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*Робота присвячена аналізу процесів запуску електромеханічних систем, зокрема процесам енергоспоживання і зносу устаткування. Отримано аналітичні вирази для визначення показників процесу запуску з урахуванням розподіленого в часі процесу споживання вхідних ресурсів. Обґрунтовано допустимість застосування спрощеної моделі ресурсоспоживання з зосередженими параметрами для оптимального управління процесом запуску. Отримані результати можуть бути використані при реалізації систем керованого запуску електродвигунів*

*Ключові слова: керований запуск, розподілене ресурсоспоживання, дрібно-раціональна апроксимація, показник ефективності запуску*

*Работа посвящена анализу процессов запуска электромеханических систем, в частности процессам энергопотребления и износа оборудования. Получены аналитические выражения для определения показателей процесса запуска с учетом распределенного во времени процесса потребления входных ресурсов. Обоснована допустимость применения упрощенной модели ресурсопотребления с сосредоточенными параметрами для оптимального управления процессом запуска. Полученные результаты могут быть использованы при реализации систем управляемого запуска электродвигателей*

*Ключевые слова: управляемый запуск, распределенное ресурсопотребление, дробно-рациональная аппроксимация, показатель эффективности запуска*

UDC 62–1/–9.007.005.1:62–503.5

DOI: 10.15587/1729-4061.2016.83203

# ANALYTICAL DETERMINATION OF THE ELECTROMECHANICAL SYSTEM STARTING PROCESS EFFICIENCY INDEX WITH REGARD TO THE DISTRIBUTED NATURE OF INPUT PRODUCTS CONSUMPTION

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## 1. Introduction

Starting processes of systems implemented without the use of special engineering solutions have an extreme impact on them. This provision is common, system-wide, and occurs in various fields of engineering. For example, direct-on-line starting of a powerful synchronous motor is equivalent to 500 hours of its normal operation [1]. In order to reduce these losses, special starting systems that allow reducing shock loads to an acceptable level are developed. However,

the desired result from the use of starting systems is achieved by increasing the starting process duration, which ultimately reduces the performance of both the starting process and the overall system.

The processes of starting an electromechanical system (EMS) are accompanied by peak power consumption, significant influence on the service life of electromechanical and processing equipment. Starting powerful electric drives, especially in commensurable capacity power supply networks, has a significant impact on a mains supply [2]. Starting voltage

dips lead to additional economic risks caused by underproduction in violations of a process environment of the started electric drive. The influence of starting processes, especially of high-power electric drives, on the economic performance of a company is underestimated and poorly studied.

Recognition of that significant effect of starting processes, especially of high-power electric drives, on the engineering and economic performance of a company has led to the emergence of a special class of devices that implement soft start of EMS. Soft start is a mandatory service function of modern variable-speed drives.

Heterogeneous, multi-vector impact of starting processes causes the high complexity of the problem of reasonable determination of the optimum starting mode. A considerable number of various engineering solutions for the controlled start of EMS further complicate the comparison of various options and selection of the optimum engineering solution for the controlled start of EMS.

One of the most promising approaches to the solution of the problem of estimating the starting process parameters and comparing design solutions of starting systems is the use of the starting process efficiency index [2], based on the resource conversion efficiency theory [3]. However, the known solutions do not take into account the continuous nature of resource consumption during the starting process, considering it as a simple lumped-parameter operation.

Accounting for the time-phased nature of resource consumption during starting will improve the accuracy of determining the starting process efficiency index, which causes the urgency of the present paper.

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## 2. Literature review and problem statement

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Modern scientific and technical literature pays much attention to various aspects of the problem of controlled start of EMS. The problems of torque formation when starting mechanisms with severe starting conditions and the ways to overcome it have been considered in [3, 4]. The effect of starting processes on the reliability and service life of drive electric motors and processing equipment has been examined in [5].

The issues of design and simulation of three-phase induction motor soft-start systems have been covered in [6].

The problems of power consumption in the induction motor soft-start have been studied in [7].

The application of fuzzy logic apparatus for the electric motor starting process control has been investigated in [8].

Implementation of the induction motor soft-start system with overload control of the engine started has been presented in [9].

The problems of dynamic identification and control of the induction motor soft-start have been studied in [10] using artificial intelligence techniques.

The above works consider, as a rule, the technical implementation of the induction motor soft-start systems. The starting process control amounts to improving the accuracy of implementation of a given starting trajectory, usually linear.

An approach to implementation of optimum starting process control systems that do not depend on the type of the electric motor started has been presented in [11]. An analytical expression for calculating the starting process efficiency index, which takes into account not only the whole range of impact of starting processes: electric power consumption,

reduced service life of all kinds of equipment, additional economic risks, but also the output product generated by the starting process and its duration has been suggested in [11].

However, the analytical expression of the starting process efficiency index has been obtained, provided that the starting process is represented by the elementary lumped-parameter operation [12].

This idea of the starting process is approximate, since the consumption of input products in the starting process is continuous and distributed over time. The problem of determining the starting process efficiency index for such conditions remains unresolved so far.

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## 3. Goals and objectives

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The goal of the paper is to obtain analytical expressions for determining the efficiency index of the controlled starting process taking into account the actual form of the input products consumption processes distributed in time.

To achieve the goal, the following objectives were set:

- determining an actual form of recording signals of input and output products of the EMS starting process in their value terms;
- obtaining an analytical expression for calculating the starting process efficiency index, taking into account the actual form of recording signals of input and output products of the starting process;
- identifying the errors in determining the efficiency index of the starting process when representing it in the form of lumped-parameter operation.

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## 4. Determination of an actual form of recording signals of input and output products consumed by the EMS during the starting process

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According to [11], the following types of input products are used during the EMS starting process: electric power and wear of the electromechanical and processing equipment. The useful output product, generated by the starting process, occurs in the form of kinetic energy of the EMS moving parts. The output product of the starting process is generated upon the completion of start-up, if the rated speed was reached.

In [11], input products were brought to cost estimates, summarized, integrated during the start. As a result, scalar cybernetic starting process characteristics – integrated cost estimates of input R and output P products are formed upon the completion of start-up. The expression [11] for determining the starting process efficiency index is obtained based on representing the starting process as the lumped-parameter operation, when the full magnitude of the integrated cost estimate of input products R is concentrated at a point in time corresponding to the beginning of the start. This greatly simplified approach allows the use of an analytical expression for determining the starting process efficiency index.

However, in actual practice, electric power consumption and wear of equipment are time-phased processes.

Using the mathematical model of a DC electric drive, developed by the author in [11], Fig. 1, we calculate the dependence of the instantaneous values of electric power and wear of equipment on time

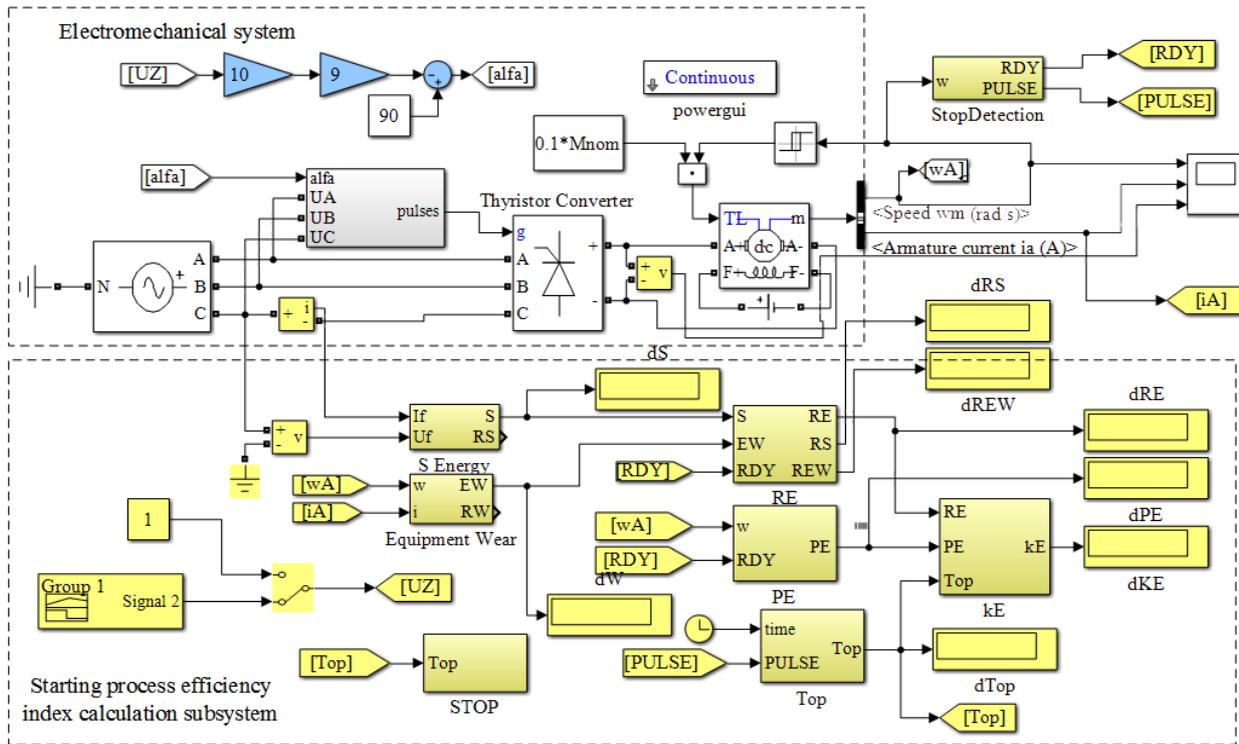


Fig. 1. The block diagram of a virtual mathematical model of the controlled starting system with a DC drive

The results are shown in Fig. 2.

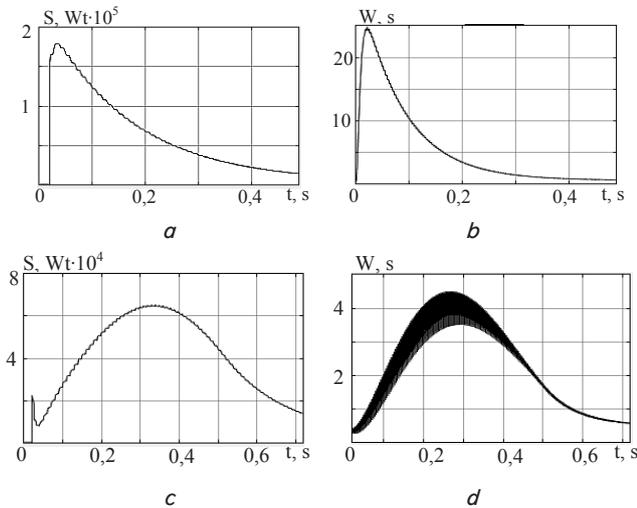


Fig. 2. The forms of dependence of electric power and wear of equipment in time during the EMS starting process (on the example of DC EMS): *a* – instantaneous power *S*, *W*, direct-on-line start; *b* – wear of equipment *W*, direct-on-line start; *c* – instantaneous power *S*, *W*, *tp*=0.5 s; *d* – wear of equipment *W*, *s*, *tp*=0.5 s

Attention is drawn to the general nature of variation of these signals over time – an increase to a maximum and then decrease to a minimum value. The presence of fluctuations in the equipment wear graphs is due to the presence of ripples. According to the analysis of the instantaneous values of the state variables of the DC electric drive during start-up, made on the mathematical model, Fig. 1, the aggregated signal of the input product cost estimate has the same characteristic shape.

To draw general conclusions, it is necessary to approximate the aggregated signal of the cost estimate of input products by an analytical expression. The approximating function shall meet the following requirements:

- the approximating function shall have sufficient accuracy in a wide range of control actions;
- the approximating function shall pass through the origin;
- the approximating function structure shall allow multiple re-integration that is needed for further calculations;
- ease of determining the approximating function parameters.

The following options of the approximating function: bell-shaped function (Gaussian function), fractional rational function, the difference between a pair of exponential functions were considered for the aggregated signal of the cost estimate of input products [13].

Based on the above requirements, the fractional rational function with the first-order numerator and the second-order denominator of the following type was taken as the approximating function

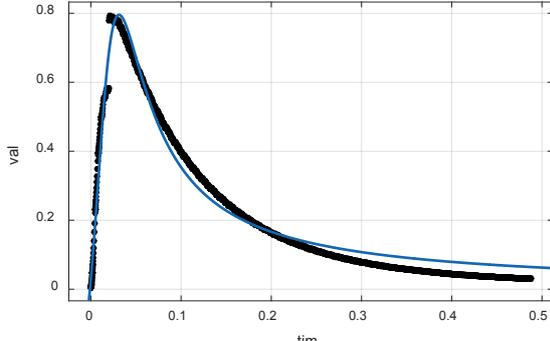
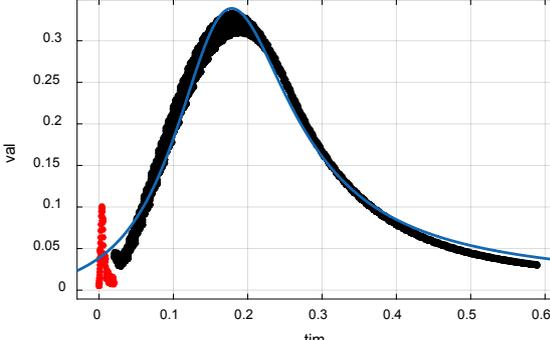
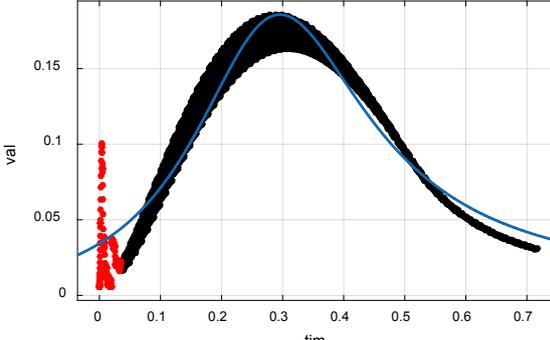
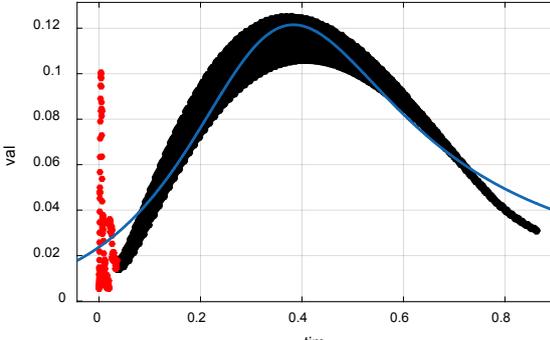
$$re(t) = \begin{cases} \frac{p_1 \cdot t}{t^2 + q_1 \cdot t + q_2}, & t \leq t_s, \\ 0, & t > t_s, \end{cases} \quad (1)$$

where  $p_1, q_1, q_2$  – fractional rational function approximation coefficients,  $t_s$  – starting process completion point.

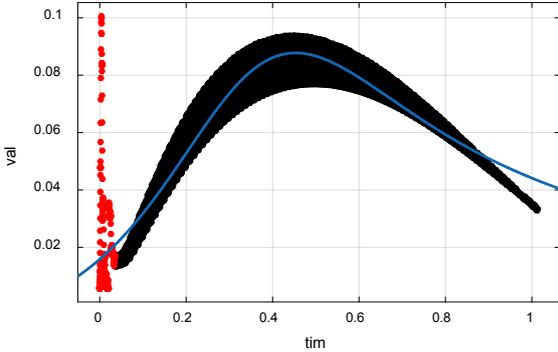
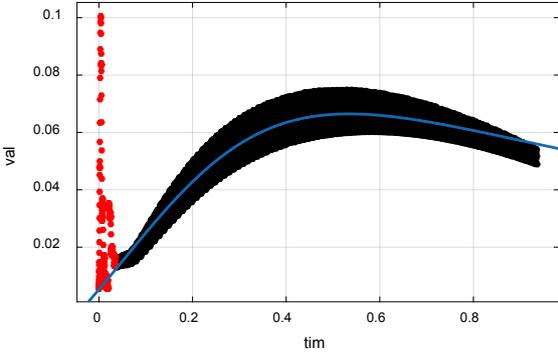
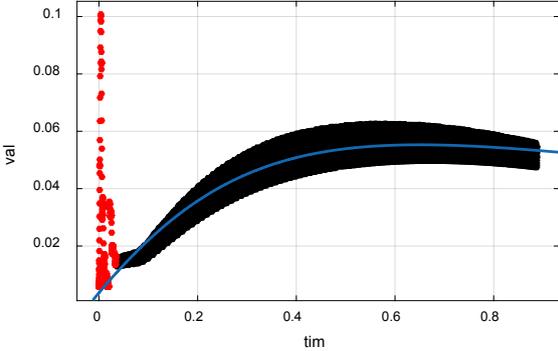
Using the mathematical model of DC electric drive (Fig. 1), valuation graphs of input products  $re(t)$  were built for various control action values. The control action  $t_u$  further refers to the sweep time of the control voltage of the adjustable rectifier from zero to a maximum value. The approximating fractional rational function coefficients (General Rat12) were obtained using the Matlab Curve Fitting Toolbox, the numerical characteristics of the resulting approximation were determined. The results are shown in Table 1.

Table 1

The results of approximation of the dependence of the cost estimate of input products  $re(t)$  for different values of the control action

tp, s ts, s	Graphs of the original signal and approximating function	Approximation quality indices
1 0.3232	<p style="text-align: center;">2</p> 	<p style="text-align: center;">3</p> <p>General model Rat12. Coefficients p1=0.02984 q1=-0.0249 q2=0.001012 Goodness of fit: SSE: 2.983 R-square: 0.9766 Adjusted R-square: 0.9766 RMSE: 0.03133</p>
0.25 0.4373		<p>General model Rat12. Coefficients p1=0.01159 q1=-0.3195 q2=0.0346 Goodness of fit: SSE: 0.8814 R-square: 0.9794 Adjusted R-square: 0.9794 RMSE: 0.01453</p>
0.5 0.5938		<p>General model Rat12. Coefficients p1=0.009799 q1=-0.5282 q2=0.09985 Goodness of fit: SSE: 0.7886 R-square: 0.9504 Adjusted R-square: 0.9504 RMSE: 0.01203</p>
0.75 0.775		<p>General model Rat12. Coefficients p1=0.01375 q1=-0.6472 q2=0.1768 Goodness of fit: SSE: 0.5787 R-square: 0.9236 Adjusted R-square: 0.9236 RMSE: 0.009158</p>

Continuation of Table 1

1	2	3
<p>1.00 0.9676</p>		<p>General model Rat12. Coefficients p1=0.02039 q1=-0.6862 q2=0.2646 Goodness of fit: SSE: 0.455 R-square: 0.8938 Adjusted R-square: 0.8937 RMSE: 0.007383</p>
<p>1.25 1.164</p>		<p>General model Rat12. Coefficients p1=0.0592 q1=-0.1786 q2=0.3105 Goodness of fit: SSE: 0.135 R-square: 0.9327 Adjusted R-square: 0.9327 RMSE: 0.004151</p>
<p>1.5 1.362</p>		<p>General model Rat12. Coefficients p1=0.09332 q1=0.3955 q2=0.4505 Goodness of fit: SSE: 0.1068 R-square: 0.9253 Adjusted R-square: 0.9253 RMSE: 0.003692</p>

According to the analysis of approximation quality indices (Table 1), the regression coefficient R in all cases is at the level of 0.9 or above, indicating a relatively high quality of the chosen method of approximation.

**5. Analytical determination of the starting process efficiency index for the case of a distributed signal of the cost estimate of input products**

After converting the recording signals of input products of the starting process into cost estimates and their aggregation, the starting process can be schematically presented as follows, Fig. 3.

Presentation of the function of the starting process output products  $pe(t)$  in the form of a pulse function in relation to the starting process is quite fair, as proved in [11].

To simplify the starting process efficiency index calculation, performed in [11], the cost estimate of input products

was also presented in the form of a pulse function, Fig. 3, *b*, pulse amplitude  $re_1$  is calculated according to the formula

$$re_1 = \int_0^{t_s} re(t) dt, \tag{2}$$

and all the starting process costs refer to the beginning of startup.

Obviously, this simplification gives us a biased estimate of the value of the starting process efficiency index.

Let us determine the exact value of the starting process efficiency index with the time-phased dependence of the cost estimate of input products, set by the above-substantiated expression (1).

To determine the starting process efficiency index with time-phased processes of consumption of input products, we use the most general expression obtained in [12]. This expression is supplemented by a term, taking into account economic risks of an enterprise associated with violations

of manufacturing processes. The economic risk function  $f_e$  has a pronounced probabilistic nature, its value is mainly due to technological environment and depends little on the starting process indices. In an extreme case, under idealized, fully deterministic conditions, the value of this function is a constant.

$$kE = \frac{A}{R} = \frac{\int_0^{t_{z+1}} (vpe(t) - vre(t)) dt - f_e}{\int_0^{t_z} (vpe(t) - vre(t)) dt} \tag{3}$$

where  $A, R$  – the potential effect and resource intensity of the starting operation, respectively [ $MU \cdot s^2$ ];  $vpe(t)$  – the resource productivity function of the starting process, determined by the expression

$$vpe(t) = \int_0^t ipe(t) dt = \int_0^t \left( \int_0^t pe(t) dt \right) dt; \tag{4}$$

$vre(t)$  – the resource consumption function of the starting process, determined by the expression

$$vre(t) = \int_0^t ire(t) dt = \int_0^t \left( \int_0^t re(t) dt \right) dt; \tag{5}$$

$t_z$  – the point of logical completion of the starting operation, determined from the equation

$$vpe(t_z) = vre(t_z); \tag{6}$$

$f_e$  – the economic risk function of the starting process.

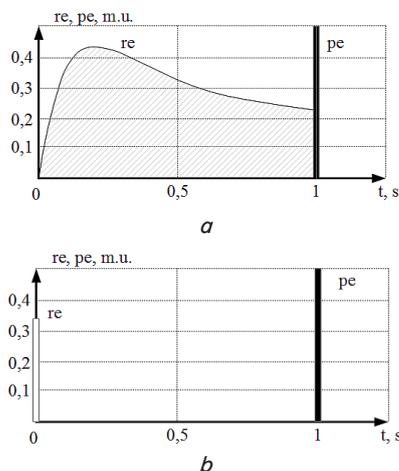


Fig. 3. The dependence of the cost estimates of input ( $re$ ) and output ( $pe$ ) products of the starting process:  
 a – actual form, distributed-parameter model;  
 b – equivalent lumped-parameter model

Variation of the form of the resource consumption signal  $re(t)$  leads to variation of absolute values of resource intensity, potential effect and efficiency index of the starting process. An important issue for optimum starting process control is only the position of the maximum efficiency index on the x-axis, rather than the absolute value of this index. This is because the optimum value of the control signal, providing the extremum does not depend on the absolute value of the starting process efficiency index.

Geometric interpretation of the expressions (3)–(6) for determining the starting process efficiency index is shown in Fig. 4.

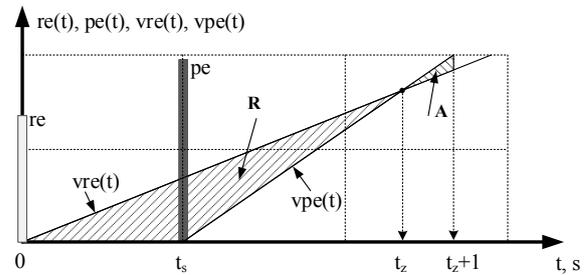


Fig. 4. Geometric interpretation of determining the starting process efficiency index

We conduct parallel computing model for options of the starting process operation model with distributed  $re(t)$  and lumped  $re_1(t)$  distribution of input products.

The function of input products of the starting process operation for lumped distribution of input products:

$$re_1(t) = \begin{cases} re_1, & t = 0, \\ 0, & t \neq 0. \end{cases} \tag{7}$$

Integral functions of input products of the starting operation

$$ire(t) = \int_0^t re(t) dt = \begin{cases} P_1 \left( \frac{-q_1 \operatorname{atan} \left( \frac{q_1 + 2t}{\sqrt{4q_2 - q_1^2}} \right) + \ln(t^2 + q_1 \cdot t + q_2)}{\sqrt{4q_2 - q_1^2}} + \frac{\ln(t^2 + q_1 \cdot t + q_2)}{2} \right) \\ -P_1 \left( \frac{-q_1 \operatorname{atan} \left( \frac{q_1}{\sqrt{4q_2 - q_1^2}} \right) + \frac{\ln(q_2)}{2}}{\sqrt{4q_2 - q_1^2}} \right), & 0 < t < t_s, \\ \beta, & t \geq t_s, \end{cases} \tag{8}$$

$$ire_1(t) = \int_0^t re_1(t) dt = \begin{cases} re_1, & t \geq t_r, \\ 0, & t < t_r \end{cases}$$

where

$$\beta = P_1 \left( \frac{-q_1 \operatorname{atan} \left( \frac{q_1 + 2t_s}{\sqrt{4q_2 - q_1^2}} \right) + \frac{\ln(t_s^2 + q_1 \cdot t_s + q_2)}{2}}{\sqrt{4q_2 - q_1^2}} \right) - P_1 \left( \frac{-q_1 \operatorname{atan} \left( \frac{q_1}{\sqrt{4q_2 - q_1^2}} \right) + \frac{\ln(q_2)}{2}}{\sqrt{4q_2 - q_1^2}} \right). \tag{9}$$

Re-integral functions of input products of the starting operation:

$$vre(t) = \int_0^t ire(t)dt, \quad vre_i(t) = \int_0^t ire_i(t)dt, \quad (10)$$

$$vre(t) = \begin{cases} \frac{P_1}{2\sqrt{4q_2 - q_1^2}} \times \\ \times \left[ \sqrt{4q_2 - q_1^2} \left[ (q_1 + t_s) \cdot \ln(t_s(q_1 + t_s) + q_2) - 2t_s \right] - \right. \\ \left. - 2[q_1^2 + q_1 \cdot t_s - 2q_2] \operatorname{atan} \frac{q_1 + 2t_s}{\sqrt{4q_2 - q_1^2}} \right] - \\ - t_s \cdot P_1 \cdot \left( \frac{\ln(q_2)}{2} - \frac{q_1 \cdot \operatorname{atan} \left( \frac{q_1}{\sqrt{4q_2 - q_1^2}} \right)}{\sqrt{4q_2 - q_1^2}} \right) - \\ - \frac{P_1}{2\sqrt{4q_2 - q_1^2}} \left[ -2(q_1^2 - 2q_2) \operatorname{atan} \frac{q_1}{\sqrt{4q_2 - q_1^2}} + \right. \\ \left. + q_1 \ln(q_2) \sqrt{4q_2 - q_1^2} \right], 0 < t < t_s, \\ \alpha + \beta(t - t_s), t \geq t_s, \end{cases} \quad (11)$$

$$vre_i(t) = \begin{cases} re_i(t - t_r), t \geq t_r, \\ 0, t < t_r, \end{cases}$$

where

$$\alpha = vre(t_s) = \frac{P_1}{2\sqrt{4q_2 - q_1^2}} \times \\ \times \left[ \sqrt{4q_2 - q_1^2} \left[ (q_1 + t_s) \cdot \ln(t_s(q_1 + t_s) + q_2) - 2t_s \right] - \right. \\ \left. - 2[q_1^2 + q_1 \cdot t_s - 2q_2] \operatorname{atan} \frac{q_1 + 2t_s}{\sqrt{4q_2 - q_1^2}} \right] - \\ - t_s \cdot P_1 \cdot \left( \frac{\ln(q_2)}{2} - \frac{q_1 \cdot \operatorname{atan} \left( \frac{q_1}{\sqrt{4q_2 - q_1^2}} \right)}{\sqrt{4q_2 - q_1^2}} \right) - \\ - \frac{P_1}{2\sqrt{4q_2 - q_1^2}} \left[ -2(q_1^2 - 2q_2) \operatorname{atan} \frac{q_1}{\sqrt{4q_2 - q_1^2}} + \right. \\ \left. + q_1 \ln(q_2) \sqrt{4q_2 - q_1^2} \right]. \quad (12)$$

The function of output products of the starting operation:

$$pe(t) = \begin{cases} pe, t = t_p, \\ 0, t \neq t_p. \end{cases} \quad (13)$$

The integral function of output products of the starting operation:

$$ipe(t) = \int_0^t pe(t)dt = \begin{cases} pe, t \geq t_p, \\ 0, t < t_p. \end{cases} \quad (14)$$

The re-integral function of output products of the starting operation:

$$vpe(t) = \int_0^t ipe(t)dt = \begin{cases} pe(t - t_p), t \geq t_p, \\ 0, t < t_p. \end{cases} \quad (15)$$

The geometric interpretation of the equation (6) for determining the point of logical completion of the operation as the point of intersection of the resource consumption and resource productivity graphs for different models of input products is shown in Fig. 5.

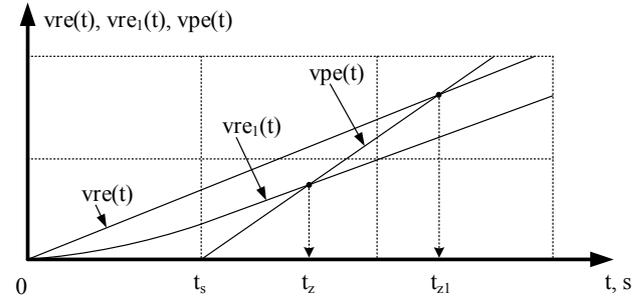


Fig. 5. Determination of the points of logical completion of the operation

In Fig. 5,  $t_s$  stands for the starting process completion point.

Let us find the point of logical completion of the distributed-parameter starting operation from the equation (6)

$$t_z = \frac{\alpha + (pe - \beta \cdot t_s)}{pe - \beta}. \quad (16)$$

Let us find the point of logical completion of the lumped-parameter starting operation from the equation (6)

$$t_{z1} = \frac{pe \cdot t_s}{pe - re_i}. \quad (17)$$

Let us determine resource consumption of the distributed-parameter starting operation according to (3) from the equation

$$R = \int_0^{t_z} (vre(t) - vpe(t))dt. \quad (18)$$

We demonstrate a geometric interpretation of the flow of consumption of input resources in the distributed-parameter starting process model, Fig. 6.

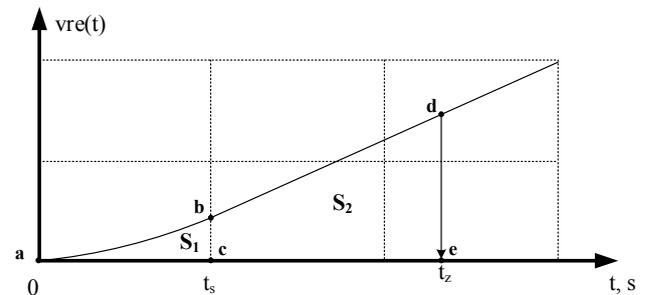


Fig. 6. Geometric interpretation of the flow of consumption of input resources

The total resource consumption of input products of the starting operation is numerically equal to the area of the curvilinear figure abdec. The area of this figure equals the sum of the area  $S_1$  of the curvilinear triangle abc and the area  $S_2$  of the rectangular trapezoid bced.

The area  $S_1$  of the curvilinear triangle abc can be calculated by the formula

$$S_1 = \int_0^{t_s} v_{re}(t) dt. \tag{19}$$

Integration in (19) provides the following expression

$$S_1 = \frac{P_1}{4\sqrt{4q_2 - q_1^2}} \left[ \sqrt{4q_2 - q_1^2} \left[ (q_1^2 + 2q_1 \cdot t_s - q_2 + t_s^2) \times \right. \right. \\ \left. \left. \times \ln(t_s(q_1 + t_s) + q_2) - t_s(2q_1 + 3t_s) \right] - \right. \\ \left. - 2 \left[ q_1^3 + 2q_1^2 t_s + q_1(t_s^2 - 3q_2) - 4q_2 \cdot t_s \right] \operatorname{atan} \frac{q_1 + 2t_s}{\sqrt{4q_2 - q_1^2}} \right] - \\ - \frac{P_1 \cdot t_s^2}{4} \left[ \ln(q_2) - \frac{2q_1 \cdot \operatorname{atan} \left( \frac{q_1}{\sqrt{4q_2 - q_1^2}} \right)}{\sqrt{4q_2 - q_1^2}} \right] - \\ - \frac{P_1 \cdot t_s}{2\sqrt{4q_2 - q_1^2}} \left[ -2(q_1^2 - 2q_2) \operatorname{atan} \frac{q_1}{\sqrt{4q_2 - q_1^2}} + \right. \\ \left. + q_1 \ln(q_2) \sqrt{4q_2 - q_1^2} \right]. \tag{20}$$

The area of the trapezoid bced, according to the equation (11) can be calculated by the formula

$$S_2 = \left( \alpha + \frac{\beta}{2}(t_z - t_s) \right) \cdot (t_z - t_s). \tag{21}$$

Finally, we obtain the following expression for the starting process resource intensity with the distributed input product:

$$R = \left( \alpha + \frac{\beta}{2}(t_z - t_s) \right) \cdot (t_z - t_s) + S_1. \tag{22}$$

Similarly, for the lumped-parameter operation

$$R_1 = \int_0^{t_{z1}} (v_{re_1}(t) - v_{pe}(t)) dt. \tag{23}$$

Integration (23) provides

$$R_1 = \frac{re_1 \cdot t_{z1}^2}{2} - \frac{pe(t_{z1} - t_s)^2}{2}. \tag{24}$$

By substituting the expression (17) in (24) for  $t_{z1}$ , we obtain

$$R_1 = \frac{re_1 \cdot pe \cdot t_s^2}{2(pe - re_1)}. \tag{25}$$

In addition to a formal approach, determined in the equation (3), the expressions for the potential effect of the

starting process can be obtained from the geometric interpretation of the resource consumption and resource productivity processes shown in Fig. 7.

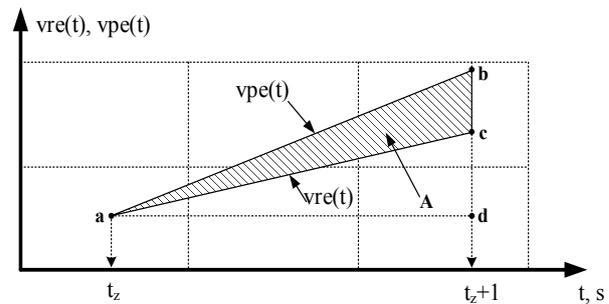


Fig. 7. Geometric interpretation of determining the potential effect of the starting process

The required potential effect of the starting process is equal to the area of the triangle  $\Delta abc$  and can be calculated as the difference between the areas of triangles  $\Delta abd$  and  $\Delta acd$ . Such an approach is absolutely accurate, since regardless of the forms of the signals  $pe(t)$  and  $re(t)$ , the signals  $vpe(t)$  and  $vre(t)$  in the region  $t > t_s$  are straight lines, which is due to conceptual features of formation of the starting operation model in cost estimates.

In general, the potential effect of the starting process can be determined by the expression

$$A = \frac{vpe(t_z + 1) - vpe(t_z)}{2} - \frac{vre(t_z + 1) - vre(t_z)}{2}. \tag{26}$$

By substituting the above expressions (11) and (15) in (26), we obtain the following expressions for the analytical determination of the potential effect of the starting process:

– for the lumped-parameter operation

$$A_1 = \frac{pe - pe_1}{2}, \tag{27}$$

– for the distributed-parameter operation

$$A = \frac{pe - \beta}{2}. \tag{29}$$

Using the expressions (3), (22), (24), we obtain the known, given in [12], expression for the lumped-parameter operation efficiency index, which confirms the correctness of the above analytical calculations.

## 6. Discussion of research results

Unfortunately, the expression (21) for the resource intensity of the distributed-parameter starting process has a fairly complex structure, since the expressions (12) and (9) are very lengthy to calculate  $\alpha$  and  $\beta$ . This does not allow performing a comparative analysis of lumped-parameter and distributed-parameter operations in general. Therefore, numerical calculations using the above formulas were carried out by means of the Matlab. Various indices of the starting process of an adjustable DC electric drive with various values of the sweep time of the control voltage were calculated. The approximation parameters of the recording signal of the

cost estimate of time-phased input products were obtained above and are presented in Table 1.

During the start, the dependence  $re(t)$  has been recorded, which was then approximated by the expression (1). After that, according to the above formulas (21), (24), (26), (27), the values of resource intensity, potential effect and efficiency indices of the lumped-parameter and distributed-parameter starting operation model were calculated. The resulting dependences of the starting process efficiency index when using different types of models are presented in Fig. 8.

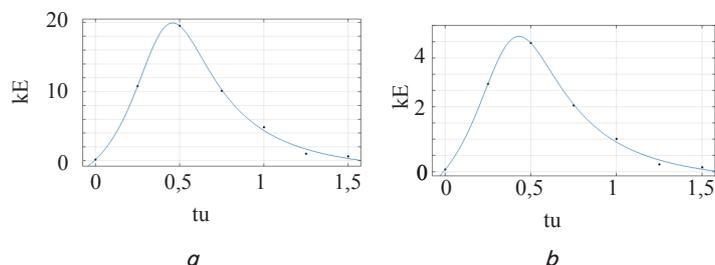


Fig. 8. Comparison of the dependence of the starting process efficiency index on the control action: *a* – in the starting process simulation by the distributed-parameter operation; *b* – in the starting process simulation by the lumped-parameter operation

Comparison of the dependencies in Fig. 7 allows drawing an important conclusion: the position of the maximum efficiency index on the x-axis when using different types of starting operation models varies slightly (within the 5 % margin of error). It should be emphasized that, although this conclusion is fully valid for the above example, its generalization for different classes of electric drives and

starters is hampered by considerable volume and complexity of numerical computation. It is the subject of a separate research, which can be done in the future.

This conclusion suggests the sufficiency of using a simpler lumped-parameter starting operation model is sufficient for optimum control of starting processes.

## 7. Conclusions

1. The actual form of the recording signals of input and output products of the EMS starting process in time, having a characteristic bell-shaped form was determined by mathematical modeling on the example of an adjustable DC electric drive. The requirements for the approximating function structure, which were the basis for selecting the approximation by the fractional rational function of the class  $rat12$  with the first-order numerator and the second-order denominator were identified.

2. The analytical expressions for determining the resource consumption, potential effect and efficiency indices of the EMS starting process, the characteristic feature of which is consideration of the time-phased nature of consumption of input products of the starting process were obtained.

3. It was found that the position of the maximum efficiency index on the x-axis, obtained when using the lumped-parameter and distributed-parameter starting operation models varies slightly (within the 5 % margin of error), which allows suggesting the sufficiency of using a simpler lumped-parameter starting operation model for optimum control of starting processes.

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