

*Отримано аналітичні вирази для приведення параметрів механічних систем резонаторів (трубчастих, пластини, циліндричних) до зосереджених. Отримано передавальну функцію нелінійного резонатора з урахуванням неізохронності. Розроблено імітаційну модель автоколивальної системи для основних типів механічних резонаторів вібросистемних датчиків, що мають змінний характер нелінійності. Проведено моделювання з метою визначення стабільності амплітуди і частоти автоколивань в системі*

*Ключові слова: вібросистемний датчик, автоколивальна система, нелінійний підсилювач, аналітичний критерій стійкості*

*Получены аналитические выражения для приведения параметров механических систем резонаторов (трубчатых, пластинных, цилиндрических) к сосредоточенным. Получена передаточная функция нелинейного резонатора с учетом неизохронности. Разработана имитационная модель автоколебательной системы для основных типов механических резонаторов вібросистемных датчиков, обладающих меняющимся характером нелинейности. Проведено моделирование с целью определения стабильности амплитуды и частоты автоколебаний в системе*

*Ключевые слова: вібросистемный датчик, автоколебательная система, нелинейный усилитель, аналитический критерий устойчивости*

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# DEVELOPMENT OF AUTO-OSCILLATING SYSTEM OF VIBRATION FREQUENCY SENSORS WITH MECHANICAL RESONATOR

**O. Oliynyk**

PhD, Associate Professor\*

E-mail: oleinik\_o@ukr.net

**Y. Taranenko**

Doctor of Technical Sciences, Professor, Head of Department\*

E-mail: taranen@rambler.ru

**A. Shvachka**

PhD, Associate Professor\*

E-mail: AleksandrShvachka@gmail.com

**O. Chorna**

Senior Lecturer\*

E-mail: mans@ua.fm

\*Department of Computer-integrated Technologies and Metrology

Ukrainian State University of Chemical Technology Gagarina ave., 8, Dnipro, Ukraine, 49005

## 1. Introduction

At the present stage of development of measurement equipment, accuracy of reproduction of the frequency standard is the highest in comparison with the standards of other physical magnitudes. Metrological characteristics of measuring transducers with a frequency-modulated output signal differ significantly from the characteristics of sensors with the amplitude-modulated signal [1].

By the type of physical phenomenon that underlies their operating principle, all sensors with frequency output can be divided into four groups:

- resonator;
- with non-resonating frequency-dependent systems;
- integrating;
- statistical.

Resonator sensors with AOS possess a number of advantages [2]:

- high quality factor of resonators makes it possible to substantially increase precision characteristics of sensors;
- there is a possibility to introduce a self-excited oscillator to the circuit for obtaining continuous output signal;
- signal to noise ratio is considerably higher than in the sensors of other groups given above.

The auto-oscillating systems of resonator sensors already developed are oriented toward a particular type of resonator [2]. However, all existing circuits of AOS commonly include

the following elements – mechanical resonator of the exciter and receiver of oscillations, feedback amplifier. All enumerated elements demonstrate a certain degree of nonlinearity, which makes their studying and modeling impossible. A problem of the generalized analysis of auto-oscillating systems with different types of mechanical resonators is crucial when selecting the type of resonator and the characteristics of positive feedback circuits.

## 2. Literature review and problem statement

Despite the fact that AOS is one of the basic components of vibration frequency sensors, the publications that address analysis and design of such systems are not numerous. Article [3] presents calculation for designing a mechanical resonator with variable section. The calculation is applicable only to the resonators of the type indicated. Authors [4] propose an analytical model of micromechanical resonator in the plane of bending vibrations. Data on that the model is applicable for calculating other types of resonators are missing. Paper [5] described the examined synchronized system of nano-mechanical resonators and study of the authors is devoted to the process of synchronization of the system of sensors. Article [6] explored modal self-excitation with the nonlinear feedback of acceleration in the class of mechanical systems

with distributed parameters. The authors [6] did not investigate methods of reduction to the lumped parameters. The analysis and synthesis of the modal and non-modal self-excited oscillations in the class of mechanical systems with nonlinear high-speed feedback is given in paper [7]. However, data on the use of results of study [7] for mechanical resonators of different types are lacking. The processes of oscillation damping in mechanical resonators were examined in [8]; data about the application of results in the imitation simulation are not given.

An analysis of publications testifies to the fact that the designed AOS of resonator sensors are very specific, which is why results of the studies cannot be used for mechanical resonators of other types. A reason for such situation is the complexity of analytical description of mechanical resonators with distributed parameters.

In articles [9, 10], author employed a method for reducing the distributed parameters of mechanical resonators to the lumped ones. This approach yields a certain generalization, simplifying the task on the subsequent analysis. But, at the same time, transition to the linear differential equations of oscillations of mechanical resonator does not consider the nonlinearity of lumped rigidity, related to the phenomena of nonisochronicity (dependence of frequency on the amplitude of oscillations of mechanical resonators). Nonlinearities in the characteristics of feedback amplifier are also not considered. If the selection of linear regions of exciter and receiver of oscillations is a simple engineering task, then examining the auto-oscillating system with a nonlinear resonator and nonlinear feedback necessitates the search for new solutions.

Thus, the study of auto-oscillating systems with two types of nonlinearity in the basic elements – of resonator and amplifier for attaining the required stability of amplitude and frequency of auto-oscillations (consequently, the accuracy of measurements) is an important scientific task.

### 3. The aim and tasks of the study

The aim of present work is to select a character of nonlinearity in the elements of feedback and mechanical resonator of the assigned type, which would ensure the assigned frequency stability and amplitude of auto-oscillations when designing a vibration frequency sensor.

To achieve the set aim, the following tasks are to be solved:

- to reduce basic, those employed in the sensors, types of mechanical resonators to the lumped mass of rigidity and oscillation damping;

- in the generalized linear differential equation of mechanical resonator, obtained with regard to the reduction, to consider nonisochronicity  $s$ , to obtain a transfer function of nonlinear resonator;

- to devise and explore on the simulation model a generalized auto-oscillating system with nonlinear resonator and nonlinear feedback with the purpose to select such character of nonlinearity that ensures maximum frequency and amplitude stability by suppressing the harmonics.

### 4. Synthesis of auto-oscillating system with mechanical resonator

In order to synthesize AOS with mechanical resonator (tubular, plate, and cylindrical) with the distributed mass  $m$  and rigidity  $c$ , they reduce it to the oscillatory system with lumped (equivalent) mass  $m_e$  and rigidity  $c_e$ . Basic conditions for this reduction [11]:

- equality of frequencies of natural oscillations of mechanical resonator;
- agreement between resonance frequency of the reduced mechanical resonator and oscillation frequency of MR, free from the analyzed mass;
- agreement between input resistances of the oscillatory systems of mechanical resonator, of that being reduced and of that already reduced, near the resonance frequencies.

In the systems with lumped parameters for the frequencies lower than the resonance one, the input resistance is of elastic character. For the frequencies higher than the resonance one, it is inertial. In this case, this is an agreement between coefficients of mechanical damping  $r_m$  and mechanical friction  $r_e$ .

Coefficient of mechanical friction  $r_e$  is determined by relationship [12]:

$$r_e = \frac{c_e}{2 \cdot \pi \cdot f \cdot Q}, \tag{1}$$

where  $f$  is the resonance frequency of oscillations;  $Q$  is the quality factor of oscillatory system (it is determined experimentally).

Taking into account given relationships for the equivalent rigidity of pipe, plate, cylinder, based on the first condition for reduction, we obtained the equivalent masses of mechanical resonators (Table 1).

For a standard experimental model of mechanical resonator, taking into account the peculiarities of fastening, we determine quality factor  $Q$  of the oscillatory system and, according to data in the table for the appropriate type of resonator, we determine parameters  $m_e$ ,  $c_e$ ,  $r_e$ .

$\lambda$  is determined by formula:

$$\lambda = \frac{1}{2\pi} \cdot \frac{\sqrt{\left[\pi^2 + 4 \cdot \left(\frac{L}{a}\right)^2\right]^2 + 12 \cdot \pi^4 \cdot \left(\frac{L}{a}\right)^4 \cdot (1 - \mu^2)}}{\left(\frac{h}{a}\right)^2 \cdot \left(\pi^2 + 4 \cdot \left(\frac{L}{a}\right)^2\right)}.$$

Table 1

Reduced parameters of mechanical resonators

Tubular resonator (fastened by two ends)		Plate resonator (fastened by perimeter)		Cylindrical resonator (fastened at edges)	
Reduced rigidity	Reduced mass	Reduced rigidity	Reduced mass	Reduced rigidity	Reduced mass
$\frac{192 \cdot E \cdot J}{L^3}$	$0,38 \cdot L \cdot m_t$	$\frac{4 \cdot \pi \cdot E \cdot h^3}{3 \cdot a^2 \cdot (1 - \mu^2)}$	$0,48 \cdot a \cdot m_p$	$\frac{7,4 \cdot E \cdot h^3}{a^2}$	$0,20 \cdot \frac{L^4}{a^2} \cdot \frac{1}{\lambda^2} \cdot m_c$

Note:  $E$  – modulus of elasticity;  $\mu$  – Poisson ratio;  $J$  – static moment of the tube’s inertia;  $L$  – length of tube or height of cylinder;  $a$  – radius of plate or middle radius of cylinder;  $h$  – mean lengthwise thickness of wall of tube or cylinder;  $m_t$  – mass of the unit of tube’s length;  $m_p$  – mass of the unit of plate’s area;  $m_c$  – mass of the unit of median surface of cylinder;  $\lambda$  – constant of cylindrical resonator

**5. Differential equation of resonator oscillations with consideration of nonisochronicity**

A differential equation of oscillations of the generalized mechanical resonator can be written down in the canonical form:

$$m_e \cdot \frac{d^2x}{dt^2} + r_e \cdot \frac{dx}{dt} + c_e \cdot x = F, \tag{2}$$

where  $m_e$ ,  $r_e$ ,  $c_e$ ,  $F$  are the reduced: mass, damping, rigidity and exciting lumped force, respectively.

The nonlinearity of resonator is considered by the introduction of relationship for the cubic elastic characteristic of beam [13]:

$$F_R = c_e \cdot x \pm \alpha \cdot x^3, \tag{3}$$

where  $c_e$  is the equivalent rigidity of resonator;  $\alpha$  is the coefficient of deviation of the elastic characteristic of resonator from the linear one.

The reduced nonlinearity displays the following character of manifestation:

- 1) frequency of auto-oscillations decreases with the growth in amplitude – the soft type of nonlinearity;
- 2) frequency of auto-oscillations increases with an increase in amplitude – the rigid type of nonlinearity;
- 3) at low amplitudes for the reduced types of mechanical resonators, the soft type of nonlinearity is manifested, which, with an increase in amplitude, passes into the rigid type of nonlinearity.

**6. Obtaining a transfer function of nonlinear resonator by the method of harmonic linearization**

The nonlinear part of rigidity  $\alpha \cdot x^3$  can be linearized by the substitution of harmonic function  $x_1 = \sin \omega t$ ; then, disregarding small terms of the series, we shall obtain coefficient of harmonic linearization  $q_1(a_1) = 0,75\alpha \cdot a_1^2$ .

Transfer function of resonator  $K_R \cdot W_{NR}(p)$  takes the form:

$$K_R \cdot W_{NR} = \frac{K_R \cdot W_R(p)}{1 - K_R \cdot W_R(p) \cdot q_1(a_1)} = \frac{K_R}{T_R^2 \cdot p^2 + 2 \cdot \xi \cdot T_R \cdot p + [1 \pm K_R \cdot q_1(a_1)]}, \tag{4}$$

where  $K_R \cdot W_R(p) = K_R / (T_R^2 \cdot p^2 + 2 \cdot \xi \cdot T_R \cdot p + 1)$  is the transfer function of linear part of mechanical resonator;  $\omega_R^2 = c_e / m_e$  is the square of circular frequency of oscillations of mechanical resonator;  $\xi = r_e / (2 \cdot m_e \cdot \omega_p)$  is the normalized attenuation factor of resonator;  $T_R = 1 / \omega_R$  is the period of resonator oscillations;  $K_R = 1 / (m_e \cdot \omega^2) = 1 / c_e$  is the static transfer coefficient of resonator.

A structure of the closed auto-oscillating system with a resonator will take the form, represented in Fig. 1.

In Fig. 1, the following designations are accepted:  $K_R \cdot W_R(p)$  – transfer function of the linear part of resonator;  $K_{ro} \cdot W_{ro}(p) = K_{ro} \cdot p$  – transfer function of the receiver of oscillations (for the basic types of receivers, capacitive, electromagnetic, magnetic-electric – differentiating element);  $K_{eo} \cdot W_{eo}(p) = K_{eo}$  – transfer function of oscillation exciter (for the basic types of oscillation exciters, capacitive, electromag-

netic, magnetic-electric – proportional element);  $F_{NA}(x_2)$  – nonlinear amplifier;  $F_{nr}$  – force, caused by nonisochronicity (nonlinearity of rigidity).

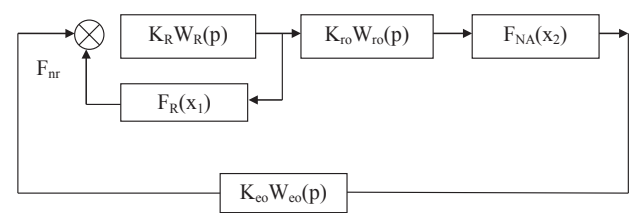


Fig. 1. Auto-oscillating system of vibration frequency sensor with mechanical resonator

**7. Imitation simulation of auto-oscillating system in the Matlab Simulink programming environment**

The main difficulty in the simulation of auto-oscillating system in the Matlab Simulink programming environment is the absence of element in the library that corresponds to a nonlinear amplifier [14, 15]. It is not possible to obtain such nonlinearity by the combination of such elements that exist in the library as, for example, clamped amplifier and dead zone or a three-position relay.

In order to search for characteristic  $F_{NA}(x_2)$  of nonlinear amplifier, we programmed a model, in which we conducted comparison of spectra of the output signals of nonlinear resonator with a standard amplifier (Fig. 2), with the examined amplifier (Fig. 3) and with a standard two-position relay.

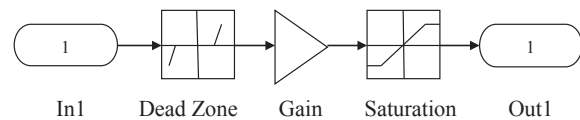


Fig. 2. Realization of amplifier with bilateral amplitude limitation in Matlab Simulink

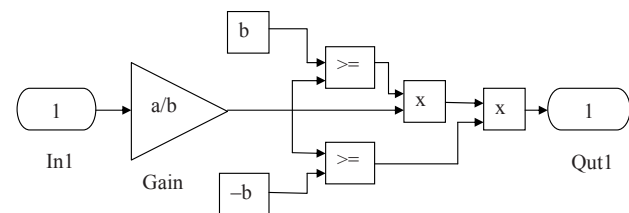


Fig. 3. Realization of amplifier with bilateral amplitude clipping in Matlab Simulink

The model is created in accordance with circuit in Fig. 1, with the use of nonlinearities shown in Fig. 2, 3 and elements from the Simulink base.

In order to reproduce results, we shall attach a fragment of the programming code

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<%1 –config_nlin =1– nonlinear amplifier bistable relay
%2 –config_nlin =2– nonlinear amplifier clipping bilateral
%3 –config_nlin =3– nonlinear amplifier clipping amplitude
config_nlin =3;
    
```

% Setting the value of the variable that determines the position of the switch

%- linear part + nonlinear amplifier –  $W_{lin} = k \cdot 1i \cdot w / ((T1 \cdot 1i \cdot w + 1) \cdot (-T2 \cdot w.^2 + 2 \cdot T2 \cdot e \cdot 1i \cdot w + 1 - m \cdot A \cdot A / w.^2) \cdot (T3 \cdot 1i \cdot w + 1))$ ;  $k=20$ ;  $m=0.01$ ;  $T1 = 0.001$ ;  $T2 = 0.0005$ ;  $T3 = 0.001$ ;  $b = 0.1$ ;  $c = 1$ ;  $e=0.001$ ;

Fig. 4 shows a simulation model of the designed auto-oscillating system.

The purpose of the simulation was a harmonic analysis of signal at the output of self-excited oscillator. Fig. 5 shows

spectra of the output signals of auto-oscillating system with a nonlinear mechanical resonator and different types of non-linearity of the amplifiers.

Results of harmonic analysis of the frequency-modulated signal (Fig. 5) demonstrate that only the nonlinear amplifier with bilateral amplitude clipping enables operation at the first harmonic and effectively suppresses noise components of measurements. Thus, the auto-oscillating system designed functions stably at the first harmonic without application of additional frequency filters.

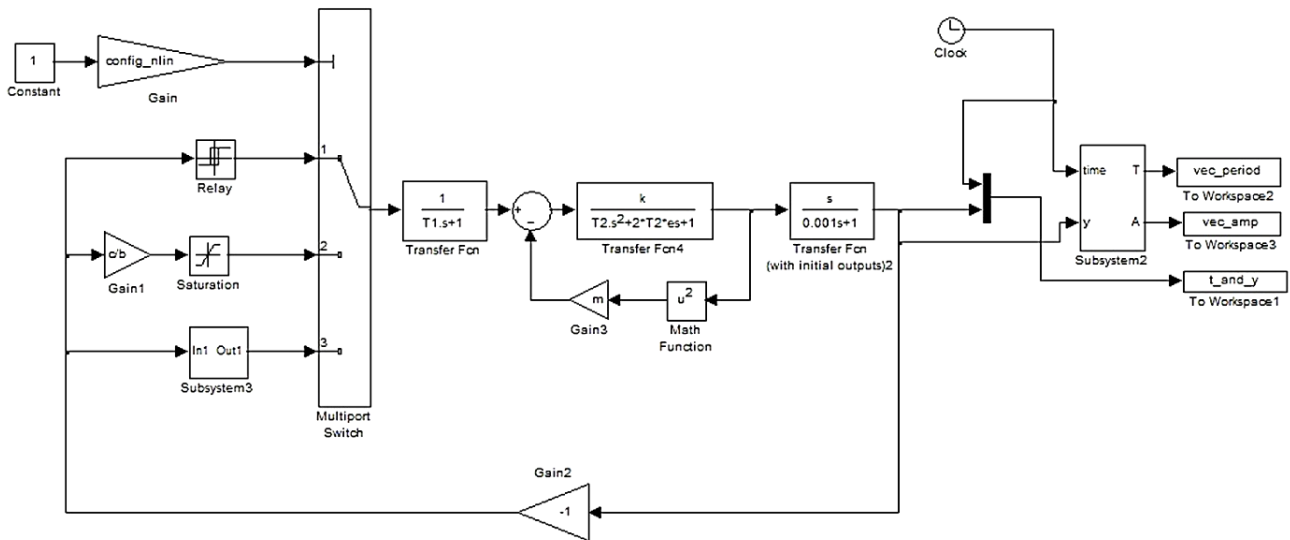


Fig. 4. Simulation model of auto-oscillating system, performed in Matlab Simulink

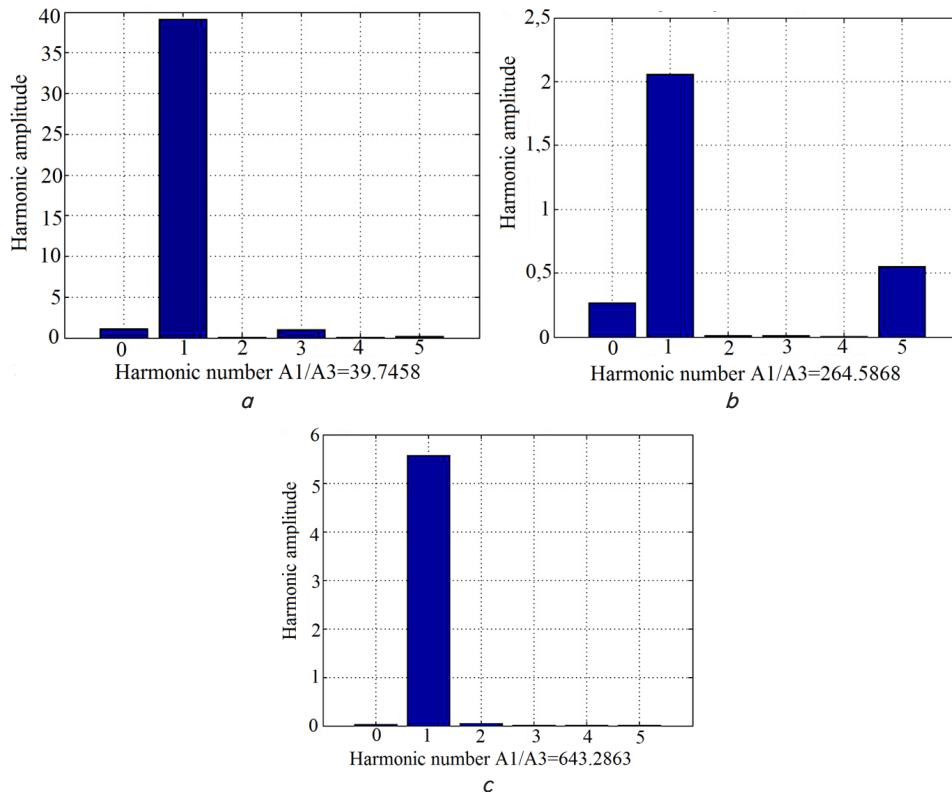


Fig. 5. Spectra of the output signals of auto-oscillating system's model: *a* – nonlinear element – a two-position relay,  $b=0.1$ ;  $s=1$ ; linear part –  $W_L(p)$ ,  $K=20$ ,  $T_1=0.001$ ,  $T_2=0.0005$ ,  $T_3=0.001$ ; *b* – nonlinear saturation amplifier,  $b=0.1$ ;  $s=1$ ; linear part –  $W_L(p)$ ,  $K=20$ ,  $T_1=0.001$ ,  $T_2=0.0005$ ,  $T_3=0.001$ ; *c* – nonlinear clipping amplifier,  $b=0.1$ ;  $s=1$ ; linear part –  $W_L(p)$ ,  $K=20$ ,  $T_1=0.001$ ,  $T_2=0.0005$ ,  $T_3=0.001$

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## 8. Discussion of results of the imitation simulation

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Research into auto-oscillating systems with two types of nonlinearity in the basic elements – resonator and amplifier – for the purpose of attaining required stability of amplitude and frequency of auto-oscillations is a new study.

Results of the imitation simulation, based on the fundamental positions of the theory of automatic regulation, indicate that nonlinear amplifier at work with a nonlinear mechanical resonator can provide for an auto-oscillating system that is stable by frequency and amplitude. This is confirmed by results of harmonic analysis of the frequency-modulated output signal, given in the present work.

The obtained transfer function of resonator, designed structure of auto-oscillating system with different types of mechanical resonators can be used in the design development of vibration frequency sensors.

A devised simulation model of the generalized auto-oscillating system with a nonlinear resonator and nonlinear feedback might be used not only for the spectral analysis of output signals, but also for conducting the studies on transient processes in AOS and stability examination.

Results of model's work confirm fully the studies, carried out in articles [9, 10, 12]. Plans for further investigations include conducting the design calculations of vibration frequency sensor with the developed

auto-oscillating system for obtaining a model sample of the sensor.

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## 9. Conclusions

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1. We obtained analytical expressions for the reduction of mechanical systems of resonators to the lumped parameters with maintaining fundamental characteristics – resonance frequency of oscillations, friction damping influence and resistance to motion. The relationships proposed make it possible to analytically describe and employ the methods of research into nonlinear systems of control in the analysis of AOS of different types of mechanical resonators, for which such a possibility was missing previously.

2. A simulation model is proposed of auto-oscillating system for the basic types of mechanical resonators of vibration frequency sensors, which is different from the known ones by the fact that it displays a changing character of nonlinearity and considers the nonisochronicity (nonlinearity of rigidity) of resonator.

3. Results of harmonic analysis of the frequency-modulated signal make it possible to assert that only the nonlinear amplifier with a bilateral amplitude clipping ensures the work at the first harmonic and effectively suppresses the noise components of measurements. Thus, the auto-oscillating system designed functions stably at the first harmonic without application of additional frequency filters.

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