

*Розглядаються особливості проведення обчислювального експерименту при дослідженні процесів перенесення тепла і маси в охолоджувачах повітря побічно-випарного типу. Передбачено розбиття проблеми на ряд задач і декомпозицію конструкції на ряд елементів. Розглядаються основні схеми руху повітря в системі каналів: перехресний струм і протитечія. Пропонується ряд вдосконалених методів спільного рішення рівнянь математичних моделей. Приведено порівняння результатів обчислювального і натурального експериментів*

*Ключові слова: непряме охолодження, випар води, тепломасоперенесення, перехресна течія, канал складного профілю, протитечія*

*Рассматриваются особенности проведения вычислительно-го эксперимента при исследовании процессов переноса тепла и массы в охладителях воздуха косвенно-испарительного типа. Предусмотрено разбиение проблемы на ряд задач и декомпозиция конструкции на ряд элементов. Рассматриваются основные схемы движения воздуха в системе каналов: перекрестный ток и противоток. Предлагается ряд усовершенствованных методов совместного решения уравнений математических моделей. Приведено сравнение результатов вычислительного и натурального экспериментов*

*Ключевые слова: косвенное охлаждение, испарение воды, тепломасоперенос, перекрестный ток, канал сложного профиля, противоток*

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# MATHEMATICAL MODELING OF AIR COOLERS OF INDIRECT EVAPORATIVE TYPE

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## 1. Introduction

Constant improvement of efficiency and minimization of energy and resource consumption of the existing and planned plants and systems are the current aspirations all over the world. In order to achieve objectives of the assigned tasks, there is an acute need in reliable methods and means for prediction of system functioning in various conditions. As a rule, full-size experiments are connected with considerable material and financial expenditures and therefore they are often virtually inapplicable.

Physical modeling is not always capable of providing reliable information on the processes in a sufficient volume.

Computing experiment based on mathematical modeling enables conduction of research in a shorter time and at smaller costs. It is particularly important to ensure sufficient reliability of the obtained results.

At present, one of priority lines of the civilized society development is bettering sanitary and hygienic conditions of habitat. For comfort air conditioning, evaporative air cooling, when water is used as a consumable, is promising. This technology is notable for its environmental friendliness and cheapness. For an effective application of the evaporative cooling technology under domestic and industrial conditions, optimization of evaporative coolers is desirable.

As it stands, development of a universal method of joint mathematical modeling of the processes taking place in the air coolers of evaporative type is of vital importance. The modeling method shall be invariant as to the structure and the conditions of operation of such coolers. Ensuring reliable results of mathematical modeling at a level of the results

common to the full-size experiments is the basic requirement to the method under development.

## 2. Literature review and problem statement

There are many papers on the subject of air coolers of evaporative type. The general approach to the problem is set forth in [1]. The theory of indirect evaporative air cooling is considered in [2]. The paper presents schemes of air flow organization and describes results of the mathematical modeling of the processes taking place in individual channels. Studies are of theoretical nature and do not ensure simulation of the entire family of the processes occurring in the coolers.

Paper [3] describes in detail techniques of evaporative air cooling (both direct and indirect ones). A number of design-related decisions were observed and the results of mathematical modeling are given. In the wet channels, film flow of evaporated water is used. Analysis of the effectiveness of the proposed solutions was made. Slot type channels were used in the cooler samples and this reduces the scope of possible designs.

Article [4] is focused on the mathematical modeling of the cross airflow scheme. The mathematical models and the research results are given. As in other studies, slot channels were only used in the cooler.

A novel regenerative air cooling method was described in [5]. The design contains movable parts, which reduces its effectiveness. Experimental studies of indirect evaporative cooling were described in [6]. Study [7] deals with the issues of optimization of the indirect evaporative air cooling pro-

cess. Mathematical modeling of the processes taking place in coolers was carried out and power efficiency of the cooler was estimated. Slot channels were only used in the cooler samples. Air cooler efficacy studies were described in [8]. Limits of temperature reduction were elucidated.

It should be noted that the channels of a simple profile (slot, tube, etc.) were considered in most papers. As a rule, there are no studies of air flows in the channels of complex profiles. Processes are typically simulated in separate groups of channels and simulation of the entire device is not carried out. Distribution of airflows in the channel system is not simulated. Therefore, there is a need for development of universal simulation method, which could be invariant to the topology of the channel system.

### 3. Research purpose and tasks

The purpose of present research consists in development of software for computational experiments in the study of air coolers of indirect evaporative type.

To realize this purpose, the following tasks were set:

- improvement of the numerical method for simulating processes taking place in the channels (in the one-dimensional formulation);
- development of a numerical method for a detailed study of the air flow processes in channels of a complex profile (in a two-dimensional formulation);
- development of a procedure of cooler decomposition to simplify unifying mathematical models;
- assessment of effectiveness of the developed modeling method through comparison of the results obtained in mathematical modeling and full-size study.

### 4. Description of the air coolers of evaporative type

Functioning of the air cooler under consideration is based on the use of the evaporative cooling effect. The main part of the cooler is the heat and mass exchange head made of a waterproof plate (film) coated with a wettable porous material. Outside air entering the dry channels is cooled thru the heat exchange with the dry surface of the plate. A part of this air is directed to the user's room and the remainder is directed to the wet channels where it is saturated with water vapor from the surface of the porous material. In some cases, the wet channels are supplied with air from the room being cooled. Warmth for water evaporation is taken from the air flowing into the dry channel. Water (fresh or slightly mineralized) is used as a consumable working agent.

In the coolers of this type, processes of heat and mass transfer are realized in a system of flat channels of a complex profile with channel width to gap ratio from 5 to 20. The airflow conditions are mostly laminar.

In the study of air coolers of indirect evaporation type, two main variants were taken into consideration:

- cross-flow scheme (Munters scheme) shown in Fig. 1;
- counterflow scheme (Neyhart scheme) shown in Fig. 2.

Irrespective of the airflow scheme, forced water feed from the top of the matrix was used.

From the viewpoint of minimizing power consumption for air pumping, Munters scheme (Fig. 1) has advantage and Neyhart scheme (Fig. 2) ensures lower temperature of the cooled air [1].

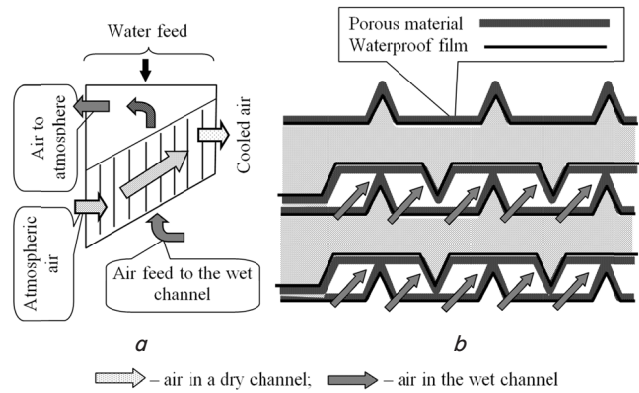


Fig. 1. Proposed channel configuration in the cross-flow scheme: *a* – general view of the scheme; *b* – detailed scheme

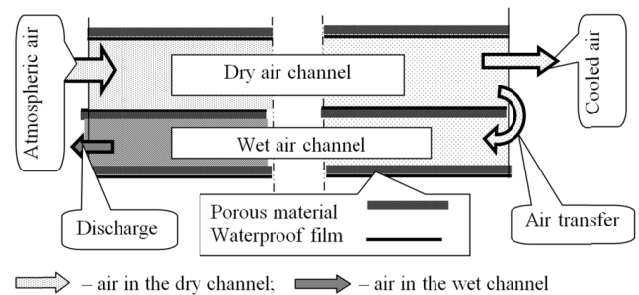


Fig. 2. Classical channel configuration in the counter-flow scheme

### 5. Mathematical modeling of the processes occurring in the cooler

In the study of the processes occurring in the heat engineering equipment, the general problem is usually split into a series of subtasks:

- study of distribution of the working media flows among the channels of the equipment elements (hydraulic problem);
- study of the processes of mass and heat transfer in the equipment elements (the problem of heat transfer).

The complexity of simulation of the processes taking place in the air coolers of indirect evaporative type consists in that the heat exchanging matrices contain a large number of channels. It should be noted that the channels (both dry and wet) are characterized by small structural dimensions from 1.5 mm to 2.5 mm for clearance and 10 mm to 30 mm for width. Furthermore, values of parameters (air temperature and humidity) differ very little in the parallel adjacent channels. A similar situation is observed in numerous plate heat exchangers.

Therefore, it is considered possible to group channels when averaging the flow parameters in the direction perpendicular to the velocity vector and to carry out simulation for the central (in the group) channel (Fig. 3).

To determine air distribution among the cooler channels, simulation was conducted using the procedure described in [9].

Study of the heat and moisture transfer processes in the calculation channels of air coolers was performed using the method of analytico-grid solution of ordinary differential equations described in [10].

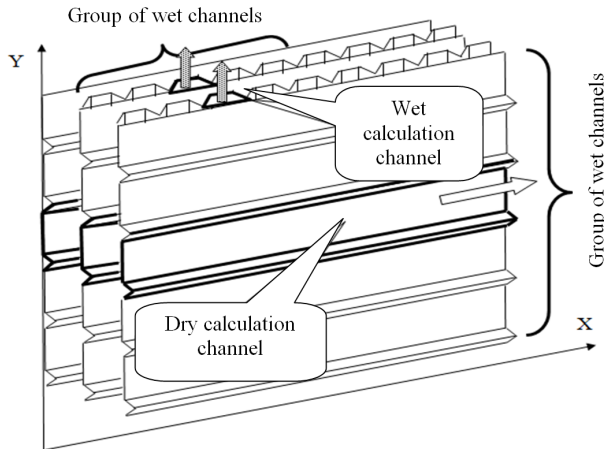


Fig. 3. Grouping of channels for the cross-flow scheme

Orientation of the media flows is shown in Fig. 4

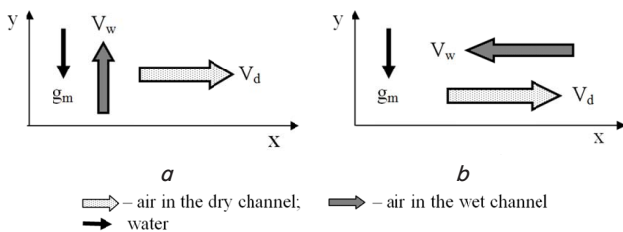


Fig. 4. Direction of air flows in the calculation channels: a – cross-flow; b – counter-flow

Mathematical models (1)–(7) were used in the study of the processes of heat and moisture transfer in the system of calculation channels.

The process of heat transfer in the dry channel was described by:

$$\delta_d \cdot v_d \cdot Cp_d \cdot \rho_d \cdot \frac{dt_d}{dx} = \sum_{n=1}^2 k_{\Sigma n} \cdot (t_{m_n} - t_d),$$

$$k_{\Sigma} = 1 / \left( \frac{1}{\alpha_d} + \frac{\delta_{wp}}{\lambda_{wp}} \right), \tag{1}$$

where  $n$  is the serial number of the enclosure surface;  $\delta_d$  is the clearance in the dry channel,  $m$ ;  $v_d$  is the calculation air velocity in the dry channel,  $m/s$ ;  $Cp_d$  is specific heat of air in the dry channel,  $J/(K \cdot kg)$ ;  $\rho_d$  is air density in the dry channel,  $kg/m^3$ ;  $t_d$  is air temperature in the dry channel,  $K$ ;  $t_m$  is air temperature in the wet channel,  $K$ ;  $k_{\Sigma}$  is the mean value of the heat transfer coefficient for the sections of the exchange surfaces;  $\alpha_d$  is calculation heat exchange coefficient in the dry channel  $W/(K \cdot m^2)$ ;  $\delta_{wp}$  is thickness of waterproofing (polyethylene film),  $m$ ;  $\lambda_{wp}$  is coefficient of the waterproofing thermal conductivity,  $W/(K \cdot m)$ .

The values of exchange coefficients are averaged for each segment of the  $x$  axis. The segments of  $x$  axis correspond to the group of wet channels.

Studies have shown that there is just a slight difference between the values of parameter on the  $x$  axis for each of the enclosure plates. So it makes sense to use the equation (1) in the form:

$$\frac{\delta_d}{2} \cdot v_d \cdot Cp_d \cdot \rho_d \cdot \frac{dt_d}{dx} = k_{\Sigma} \cdot (t_m - t_d). \tag{2}$$

A similar approach was used in subsequent models.

The process of heat transfer in the wet plate from which water is running off was described by equation:

$$g_m \cdot C_m \cdot \frac{dt_m}{dy} = -(t_d - t_m) \cdot k_{\Sigma} - \alpha_w \cdot (t_w - t_m) + m \cdot r, \tag{3}$$

where  $g_m$  is the linear water consumption,  $kg/(s \cdot m)$ ;  $\alpha_w$  is the calculation coefficient of heat exchange in the wet channel,  $W/(K \cdot m^2)$ .

Water flow in the wet plate was described by equation (4)

$$\frac{dg_m}{dy} = m, \tag{4}$$

where

$$m = \sigma (d_m - d_w);$$

$$\sigma = (\alpha_w \cdot D) / (Cp_w \cdot a_w),$$

is the calculation coefficient of mass exchange on the plate surface,  $kg/(s \cdot m)$ ;  $Cp_w$  is specific heat of air in the wet channel,  $J/(K \cdot kg)$ .

The process of heat and moisture transfer in the wet channel for the cross-flow scheme was described by equations:

$$F_w \cdot v_w \cdot Cp_w \cdot \rho_w \cdot \frac{dt_w}{dy} =$$

$$= P_w \cdot (\alpha_w \cdot (t_m - t_w) + m \cdot Cp_d \cdot t_m), \tag{5}$$

$$F_w \cdot v_w \cdot \rho_d \cdot \frac{dd_w}{dx} = P_w \cdot \sigma \cdot (d_m - d_w), \tag{6}$$

where  $F_w$ ,  $P_w$  are the passage area and the wet channel perimeter respectively;  $v_w$  is the calculation air velocity in the wet channel,  $m/s$ ;  $d_d$  is absolute humidity of air in the dry channel,  $kg/kg$ ;  $d_w$  is absolute humidity of air in the wet channel  $kg/kg$ .

The process of heat and moisture transfer in the wet channel for the counter-flow scheme was described by equations:

$$-\frac{\delta_w}{2} \cdot v_w \cdot Cp_w \cdot \rho_w \cdot \frac{dt_w}{dx} =$$

$$= \alpha_w \cdot (t_m - t_w) + m \cdot Cp_d \cdot t_m, \tag{7}$$

$$-\frac{\delta_w}{2} \cdot v_w \cdot \rho_d \cdot \frac{dd_w}{dx} = \sigma \cdot (d_m - d_w). \tag{8}$$

Thermal-physical properties of materials and working media (water, air) were assumed according to the literature sources, in particular from work [11] for wet air.

In the first approximation, the values of coefficients of heat exchange in the channels were assumed according to the recommendations of [12, 13].

To solve equations (1)–(8), an absolutely stable analytical-grid method [10] was used. The general approach was considered by the example of solution of equation (2). Equation (2) reduces to:

$$A \cdot \frac{dt_d}{dx} + t_d = t_m, \tag{9}$$

where

$$A = \frac{\delta_d \cdot v_d \cdot C_{p_d} \cdot \rho_d}{2 \cdot k_z}$$

The calculation formula is as follows:

$$t_{d_i} = D_1 \cdot t_{d_{i-1}} + D_2 \cdot (t_{m_i} + t_{m_{i-1}}), \quad (10)$$

where

$$D_1 = \exp\left(-\frac{\Delta x}{A}\right); \quad D_2 = 0.5 \cdot (1 - D_1).$$

Determination of values of the heat exchange coefficients in the dry channel having a complex shape (Fig. 5) was of especial difficulty.

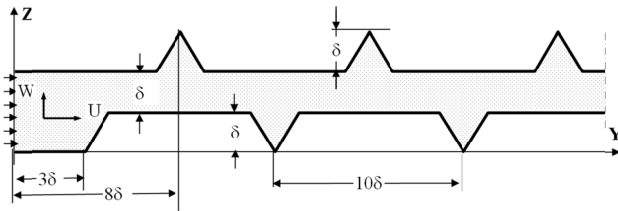


Fig. 5. Dry channel scheme

For the numerical simulation of a viscous gas flow in the dry channel at laminar- and transit-flow conditions, a modified difference method has been developed. The problem was formulated in a two-dimensional statement. The mathematical model included continuity equations (11), Navier-Stokes equation in the projection to coordinate axis (12), (13), pressure equation (14) and heat transfer equation (15):

$$\frac{\partial U}{\partial Y} + \frac{\partial W}{\partial Z} = 0, \quad (11)$$

$$\frac{1}{v} \cdot \frac{\partial U}{\partial \tau} + \frac{U}{v} \cdot \frac{\partial U}{\partial Y} + \frac{W}{v} \cdot \frac{\partial U}{\partial Z} = -\frac{1}{v \cdot \rho} \cdot \frac{\partial P}{\partial Y} + \left( \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right), \quad (12)$$

$$\frac{1}{v} \cdot \frac{\partial W}{\partial \tau} + \frac{U}{v} \cdot \frac{\partial W}{\partial Y} + \frac{W}{v} \cdot \frac{\partial W}{\partial Z} = -\frac{1}{v \cdot \rho} \cdot \frac{\partial P}{\partial Z} + \left( \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right), \quad (13)$$

$$\frac{\partial^2 P}{\partial Y^2} + \frac{\partial^2 P}{\partial Z^2} = 2 \cdot \rho \cdot \left( \frac{\partial W}{\partial Z} \cdot \frac{\partial U}{\partial Y} - \frac{\partial W}{\partial Y} \cdot \frac{\partial U}{\partial Z} \right), \quad (14)$$

$$c_p \cdot \rho \cdot \left( \frac{\partial T}{\partial \tau} + U \cdot \frac{\partial T}{\partial Y} + W \cdot \frac{\partial T}{\partial Z} \right) = \lambda \cdot \left( \frac{\partial^2 T}{\partial Y^2} + \frac{\partial^2 T}{\partial Z^2} \right). \quad (15)$$

The following notations were used: Y, Z, τ are coordinates and time respectively; U, W, P, T are the components of velocity vector, pressure and temperature; c<sub>p</sub>, λ are specific heat and thermal conductivity of air; ρ, v are density and coefficient of kinematic viscosity of air.

The following boundary conditions were taken:

– at the heat exchanging surfaces:

$$U=0, W=0, dP/dN=0, T_s=f(Y);$$

– at the channel inlet:

$$W=0, U=f(Z), T=T_0, P=P_{in};$$

– at the channel outlet:

$$P=P_{out}, W=0, d^2U/dY^2=0, d^2T/dY^2=0,$$

where P<sub>in</sub>, P<sub>out</sub> are pressures at the channel inlet and outlet respectively, N is normal to the surface.

The task of modeling the process of airflow in the channel was split into two sub-tasks: determination of the velocity field and determination of the pressure field.

Grid method was used. The diagram of indexing of the grid nodes is shown in Fig. 6.

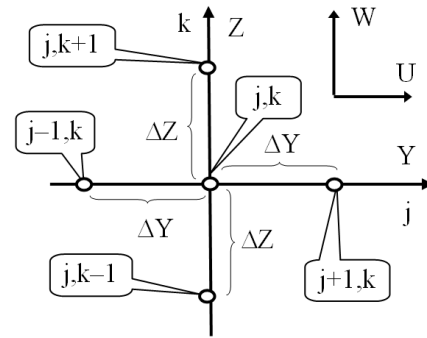


Fig. 6. Diagram of grid node indexing

In the study of the fields of velocity and temperature, a modified finite-difference method was used in which the complex including members with the second and first derivatives is approximated according to Patankar [14] by the three-point scheme:

$$\frac{\partial^2 f}{\partial x^2} - D \cdot \frac{\partial f}{\partial x} \approx \frac{(A_1 \cdot f_{i-1} + A_2 \cdot f_i + A_3 \cdot f_{i+1})}{\Delta x^2}. \quad (16)$$

The D quantity can be a solution function. Then its value is adjusted using iteration method (local iterations). The calculation relations for coefficients A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub> can be obtained by analytical solution of equation (17) at a constant value of the right side in the axis section limited by the grid nodes with indices i-1 and i+1:

$$\frac{\partial^2 f}{\partial x^2} - D \cdot \frac{\partial f}{\partial x} = \text{const}. \quad (17)$$

At the same time, it is possible to obtain (by analytical solution of (17)) a similar approximating relation for the first derivative. This is important for approximation of the continuity equation:

$$\frac{\partial f}{\partial x} \approx \frac{B_1 \cdot f_{i-1} + B_2 \cdot f_i + B_3 \cdot f_{i+1}}{\Delta x}, \quad (18)$$

where

$$A_1 = \frac{D \cdot \Delta x \cdot (C_2 - C_1)}{C_3}; \quad A_2 = -\frac{D \cdot \Delta x \cdot C_2}{C_3}; \quad A_3 = \frac{D \cdot \Delta x \cdot C_1}{C_3};$$

$$B_1 = \frac{D \cdot \Delta x \cdot (C_1 + 1) - C_2 + C_1}{C_3};$$

$$B_2 = \frac{C_2 - 2 \cdot D \cdot \Delta x \cdot (C_1 + 1)}{C_3};$$

$$B_3 = \frac{D \cdot \Delta x \cdot (C_1 + 1) - C_1}{C_3};$$

$$C_1 = \exp(D \cdot \Delta x) - 1;$$

$$C_2 = \exp(2 \cdot D \cdot \Delta x) - 1;$$

$$C_3 = C_2 - 2C_1.$$

Using above approximation schemes (16), (18), the system of equations (11)–(13) can be written down as (15)–(19). The coefficient indices  $A_1$ – $A_3$ ,  $B_1$ – $B_3$  reflect the component of the velocity vector ( $u, w$ ) and the coordinate axis ( $y, z$ ). The first time derivative is reflected by an implicit scheme.

The continuity equation (11) takes the form:

$$U_{j,k} \cdot a_1 + W_{j,k} \cdot b_1 = c_1, \tag{19}$$

where

$$a_1 = \frac{B_{2uy}}{\Delta Y};$$

$$b_1 = \frac{B_{2wz}}{\Delta Z};$$

$$c_1 = -\frac{1}{\Delta Y} \cdot (B_{1uy} \cdot U_{j-1,k} + B_{3uy} \cdot U_{j+1,k}) - \frac{1}{\Delta Z} \cdot (B_{1wz} \cdot W_{j,k-1} + B_{3wz} \cdot W_{j,k+1}).$$

Equation (12) is put in the form:

$$\left( \frac{\partial^2 U}{\partial Y^2} - D_u \cdot \frac{\partial U}{\partial Y} \right) + \left( \frac{\partial^2 U}{\partial Z^2} - D_w \cdot \frac{\partial U}{\partial Z} \right) = \frac{1}{v \cdot \rho} \cdot \frac{\partial P}{\partial Y} + \frac{1}{v} \cdot \frac{\partial U}{\partial \tau}, \tag{20}$$

where

$$D_u = U/v; \quad D_w = W/v.$$

In a compact form:

$$U_{j,k} \cdot a_2 + W_{j,k} \cdot b_2 = c_2, \tag{21}$$

where

$$a_2 = \frac{A_{2uy}}{(\Delta Y)^2} + \frac{A_{2uz}}{(\Delta Z)^2} / -\frac{1}{v \cdot \Delta \tau}; \quad b_2 = 0;$$

$$c_2 = -\frac{A_{1uy} \cdot U_{j-1,k} + A_{3uy} \cdot U_{j+1,k}}{(\Delta Y)^2} - \frac{A_{1uz} \cdot U_{j,k-1} + A_{3uz} \cdot U_{j,k+1}}{(\Delta Z)^2} + \frac{P_{j+1,k} - P_{j-1,k}}{2 \cdot v \cdot \rho \cdot \Delta Y} - \frac{U_{j,k}^*}{v \cdot \Delta \tau}.$$

Equation (13) is put in the form:

$$\left( \frac{\partial^2 W}{\partial Y^2} - D_u \cdot \frac{\partial W}{\partial Y} \right) + \left( \frac{\partial^2 W}{\partial Z^2} - D_w \cdot \frac{\partial W}{\partial Z} \right) = \frac{1}{v \cdot \rho} \cdot \frac{\partial P}{\partial Z} - \frac{1}{v} \cdot \frac{\partial W}{\partial \tau}. \tag{22}$$

Compact form:

$$U_{j,k} \cdot a_3 + W_{j,k} \cdot b_3 = c_3, \tag{23}$$

where

$$a_3 = 0; \quad b_3 = \frac{A_{2uy}}{(\Delta Y)^2} + \frac{A_{2uz}}{(\Delta Z)^2} - \frac{1}{v \cdot \Delta \tau};$$

$$c_3 = -\frac{A_{1uy} \cdot W_{j-1,k} + A_{3uy} \cdot W_{j+1,k}}{(\Delta Y)^2} - \frac{A_{1uz} \cdot W_{j,k-1} + A_{3uz} \cdot W_{j,k+1}}{(\Delta Z)^2} + \frac{P_{j,k+1} - P_{j,k-1}}{2 \cdot v \cdot \rho \cdot \Delta Z} - \frac{W_{j,k}^*}{v \cdot \Delta \tau}.$$

Using a three-point difference scheme, equation (13) is reduced to the calculation formula:

$$P_{j,k} = \frac{\Delta Z^2 \cdot (P_{j-1,k} + P_{j+1,k})}{2 \cdot (\Delta Z^2 + \Delta Y^2)} + \frac{\Delta Y^2 \cdot (P_{j,k-1} + P_{j,k+1})}{2 \cdot (\Delta Z^2 + \Delta Y^2)} + \frac{\rho \cdot \Delta Z \cdot \Delta Y}{4 \cdot (\Delta Z^2 + \Delta Y^2)} \cdot (U_{j,k+1} - U_{j,k-1}) \cdot (W_{j+1,k} - W_{j-1,k}) - \frac{\rho \cdot \Delta Z \cdot \Delta Y}{4 \cdot (\Delta Z^2 + \Delta Y^2)} \cdot (W_{j,k+1} - W_{j,k-1}) \cdot (U_{j+1,k} - U_{j-1,k}). \tag{24}$$

In simulation of the velocity-component fields  $W, U$ , it is necessary to solve the overdetermined system of equations (19), (21), (23). By minimizing functional  $\delta$  (25) according to (26) to calculate values of  $W, U$ , the system of equations (27), (28) is obtained.

$$\delta = (U_{j,k} \cdot a_1 + W_{j,k} \cdot b_1 - c_1)^2 + (U_{j,k} \cdot a_2 + W_{j,k} \cdot b_2 - c_2)^2 + (U_{j,k} \cdot a_3 + W_{j,k} \cdot b_3 - c_3)^2, \tag{25}$$

$$\frac{\partial \delta}{\partial U_{j,k}} = 0, \quad \frac{\partial \delta}{\partial W_{j,k}} = 0, \tag{26}$$

$$U_{j,k} \cdot d_1 + W_{j,k} \cdot f_1 = m_1, \tag{27}$$

$$U_{j,k} \cdot f_1 + W_{j,k} \cdot d_1 = m_2. \tag{28}$$

Taking into account  $b_2=0$  and  $a_3=0$ , obtain:

$$d_1 = a_1^2 + a_2^2, \quad f_1 = a_1 \cdot b_1,$$

$$m_1 = a_1 \cdot c_1 + a_2 \cdot c_2,$$

$$m_2 = b_1 \cdot c_1 + b_3 \cdot c_3.$$

In simulation of pressure and velocity fields, an agreed solution of equations (27), (28), (24) is required.

Modeling algorithm consists in assignment in the first approximation of  $P, W, U$  values. Then attainment of the steady-state flow conditions is modeled by an iterative link-

age of solutions (27), (28) and (24) with refining values  $d_1$ ,  $f_1$ ,  $m_1$ ,  $m_2$  at each iteration.

Later on, equation (15) is written down in the form of (29) for modeling temperature fields.

$$\left(\frac{\partial^2 T}{\partial Y^2} - D_{Ty} \cdot \frac{\partial T}{\partial Y}\right) + \left(\frac{\partial^2 T}{\partial Z^2} - D_{Tz} \cdot \frac{\partial T}{\partial Z}\right) = D_{T\tau} \cdot \frac{\partial T}{\partial \tau}, \quad (29)$$

where

$$D_{Ty} = \frac{c_p \cdot \rho \cdot U}{\lambda}; \quad D_{Tz} = \frac{c_p \cdot \rho \cdot W}{\lambda}; \quad D_{T\tau} = \frac{c_p \cdot \rho}{\lambda}.$$

In approximation of the left side of the equation (29), relations of form (16) are used. The right side of (29) is approximated by the implicit difference scheme. The calculation formula takes the form of (30).

$$T_{j,k} = \frac{\left( (A_{1ty} \cdot T_{j-1,k} + A_{3ty} \cdot T_{j+1,k}) / (\Delta Y)^2 + (A_{1tz} \cdot T_{j,k-1} + A_{3tz} \cdot T_{j,k+1}) / (\Delta Z)^2 - D_{T\tau} \cdot T_{j,k}^{n-1} \right)}{\left( A_{2ty} / (\Delta Y)^2 + A_{2tz} / (\Delta Z)^2 - D_{T\tau} / \Delta \tau \right)}, \quad (30)$$

where  $\Delta Y$ ,  $\Delta Z$ ,  $\Delta \tau$  are Y axis step, Z axis step and time step respectively.

The obtained temperature fields ensured determination of how the heat exchange coefficient values vary along the channel length. For each channel section, equal temperature values  $T_S$  on the channel walls were assumed.

Variation of the average air temperature  $T_{dj}$  was calculated by the channel cross-sections (31).

$$T_{d_j} = \left( \sum_{k=1}^{k_{max}} T_{j,k} \cdot \Delta Z \right) / \left( \sum_{k=1}^{k_{max}} \Delta Z \right). \quad (31)$$

Because of a complex periodic shape of the channel walls (Fig. 5) and taking into account the algorithm for solving equation (2), smoothing of the average temperature variation on Y coordinate was done. At the same time, the smoothed values of derivative  $dT_d/dY$  were automatically computed.

Calculation of the heat exchange coefficients  $\alpha_d$  for each channel section with index j was conducted using relation (32).

$$\alpha_{dj} = \frac{\delta_d \cdot v_d \cdot C_{p_d} \cdot \rho_d}{2 \cdot (T_{sj} - T_{dj})} \cdot \left( \frac{dT_d}{dY} \right)_j. \quad (32)$$

The resulting relations  $Nu=f(Y/\delta)$  for some flow conditions are shown in Fig. 7.

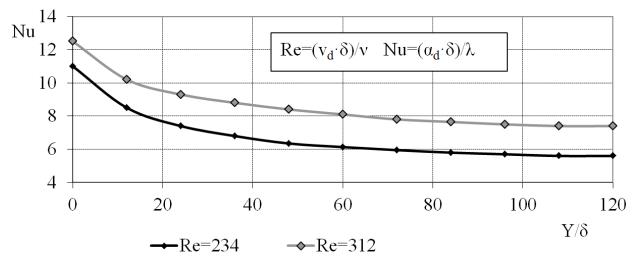


Fig. 7. Relation  $Nu=f(Y/\delta)$

In the first approximation, the values of the heat exchange coefficients in the channels were taken according to

the recommendations in [12, 13]. Next, iterative refinement of the modeling results was performed. Values of the heat exchange coefficients and the process parameters were refined in all iterations. Calculation of the two-dimensional fields of velocity and temperature was performed for the channel with a complex profile. At the same time, variation of the heat exchange coefficient values in the direction of air flow was taken into account.

## 6. Results of modeling of air coolers

The developed approach to the mathematical modeling of the processes in the air coolers enables detailed study of air temperature and humidity variation in the channels and on the wetted separation plate for various air movement schemes. Studied were carried out for the processes of a joint heat and moisture transfer in a matrix consisting of  $0.3 \times 0.3$  m plates with 0.0025 m gaps in the dry and wet channels. Thickness of the porous layer (fleece) of the separating plate was 0.3 mm and thickness of the waterproofing polyethylene film was 0.15 mm.

Atmospheric air with temperature of 40 °C and absolute humidity of 0.02 kg/kg was fed to the dry channels. The temperature of water supplied to the top of the separating plate was 25 °C. Water consumption rate was 0.00002 kg/(s·m).

Results of numerical simulation for cross scheme are given in Fig. 8, 9.

Average (consumed) velocity of air was 2.0 m/s in dry channels and 1.0 m/s in wet channels. Temperature of air pumped to the wet channels was 28 °C.

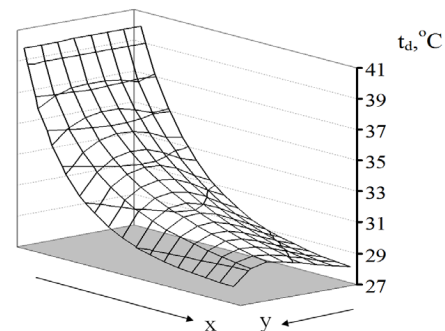


Fig. 8. Field of air temperature in the dry channel (cross-flow scheme)

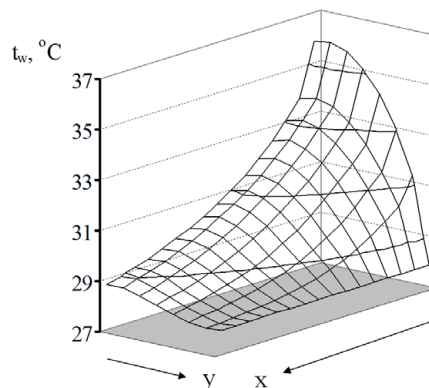


Fig. 9. Field of air temperature in the wet channel (cross-flow scheme)

The results for the counter-flow scheme are given in Fig. 10, 11.

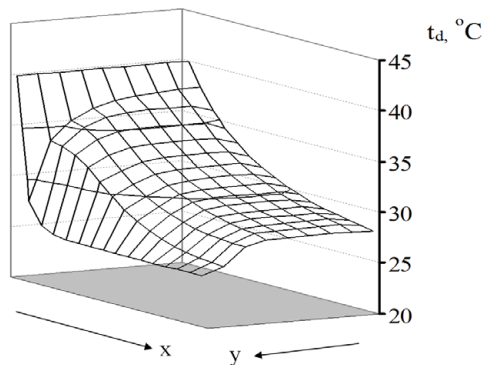


Fig. 10. Field of air temperature in the dry channel (counter-flow scheme)

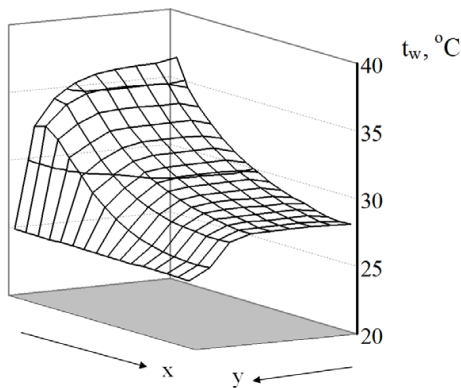


Fig. 11. Field of air temperature in the wet channel (counter-flow scheme)

A marked influence of the incoming water temperature on the temperature fields in the dry and wet channels was observed.

Check of the effectiveness of the proposed approach was carried out by comparing results of computational and full-size experiments. Tests of full-size specimens of air coolers of indirect evaporative type were carried out in laboratory of Prof. Merkt Technical Cybernetics Department, National Maritime University, Odessa, Ukraine.

It was impossible to measure temperature, humidity and air velocity directly in the channels, therefore comparison of mean values of the cooled air parameters (temperature, humidity, speed) at the cooler outlet was carried out. To ensure necessary heat and humidity condition of atmospheric air pumped to the air cooler inlet, a vessel with capacity  $V_k=32 \text{ m}^3$  was used.

The chamber was equipped with independent fans, heaters, humidifiers and dehumidifiers. At the beginning of each test cycle, the water supply unit of the air cooler was filled with consumable water from the water supply pipeline network. The values of air parameters at the cooler inlet and outlet were measured and averaged not less than 7 times for each variant of the outside air parameter specification. Measurements were carried out when the cooler reached the stationary operation mode but not earlier than after 15 minutes of use in each mode.

The inlet air parameters were measured at 5 points uniformly distributed in the inlet cross-sectional area. The

outlet air parameters were measured at 9 points uniformly distributed in the outlet cross-sectional area. The measurement results were averaged. The air velocity at the cooler inlet and outlet were measured at a distance not less than 50 mm from the inlet (outlet) cross-section.

Studies were carried out for the full-scale cooler sample with a cross-flow air movement scheme. Results of numerical and full-size experiments are given in Tables 1, 2. The following notations were used:

- $t_a, d_a$  for the atmospheric air temperature and absolute humidity respectively;
- $t_m, t_e$  for the temperatures of the cooled air obtained by mathematical modeling and experimentally, respectively;
- $\Delta t$  for the difference between the results of the full-scale and computational experiments.

Table 1

Results of mathematical and full-size experiments at lower air humidity

No.	$d_a=0.0122 \text{ kg/kg}$				$d_a=0.0154 \text{ kg/kg}$			
	$t_a, ^\circ\text{C}$	$t_m, ^\circ\text{C}$	$t_e, ^\circ\text{C}$	$\Delta t, ^\circ\text{C}$	$t_a, ^\circ\text{C}$	$t_m, ^\circ\text{C}$	$t_e, ^\circ\text{C}$	$\Delta t, ^\circ\text{C}$
1	25.2	19.2	19.5	0.3	26.0	20.9	21.2	0.3
2	27.0	19.6	19.7	0.1	27.5	21.3	21.3	0.0
3	30.4	20.4	20.2	-0.2	30.4	22.0	22.1	0.1
4	35.0	21.3	21.2	-0.1	36.0	23.2	23.0	-0.2
5	39.2	22.2	22.2	0.0	38.5	23.8	23.7	-0.1
6	42.5	22.8	22.6	-0.2	40.5	24.2	24.0	-0.2
7	44.8	23.2	22.7	-0.5	45.2	25.1	24.7	-0.4

Table 2

Results of mathematical and full-size experiments at higher air humidity

No.	$d_a=0.0182 \text{ kg/kg}$				$d_a=0.022 \text{ kg/kg}$			
	$t_a, ^\circ\text{C}$	$t_m, ^\circ\text{C}$	$t_e, ^\circ\text{C}$	$\Delta t, ^\circ\text{C}$	$t_a, ^\circ\text{C}$	$t_m, ^\circ\text{C}$	$t_e, ^\circ\text{C}$	$\Delta t, ^\circ\text{C}$
1	25.0	21.8	22.2	0.4	25.0	23.2	23.5	0.3
2	26.5	22.2	22.3	0.1	26.0	23.5	23.6	0.1
3	29.5	23.0	22.9	-0.1	30.0	24.6	24.6	0.0
4	32.0	23.6	23.5	-0.1	32.0	25.1	24.9	-0.2
5	33.5	24.0	23.9	-0.1	40.2	27.1	26.9	-0.2
6	39.4	25.3	25.1	-0.2	44.5	28.0	27.7	-0.3
7	42.5	25.9	25.7	-0.2	45.0	28.1	27.8	-0.3

The studies have shown that the agreement between the results of mathematical modeling and the full-size experiment was acceptable for practical application. The difference  $\Delta t$  in the values of the cooled air temperatures at the cooler outlet obtained in the full-size experiment ( $t_e$ ) and in simulation ( $t_m$ ) was not more than  $0.5 \text{ }^\circ\text{C}$ .

It was noted that the values of  $t_e$  exceeded the values of  $t_m$  in the region of lower values of atmospheric air temperature  $t_a$ , and in the region of higher values of  $t_a$  it was just opposite. At middle temperature values  $t_a$ , values  $t_e$  and  $t_m$  were almost identical.

Studies of the effect of the cooled air consumption on its temperature were conducted as well. The results are shown in Table 3. Relative value of air consumption  $G$  was determined with respect to the basic one when the calculation air velocity in the dry channels was  $1.5 \text{ m/s}$ . Absolute air humidity during the experiment was  $d=0.02 \text{ kg/kg}$ .

**Table 3**  
Dependence of the cooled air temperature on the cooler capacity

$t_{a,} \text{ } ^\circ\text{C}$	Relative capacity of the cooler, G					
	1		0.7		1.25	
	$t_{m,} \text{ } ^\circ\text{C}$	$t_e, \text{ } ^\circ\text{C}$	$t_{m,} \text{ } ^\circ\text{C}$	$t_e, \text{ } ^\circ\text{C}$	$t_{m,} \text{ } ^\circ\text{C}$	$t_e, \text{ } ^\circ\text{C}$
25.0	22.4	22.8	20.8	21.1	23.6	23.9
28.0	23.3	23.5	21.5	21.7	24.5	24.7
32.0	24.3	24.3	22.5	22.4	25.6	25.7
38.0	25.8	26.0	23.8	23.9	27.1	26.9
42.0	26.6	26.4	24.6	24.5	28.0	27.7
45.0	27.2	27.0	25.2	24.9	28.6	28.2

As it follows from Table 3, when the cooler capacity was changing (in the relative consumption of cooled air G), difference between temperatures  $t_e$  and  $t_m$  was also insignificant and did not exceed  $0.3 \text{ } ^\circ\text{C}$ .

### 7. Discussion of results of modeling

The results of computational and full-size experiments have shown that the proposed approach to mathematical modeling of air coolers of indirect evaporative type enables achievement of simulation reliability at the level of the full-size experiments.

Effectiveness of the analytical-grid method in the integrated simulation of interconnected transfer processes proceeding in the channel system of air coolers of indirect evaporative type should be pointed out.

The proposed method of modeling processes of media flow and heat transfer in the channels (in the two-dimensional formulation) enables obtaining of detailed fields of velocity and temperature and calculation of the heat transfer coefficients. The method is invariant to the channel profile and the working media properties.

The developed mathematical provision ensures detailed investigation of the transfer processes in the heat and mass exchange devices designed for various purposes, especially in the plate-type heat exchangers. This is especially import-

ant for optimization of operating conditions when designing heat engineering equipment.

The next step to make is development of methods of mathematical modeling of related transfer processes in the channels of various configurations (in the three-dimensional formulation) which should expand the field of application of this method.

### 8. Conclusions

1. Analytical-grid method of numerical simulation of interconnected heat and mass transfer processes in the channel system has been improved. The difference scheme corresponds to the trapezium scheme but the coefficients of the calculation formula are determined based on the analytical solution. The method is notable for its absolute stability and a minute method error.

2. The method of modeling heat and mass transfer in a channel of complex profile in the two-dimensional formulation (velocity, pressure and temperature fields) has been developed. Joint solution of continuity equations, Navier-Stokes equation (in projections to the coordinate axes) and pressure equation was realized. A new difference scheme was proposed. The method is invariant as to the channel calculation and the working medium type.

3. An efficient procedure of decomposition of the channel system topology was proposed. Its characteristic feature consists in that a single calculation channel is substituted for a group of adjacent channels. This approach reduces requirements to the simulation tool resources but it practically does not reduce validity of the information obtained.

4. Computational and full-size experiments for air cooler samples were carried out. Comparison of the results obtained in full-scale experiments and mathematical modeling has shown the possibility of ensuring mathematical modeling reliability at the level of full-scale experiments. Disagreement between the calculated and experimental values of the cooled air temperature did not exceed  $0.5 \text{ } ^\circ\text{C}$ .

The proposed method of simulation of interconnected heat and mass transfer processes in the channel system can be used in investigation of the processes taking place in various thermal engineering devices.

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*Створено комп'ютерну імітаційну модель дослідження режимів динамічної навантаженості механічних коливальних систем. Розроблена модель орієнтована на дослідження динаміки одноступінчастої евольвентної зубчастої передачі, за умови дії зовнішнього зусилля змінного характеру. Комп'ютерна модель реалізована засобами моделюючого середовища MATLAB-Simulink, з використанням принципів електронного моделювання. На основі результатів моделювання отримано оцінку динамічних зусиль у вузлах зубчастої передачі, залежно від виду функції навантаженості*

*Ключові слова: Simulink-модель, електронне моделювання, динамічне навантаження, коливальна система, зубчаста передача*

*Создана компьютерная имитационная модель исследования режимов динамической нагруженности механических колебательных систем. Разработанная модель ориентирована на исследование динамики одноступенчатой эвольвентной зубчатой передачи, при условии воздействия внешнего усилия переменного характера. Компьютерная модель реализована средствами моделирующей среды MATLAB-Simulink с использованием принципов электронного моделирования. На основе результатов моделирования получена оценка динамических усилий в узлах зубчатой передачи, в зависимости от вида функции нагруженности*

*Ключевые слова: Simulink-модель, электронное моделирование, динамическая нагрузка, колебательная система, зубчатая передача*

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# DEVELOPMENT AND APPLICATION OF COMPUTER MODEL TO STUDY THE MODES OF DYNAMIC LOADING IN MECHANICAL OSCILLATORY SYSTEMS

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## 1. Introduction

One of the most complex mechanical systems that are widely used in machine building and instrument making is different types of reducers. The functionality of such mechanical devices may be different, depending on the

type of application tasks. The most commonly used are the gearboxes with toothed gears that can be considered the typical nodes of mechanisms [1]. The main and the most responsible element of the tooth gear design is the kinematic couples with shaft sections that rotate on bearings. Weights on bearings may be considered as localized or distributed,