathrm{rPF}}\right)$ are the initial information for determining root- mean-square errors of the calculation of unknown parameters $У$, volumes of works.

In order to take into consideration the influence of time, let us introduce the derivatives with respect to time, interpreted as the rate of change in the value of function $\dot{\mathrm{X}}_{1}=\mathrm{f}\left(\mathrm{t}_{1}\right) ; \dot{\mathrm{X}}_{2}=\mathrm{f}\left(\mathrm{t}_{2}\right) ; \dot{\mathrm{X}}_{3}=\mathrm{f}\left(\mathrm{t}_{3}\right)$. Let us select, for example, the function of the form: $\dot{X}_{i}=a t_{i}^{k}$. The function reflects the rate of the consumption of the $\mathrm{i}^{\text {th }}$ resource. Function $\dot{X}_{\mathrm{i}}=\mathrm{at}_{\mathrm{i}}^{\mathrm{k}}$ is selected on conditions of simplification of integrating process. For specific calculations, the form of the function is determined on the basis of possibilities of obtaining initial information, specific character of PP, necessary for accuracy and other conditions. In our case, these will be the functions:

$$
\dot{\mathrm{X}}_{1}=\mathrm{a}_{1} \sqrt{\mathrm{t}_{1}} ; \quad \dot{\mathrm{X}}_{2}=\mathrm{a}_{2} \sqrt{\mathrm{t}_{2}} ; \quad \dot{\mathrm{X}}_{3}=\mathrm{a}_{3} \sqrt{\mathrm{t}_{3}} .
$$

Table 1
Calculations of perspective volume of works considering the factor of time

| № of entry | Fixed assets $\mathrm{X}_{1}$ mln , Ru | $\begin{gathered} \text { Mate- } \\ \text { rials } \\ \mathrm{X}_{2} \mathrm{mln} \\ \text { RUB } \end{gathered}$ | Labour $\mathrm{X}_{3} \mathrm{mln}$, RUB | Volume of work of actual $\mathrm{Y}_{\mathrm{f}}$ $m \ln R U B$ | Volume of work of calculated $\mathrm{Y}_{\mathrm{rPF}}$ $m \ln R U B$ | $\begin{gathered} \Sigma \mathrm{X}_{\mathrm{i}} \mathrm{mln}, \\ \text { RUB } \end{gathered}$ | $\mathrm{K}_{\mathrm{q}}$ | $\beta_{1}$ | $\beta_{2}$ | $\beta_{3}$ | $\begin{aligned} & \mathrm{Y}_{\mathrm{r} 4 \mathrm{D}} \\ & \mathrm{mln}, \\ & \text { RUB } \end{aligned}$ | $\mathrm{Y}_{\mathrm{f}}-\mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}=\Delta$ |  | $\mathrm{Y}_{\mathrm{f}}-\mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}=\Delta$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  | $\begin{array}{\|c\|} \text { thousand } \\ \text { RUB } \end{array}$ | $\Delta^{2}$ | thousand RUB | $\Delta^{2}$ |
| 1 | 8767 | 21042 | 2014 | 35071 | 35009 | 31823 | 1,1 | 1,01 | 0,98 | 0,96 | 35078 | -7,4 | 54,8 | 62 | 3844 |
| 2 | 8792 | 21318 | 2019 | 35218 | 35111 | 32129 | 1,1 | 1,01 | 0,99 | 0,97 | 35362 | -145,6 | 21199,4 | 105 | 11025 |
| 3 | 8657 | 20047 | 1995 | 34113 | 34608 | 30729 | 1,1 | 0,99 | 0,93 | 0,95 | 34128 | -15 | 225 | -495 | 245025 |
| 4 | 8798 | 22109 | 2102 | 35307 | 35397 | 33009 | 1,07 | 1,01 | 1,02 | 1,00 | 35150 | 150,7 | 22710,5 | -90 | 8100 |
| 5 | 8771 | 21126 | 2017 | 35111 | 35940 | 31914 | 1,1 | 1,01 | 0,98 | 0,96 | 35101 | 9,6 | 92,2 | 71 | 5041 |
| 6 | 9026 | 23874 | 2115 | 36208 | 36248 | 35015 | 1,04 | 1,04 | 1,1 | 0,92 | 36062 | 146,5 | 21462,2 | -40 | 1600 |
| 7 | 9134 | 23525 | 2127 | 36451 | 37030 | 34786 | 1,04 | 1,05 | 1,09 | 1,02 | 36145 | 306 | 93606 | -571 | 326041 |
| 8 | 8992 | 22457 | 2118 | 35824 | 36486 | 33567 | 1,07 | 1,02 | 1,04 | 1,01 | 35733 | 91 | 8281 | -662 | 438244 |
| 9 | 8974 | 22311 | 2115 | 35626 | 35410 | 33499 | 1,07 | 1,03 | 1,03 | 1,01 | 35595 | 31 | 961 | 216 | 46656 |
| 10 | 8949 | 22186 | 2103 | 35317 | 35234 | 33238 | 1,06 | 1,03 | 1,03 | 1,00 | 35240 | -6 | 36 | 83 | 6889 |
| 11 | 7537 | 17526 | 1823 | 28614 | 28328 | 26936 | 1,06 | 0,87 | 0,81 | 0,87 | 28451 | 162,8 | 26503,8 | 291 | 84681 |
| 12 | 7849 | 19004 | 1902 | 30417 | 29969 | 28755 | 1,06 | 0,90 | 0,88 | 0,91 | 30416 | 1,2 | 1,4 | 448 | 00704 |
| 13 | 9124 | 23577 | 2219 | 38668 | 38503 | 34920 | 1,1 | 1,05 | 1,09 | 1,06 | 38322 | 345,7 | 119508,5 | 165 | 7225 |
| 14 | 8013 | 19976 | 1958 | 31118 | 30878 | 29947 | 1,04 | 0,92 | 0,93 | 0,94 | 31209 | -91,1 | 8299,2 | 240 | 7600 |
| 15 | 7916 | 18878 | 1897 | 29875 | 30341 | 28691 | 1,04 | 0,81 | 0,88 | 0,91 | 29932 | -57 | 3249 | -466 | 17156 |
| 16 | 8527 | 20007 | 1999 | 34187 | 33922 | 30533 | 1,1 | 0,98 | 0,93 | 0,96 | 33629 | 55,8 | 3113,6 | 265 | 70225 |
| 17 | 8391 | 19875 | 1876 | 33211 | 33485 | 30141 | 1,1 | 0,97 | 0,92 | 0,90 | 33158 | 52 | 2704 | -274 | 75076 |
| 18 | 9318 | 24119 | 2508 | 40506 | 40131 | 35945 | 1,1 | 1,07 | 1,12 | 1,2 | 39547 | 95,9 | 9196 | 375 | 40625 |
| 19 | 9105 | 23965 | 2417 | 37782 | 38403 | 35487 | 1,06 | 1,05 | 1,11 | 1,16 | 37607 | 175 | 30625 | -621 | 85641 |
| 20 | 8579 | 20196 | 1993 | 34525 | 34117 | 30768 | 1,1 | 0,99 | 0,94 | 0,95 | 33939 | 586 | 343396 | 408 | 66464 |
| 21 | 7958 | 19994 | 1915 | 29976 | 30342 | 29867 | 1,01 | 0,91 | 0,93 | 0,92 | 30179 | -203 | 41209 | -366 | 33956 |
| 22 | 9213 | 23341 | 2203 | 38715 | 37998 | 34757 | 1,1 | 1,06 | 1,08 | 1,05 | 38158 | 557 | 310249 | 417 | 173889 |
| 23 | 8979 | 22513 | 2107 | 35826 | 36334 | 33599 | 1,07 | 1,03 | 1,04 | 1,01 | 35826 | 0 | 0 | -508 | 258064 |
| 24 | 9452 | 24768 | 2617 | 41532 | 41344 | 36837 | 1,13 | 1,00 | 1,15 | 1,25 | 40701 | 831 | 553536 | 188 | 35344 |
| 25 | 7923 | 18986 | 1926 | 29868 | 29506 | 28835 | 1,04 | 0,91 | 0,88 | 0,92 | 29954 | -86 | 7396 | 362 | 131044 |
| 26 | 8122 | 19834 | 1983 | 31415 | 31648 | 29939 | 1.05 | 0,93 | 0,92 | 0,95 | 31996 | 19 | 361 | -233 | 54289 |
| 27 | 9247 | 23452 | 2218 | 38719 | 38250 | 34917 | 1,1 | 1,06 | 1,09 | 1,07 | 38441 | 278 | 77284 | 469 | 219961 |
| 28 | 8255 | 20189 | 2004 | 32814 | 32389 | 30448 | 1,08 | 0,95 | 0,94 | 0,96 | 32969 | -155 | 24025 | 425 | 180625 |
| 29 | 9008 | 22989 | 2128 | 37699 | 37482 | 34125 | 1,1 | 1,03 | 1,07 | 1,02 | 37565 | 134 | 17956 | 217 | 47089 |
| 30 | 9296 | 23577 | 2300 | 38589 | 38982 | 35173 | 1,1 | 1,07 | 1,09 | 1,10 | 38605 | -155 | 24025 | -393 | 154449 |

Let the planned time $\mathrm{t}_{\mathrm{pl}}$ of completing the works makes up 12 decades.

1. Let us determine the kind of function:

$$
X_{1}=\int_{0}^{t_{01}} \mathrm{a}_{1} \sqrt{\mathrm{t}_{1}} \mathrm{dt}_{1}=8689
$$

$\mathrm{X}_{1}$ is the average value of volume of fixed assets within the analyzed period (determined from Table 1, column 2). Then

$$
\left.\frac{2}{3} \mathrm{a}_{1} \sqrt{\mathrm{t}_{1}^{3}}\right|_{0} ^{12}=8689
$$

Then

$$
a_{1}=\frac{3}{2} \cdot 8689 \frac{1}{12 \sqrt{12}}=313,57 .
$$

So: $\dot{X}_{1}=313,57 \sqrt{\mathrm{t}_{1}}$.
2. Similarly:

$$
\mathrm{X}_{2}=\int_{0}^{\mathrm{t}_{\mathrm{tl}}} \mathrm{a}_{2} \sqrt{\mathrm{t}_{2}} \mathrm{dt}_{2}=21558,7
$$

$X_{2}$ is the average value of costs of materials.

$$
\begin{aligned}
& \left.\frac{2}{3} \mathrm{a}_{2} \sqrt{\mathrm{t}^{3}}{ }_{2}\right|_{0} ^{12}=21558,7 \\
& \mathrm{a}_{2}=\frac{3}{2} 21558,7 \frac{1}{12 \sqrt{12}}=778 \\
& \dot{\mathrm{X}}_{2}=778 \sqrt{\mathrm{t}_{2}} \\
& 3 . \mathrm{X}_{3}=\int_{0}^{\mathrm{t}_{10}} \mathrm{a}_{3} \sqrt{\mathrm{t}_{3}} \mathrm{dt}_{3}=2090,6 \\
& \left.\frac{2}{3} \mathrm{a}_{3} \sqrt{\mathrm{t}_{3}^{3}}\right|_{0} ^{12}=2090,6
\end{aligned}
$$

are characterizes the average value of labor costs:

$$
\begin{aligned}
& \mathrm{a}_{3}=\frac{3}{2} 2090,6 \frac{1}{12 \sqrt{12}}=75,4 ; \\
& \dot{\mathrm{X}}_{3}=75,4 \sqrt{\mathrm{t}_{3}} .
\end{aligned}
$$

Let us determine the coefficients $\beta_{1 \mathrm{i}}, \beta_{2 \mathrm{i}}, \beta_{3 \mathrm{i}}$. included in the formula (4).

For example:

$$
\begin{aligned}
& \beta_{1 \mathrm{i}}=\frac{8767}{8689}=1,14 ; \\
& \beta_{2 \mathrm{i}}=\frac{21042}{21558,7}=0,98 \\
& \beta_{3 \mathrm{i}}=\frac{2014}{2090,6}=0,96
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}=\mathrm{K}_{\mathrm{q}} \sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\beta_{\mathrm{i}} \int_{0}^{\mathrm{t}_{\mathrm{i}}} \dot{\mathrm{X}}_{\mathrm{i}} \mathrm{dt}\right)=\mathrm{K}_{\mathrm{i}} \beta_{1} \int_{0}^{\mathrm{t}_{1}} \dot{X}_{1} \mathrm{dt}_{1}+ \\
& +\mathrm{K}_{2} \beta_{2} \int_{0}^{\mathrm{t}_{\mathrm{t}}} \dot{X}_{2} \mathrm{dt}_{2}+\mathrm{K}_{3} \beta_{3} \int_{0}^{\mathrm{t}_{\mathrm{i}}} \dot{X}_{3} \mathrm{dt}_{3}+\mathrm{K}_{\mathrm{n}} \beta_{\mathrm{n}} \int_{0}^{\mathrm{t}_{1}} \dot{X}_{\mathrm{n}} \mathrm{dt}_{\mathrm{n}} .
\end{aligned}
$$

The results of calculations $\mathrm{Y}_{\mathrm{r} 4 \mathrm{D}}$ are given in column 14 of Table 1. Column 16 contains data of comparison of the results, obtained by the developed method, and those, obtained by the traditional PF $-\mathrm{Y}_{\mathrm{pf}}$.

The evaluation of accuracy of the results showed that root-mean-square error of determining volumes of works by the 4 D model: $\sigma_{4 \mathrm{D}}= \pm 252$ thousand RUB. Accuracy of calculation according to the model, which does not consider mutual and interdependent influence of time and resources, comprised $\sigma_{\mathrm{PF}}= \pm 361$ thousand RUB. The difference comprises 109 thousand RUB, or $43 \%$.

The advantage of the proposed development is an increase in accuracy of prognostic calculations, which consider the systems development of the production process overtime, approximated to actual conditions of resources consumption.

An increase in accuracy of calculations contributes to an improvement in the quality of planning the implementation of advanced technologies, to an increase in effectiveness of using resources and, therefore, to an increase in the effectiveness of the production process.

The conducted studies, based on the previously completed works [19, 22], dedicated to a qualitatively new approach to engineering development of 4D space concept of the special theory of relativity, result into the need for the solution of the problems of actual production process, connected with appearance of non-collinearity of vectors of time and gains in volume of works.

## 6. Conclusions

1. As a result of the conducted studies, we developed the 4D model, including basic production resources and the outcome of production activity, which makes it possible to systemically analyze the development of production process over time from the positions of special theory of relativity.
2. It was established that the essence of time in a specific production process is manifested in the fact that the system of vectors of an increment in time $\Delta \mathbf{T}_{i}$, is collinear to the corresponding vectors of an increment in volume of works $\Delta \mathbf{Y}_{i}$ $(\mathrm{i}=1,2, \ldots \infty)$. In this case, the vector of an increment in time $\Delta \mathbf{T}_{i}$ corresponds to each vector of an increment in volume of works $\Delta \mathbf{Y}_{\mathrm{i}}$. The infinite number of vectors of increments in time $\Delta \mathbf{T}_{\mathbf{i}}$ corresponds to the infinite number of vectors of increments in volume of works $\Delta \mathbf{Y}_{\mathrm{i}}$. It was shown that the non-uniformity of production process causes "compressibility" and "stretching" of time parameter along with changes in increments in volume of work $\Delta \mathrm{Y}$ and productivity $\mathrm{Q}(\mathrm{t})$.
3. The performed example of calculation, connected with determining the prospective volume of works in 4 D space, including basic production resources and time, showed the possibility of a considerable (up to $40 \%$ ) increase in accuracy of determining the required parameter.

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