

UDC 681.5 : 004.94

DOI: 10.15587/1729-4061.2017.90291

*Розроблено метод математичного моделювання динаміки гнучкого зв'язку (ГЗ) на основі автоматичного контролю осевого руху його елементів. Синтезовано регулятор відстаней між елементами ГЗ як складова математичної моделі. Запропоновано спосіб моделювання ГЗ зі змінною довжиною. Показано ефективність розробленого методу у порівнянні з методом зосереджених мас та еластичних зв'язків при моделюванні нерозтяжних ГЗ*

*Ключові слова: моделювання гнучкого зв'язку, кабель-трос, телекерований підводний апарат, підводний комплекс*

*Разработан метод математического моделирования динамики гибкой связи (ГС) на основе автоматического контроля осевого движения его элементов. Синтезирован регулятор расстояний между элементами ГС как составляющая математической модели. Предложен способ моделирования ГС с изменяемой длиной. Показана эффективность разработанного метода в сравнении с методом сосредоточенных масс и эластичных связей при моделировании нерастяжимых ГС*

*Ключевые слова: моделирование гибкой связи, кабель-трос, телеуправляемый подводный аппарат, подводный комплекс*

# DEVELOPMENT OF THE MATHEMATICAL MODELING METHOD FOR DYNAMICS OF THE FLEXIBLE TETHER AS AN ELEMENT OF THE UNDERWATER COMPLEX

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## 1. Introduction

Underwater complexes with flexible tethers (UCFT) consist of two types of elements: those with lumped and distributed parameters. The former include marine mobile objects (MMO): surface vessels, submarines, remotely operated vehicles (ROV) and others. The latter include flexible tethers (FT): umbilical cables, towing cables, anchor chains, etc. An example of a two-linked UCFT is shown in Fig. 1 [1].

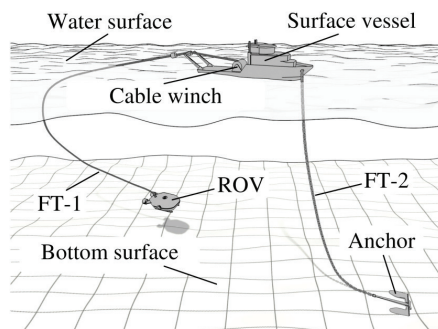


Fig. 1. Two-linked underwater complex with flexible tethers

Mathematical modeling is one of the main methods of the UCFT studies. In this regard, the UCFT motion simulators should include corresponding MMO and FT mathematical models. It is known that their motion equations are non-linear and their dynamic behaviors are mutually dependent. As

a result, these equations are strongly coupled, which has to be taken into account when simulating UCFTs. Due to equations nonlinearity usually it is impossible to find an analytical solution of such a complicated problem, hence numerical methods are employed.

The motion of MMOs as of solid bodies in the water flow has been well studied [2, 3]. The MMO mathematical models include mathematical models of their hulls, hydrodynamic drags, propulsive complexes, bearing surfaces, and other elements. As a rule, it is possible to develop an MMO mathematical model in the form of ordinary differential equations system, despite complex dependencies between parameters of model elements. This allows applying effective methods of numerical solution of differential equations to the MMO simulation.

The flexible tether modeling is a more challenging problem, as the FT is an object with distributed parameters which is described by a system of nonlinear differential equations with partial derivatives. Two approaches were evolved for modeling FT dynamics [4]. These are segmental and lumped-mass-spring (LMS) methods. For the former the FT is modeled as a continuous system and resulting partial differential equations are solved numerically by finite difference or any suitable approximation method. In the LMS approach the cable is modeled as mass points joined together by massless elastic elements of finite length, which makes it possible to express the FT model in the form of ordinary differential equations. All the forces along the cable are assumed to be concentrated at the mass points.

The UCFT mainly employs inextensible FTs (anchor chains, steel cables) and poorly extensible FTs (umbilical cables with a Kevlar load-bearing element). To model the latter, the assumption of their inextensibility is usually adopted.

The segmental method makes it possible to naturally simulate inextensible FTs by taking into account the relevant inextensibility equation. But iterative solution search makes it demanding to computational resources and suitable only for rough estimations. The LMS method is well suited for simulating elastic FTs. But its usage for inextensible FTs is followed by a significant increase of computational resources demands. The development of the FT mathematical modeling method endowed with classic methods merits and deprived of their drawbacks is the pressing scientific problem. Its solution will make the practice of flexible tethers simulating available for underwater complexes researchers without specialized computational equipment.

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## 2. Literature review and problem statement

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Usually researches neglect the umbilical cable effect when developing simulators of tethered MMOs such as ROVs. At that, the ROV is a part of a UCFT by definition. The main reason is that it causes the MMO numerical model to be very complicated and difficult to solve. Thus, the ROV mathematical models presented in [5, 6] don't include the cable influence and are useful for studying only slow ROV motions or positioning without current.

The ROV model presented in [7] is provided with an input for external disturbances simulation, which could be the cable effect. But the cable model is not presented in the paper.

There are known ROV simulators with cable effect simulation based on stationary cable motion models. The usage of such models simplifies the ROV simulators development and decreases the needed computational resources. Thus, in [8] a hydrodynamic model is developed to simulate the six degrees of freedom motions of the ROV including the stationary umbilical cable effect. The multi-step shooting method is suggested to solve the two-end boundary-value problem on the umbilical cable with respect to a set of first-order ordinary differential equation system. In [9] there is suggested the umbilical cable mathematical model which takes into account the water stream unevenness and makes it possible to estimate the cable influence on the ROV maneuverability. In [10] there is suggested the umbilical cable mathematical model with variable length of its released part. The assumption of flat cable line and three cable segments is adopted. The variability of cable length is simulated at the expense of its first segment. But the steady motion FT models don't take into account its dynamics, which limits their usage with certain UCFT motion modes. Such are synchronous motion of a surface vessel and an ROV, and also the ROV positioning within water stream.

The paper [11] presents a three-dimensional hydrodynamic model to simulate an underwater towed system. The governing equations of cable were established based on the segmental approach. The cable motion equations were approximated using a central implicit finite difference method, which formed the assembly of non-linear algebraic equations. The established equations were solved using Newton's iterative method. Since the model uses implicit time integration, it is stable for large time steps and is effective for the simulation of large-scale towed systems. Because of the iterative

solving algorithm the rough cable approximation was used. The paper presents six-segment cable model with nonlinear segment length distribution from 8.23 m to 723 m with a total cable length of approximately 1 km. Such simulation results have limited validity and are acceptable for large scaled UCFTs. To increase simulation accuracy, it is needed to increase the number of cable segments which will dramatically increase needed computational resources.

In paper [12] there is suggested the mathematical model of the FT dynamics of an underwater branched cable system on the basis of virtual work principle generalization. The obtained flexible tether differential equations system includes partial derivatives and needs either significant simplification or significant computational resources to find its solution. In [13] the flexible tether equations were approximated with the help of splines and reduced to non-linear Cauchy form by time axis which could be solved numerically. Such approximation limits the model usage with certain motion modes and UCFT configurations, which significantly narrows the model practical value.

In [14] a hydrodynamic mathematical model for simulating the motions of a ship moored near the quay in waves is presented. The mooring force is simulated using the linear LMS model. In [15] the lumped parameter cable model with consideration of the bending and torsional effects was used to simulate a circular maneuver of a towed horizontal array.

The paper [16] studies the dynamic performance of subsea umbilical cable laying system. The cable is approximated with a multi-body kinematic chain consisting of rigid finite cable elements which are connected with each other by spring-damping elements. So, the LMS method was used for simulation.

In [17] a dynamic model for estimating the position of ship towed array during the U-turn maneuver is developed. The towed array system is approximately assumed as a uniform towed cable neglecting the difference of modules' material and diameter. And starting with the stern, the towed cable was equally divided into several sections, each of which was modeled as a mass-spring by LMS method.

In [18] a numerical model of a spar platform, tethered by a mooring cable with a spherical joint, was developed for the dynamic simulation of the floating structure in ocean. The geometry modeling of the cable was established based on the LMS approach through which the cable was divided into 10 elements.

In [19] a model of semi-submersible autonomous underwater vehicle consisting of a towing vehicle, a tow cable, and a towfish was developed. The internal cable forces were considered as elastic forces so the LMS cable model was used. The torsional effect was counted as not significant, so the authors considered only elastic axial and bending force effects. The stiffness matrix for the bending effect was derived from the curvature definition. The stiffness matrix of the axial effect was composed of nodal coordinates which include the 3rd order polynomial functions of the relative element elongation. So, the nonlinear tension effect was taken into account.

The research articles analysis shows that the main disadvantage of the segmental approach is the need of significant computer resources to simulate the cable dynamics. Simulation and study of multilinked UCFTs, which consist of several rigid bodies interconnected with flexible tethers, become almost impossible without specialized high-performance computing systems. Besides, the segmental approach usage

for multilinked UCFT model development is complicated. The reason is that the rigid and flexible bodies of which the UCFT consists must be expressed as a single non-linear differential equation system with partial derivatives. Thus, different UCFT configurations need development of appropriate mathematical models. Therefore, the segmental approach is used mostly for single-linked UCFTs simulation. And the number of segments usually doesn't exceed 6–10 due to necessity of high computing performance.

The LMS method is quite suitable for simulation of multilinked UCFTs of different configurations. But it becomes demanding to computational resources when simulating inextensible flexible tethers. The reason is that the high cable axial stiffness coefficient leads to inappropriate numerical integration and computational instability. Decreasing the integration step causes the computational time increasing.

At last, the presented mathematical models of flexible tether dynamics are designed for constant tether length, though real UCFTs are equipped with cable winches and mooring windlasses. Flexible tether veering and hauling change its influence on an ROV and surface vessel and need to be taken into account in its modeling.

### 3. The aim and objectives of the study

The study aim is to develop a mathematical modeling method for the dynamics of spatial motion of the inextensible flexible tether of variable length in the flow of liquid on the basis of the automatic control of its elements axial motion. This will significantly enhance the productivity of the calculation process during a computer-aided study of the dynamics of underwater complexes with flexible tethers.

To achieve this aim, the following objectives are solved:

- the flexible tether motion governing equation and the equations of the external forces affecting it are provided;
- the fundamentals of the FT dynamics mathematical modeling method with automatic control of its elements axial motion are formulated;
- the regulator of distances between the FT elements is synthesized as a part of the FT mathematical model;
- the method of FT simulation considering that its length varies during its operation is developed;
- the FT motion dynamics is simulated, and the results are compared to the results obtained with known modeling method.

### 4. Materials and methods of development of the mathematical modeling method for the flexible tether dynamics

#### 4.1. The flexible tether motion governing equation

Simulation of an object with distributed parameters is carried out through its approximation by finite elements. In this work let's adopt an assumption that the flexible tether consists of  $N < \infty$  rigid bodies connected serially by  $N-1$  massless non-stretchable links (Fig. 2).

The FT elements move under the influence of external and internal forces. The external forces are the gravity force, the buoyancy force, the hydrodynamic drag force, and the environment inertial force that is expressed through added masses. The internal forces are the axial tension force, the twist and bend forces. The latter two forces become signif-

icant only if the flexible tether is twisted or bend much, so usually only the axial tension force is taken into account.

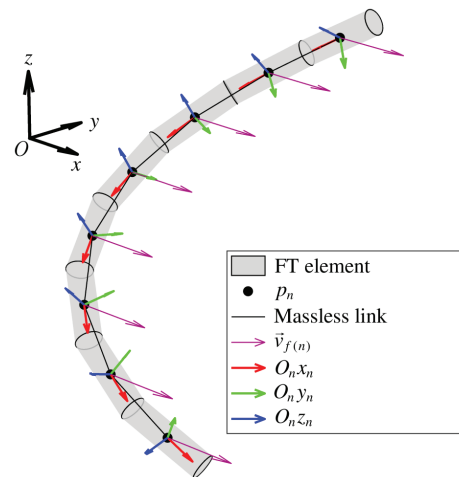


Fig. 2. Flexible tether elements and coordinate systems

On the basis of the Newton's second law, let us write down the equation of the n-th FT element motion as of a solid body in the vectoral form:

$$\mu_n \vec{a}_n = \vec{F}_{g(n)} + \vec{F}_{b(n)} + \vec{F}_{f(n)} + \vec{F}_{\tau(n)} - \vec{F}_{\tau(n-1)}; \quad n=1, 2, \dots, N, \quad (1)$$

where  $\mu_n$  – the flexible tether element mass with added mass of liquid,  $\vec{a}_n$  – the flexible tether element acceleration given in basic inertial coordinate system  $Oxyz$ ,  $\vec{F}_{g(n)}$  – the flexible tether element gravity force,  $\vec{F}_{b(n)}$  – the flexible tether element buoyancy force,  $\vec{F}_{f(n)}$  – the flexible tether element hydrodynamic drag force,  $\vec{F}_{\tau(n-1)}$  and  $\vec{F}_{\tau(n)}$  – the tension forces from previous and next links respectively. In designating some of the variables, the enumeration index  $n$  is enclosed in parentheses to distinguish it from the identification index of the variables.

At  $n=1$  the FT element is basically its inboard (root) end, and the force  $\vec{F}_{root} = -\vec{F}_{\tau(0)}$  is the propulsive force at the inboard FT end. At  $n=N$  the FT element is basically its running (fast line) end, and the force  $\vec{F}_{fast} = \vec{F}_{\tau(N)}$  is the propulsive force at the FT running end.

The basic coordinate system  $Oxyz$  is determined as follows: its origin is connected to a point on the Earth's surface, the  $Ox$  axis is directed to the north, the  $Oz$  axis is directed vertically upwards, and the  $Oy$  axis forms with the first two the right-handed rectangular (Cartesian) coordinate system.

Kinematic parameters of the FT elements are calculated through integrating the components of the vectors  $\vec{a}_n$ . The velocity of each FT element  $\vec{v}_n$  in relation to the basic coordinate system  $Oxyz$  is calculated through integrating by the time of the accelerations  $\vec{a}_n$ . The FT elements coordinates  $p_n = \{x_n, y_n, z_n\}$  are calculated through integrating by the time of the elements of the vectors  $\vec{v}_n$ . To designate the FT elements coordinates, let us also use the vector form:  $\vec{p}_n$  – the radius vector which connects the origin of the basic coordinate system with the point  $p_n$ .

#### 4.2. External forces causing the flexible tether motion

The FT element gravity force is calculated on the basis of the free fall acceleration:

$$\vec{F}_{g(n)} = m_n g \vec{e},$$

where  $m_n$  – the FT element mass (without added masses of liquid),  $g$  – the free fall acceleration,  $\vec{e}$  – the unit vector directed vertically downwards.

The external forces  $\vec{F}_{b(n)}$  and  $\vec{F}_{f(n)}$  depend on the kinematic parameters of the FT element, namely, its velocity and coordinates.

The buoyancy force is calculated on the basis of the liquid density  $\rho$  and the volume of the immersed FT element  $\Omega_n$ :

$$\vec{F}_{b(n)} = -\rho \Omega_n g \vec{e}; \quad \Omega_n = f(\vec{p}_n, \vec{p}_{n-1}, \vec{p}_{n+1}).$$

The  $\Omega_n$  value depends on the current FT element coordinates, as well as on the coordinates of the previous and next FT elements, since they determine the FT element position.

When determining the hydrodynamic drag forces for the FT motion in a liquid, one usually proceeds from the assumption of their dependence on the FT elements velocity in relation to the liquid  $\vec{v}_{f(n)}$  [2]:

$$\vec{v}_{f(n)} = \vec{v}_n - \vec{v}_{c(n)}; \quad \vec{v}_{c(n)} = V(\vec{p}_n),$$

where  $\vec{v}_n$  – the vector of the FT element velocity in relation to the basic coordinate system Oxyz,  $\vec{v}_{c(n)}$  – the vector of the stream velocity in relation to the basic coordinate system Oxyz in the point of the FT element location,  $V$  – the vector field of the stream velocity.

To calculate the hydrodynamic drag force  $\vec{F}_{f(n)}$ , let us introduce the natural coordinate system  $O_n x_n y_n z_n$  for each FT element and designate its basis with the unit vectors  $\vec{i}_n$ ,  $\vec{j}_n$  and  $\vec{k}_n$ . The  $O_n x_n$  axis is tangent to the axial line of the flexible tether and is directed from the point  $p_n$  to the point  $p_{n+1}$ . The  $O_n y_n$  and  $O_n z_n$  axes will be determined from the unit vector  $\vec{i}_n$  and the vector of the FT element velocity in relation to the flow of liquid  $\vec{v}_{f(n)}$ :

$$\vec{i}_n = \{i_{x(n)}, i_{y(n)}, i_{z(n)}\};$$

$$\vec{k}_n = \{k_{x(n)}, k_{y(n)}, k_{z(n)}\} = \frac{\vec{i}_n \times \vec{v}_{f(n)}}{|\vec{i}_n \times \vec{v}_{f(n)}|};$$

$$\vec{j}_n = \{j_{x(n)}, j_{y(n)}, j_{z(n)}\} = \frac{\vec{k}_n \times \vec{i}_n}{|\vec{k}_n \times \vec{i}_n|}.$$

Then on the basis of the matrix equation of the coupling between the basic and natural coordinate systems, the normal  $v_{fn(n)}$ , tangential  $v_{ft(n)}$  and lateral  $v_{fl(n)}$  components of the vector  $\vec{v}_{f(n)}$  are determined:

$$\begin{pmatrix} v_{ft(n)} \\ v_{fn(n)} \\ v_{fl(n)} \end{pmatrix} = K_n^{-1} \begin{pmatrix} v_{fx(n)} \\ v_{fy(n)} \\ v_{fz(n)} \end{pmatrix}; \quad K_n = \begin{pmatrix} i_{x(n)} & j_{x(n)} & k_{x(n)} \\ i_{y(n)} & j_{y(n)} & k_{y(n)} \\ i_{z(n)} & j_{z(n)} & k_{z(n)} \end{pmatrix},$$

where  $K_n$  – the coupling matrix for the n-th FT element.

The components of the vector  $\vec{F}_{f(n)}$  in the natural coordinate system for each FT element are calculated on the basis of the known equations [20]:

$$F_{ft(n)} = -0,5\rho D_{t(n)} c_{t(n)} v_{ft(n)}^2;$$

$$F_{fn(n)} = -0,5\rho D_{n(n)} c_{n(n)} v_{fn(n)}^2;$$

$$F_{fl(n)} = -0,5\rho D_{l(n)} c_{l(n)} v_{fl(n)}^2,$$

where  $F_{ft(n)}$ ,  $F_{fn(n)}$ ,  $F_{fl(n)}$  – the tangential, normal and lateral components of the vector  $\vec{F}_{f(n)}$  respectively,  $\rho$  – the water density,  $D_{t(n)}$ ,  $D_{n(n)}$ ,  $D_{l(n)}$  – the characteristic areas of the FT element, which depend on its length and the diameter  $d_{FT}$ ,  $c_{t(n)}$ ,  $c_{n(n)}$ ,  $c_{l(n)}$  – the FT tangential, normal and lateral hydrodynamic coefficients respectively.

Using the coupling matrix equation, let us determine the components of the vector  $\vec{F}_{f(n)} = \{F_{fx(n)}, F_{fy(n)}, F_{fz(n)}\}$  in the basic coordinate system:

$$\begin{pmatrix} F_{fx(n)} \\ F_{fy(n)} \\ F_{fz(n)} \end{pmatrix} = K_n \begin{pmatrix} F_{ft(n)} \\ F_{fn(n)} \\ F_{fl(n)} \end{pmatrix}.$$

Thus, all the vectors of external forces are obtained in the basic coordinate system.

### 4.3. The fundamentals of the flexible tether modeling method with automatic control of its elements axial motion

To model the FT elements motion, both external and internal forces (constraint reaction forces) should be set. The major problem in the FT modeling is determining the constraint reaction forces. Therefore, if we determine the constraint reaction forces, we will obtain the complete information for the FT modeling.

To determine the constraint reaction forces, let us view the FT as a multi-dimensional object of control. The controlled values are its elements coordinates  $\vec{p}_n$ , the controlling values are the vectors of the constraint reaction forces  $\vec{F}_{\tau(n)}$  except the forces at  $n=1$  and  $n=N$ , the disturbing impacts are the vectors of the externals forces  $\vec{F}_{ext(n)}$ :

$$\mu_n \vec{a}_n = \vec{F}_{\tau(n)} - \vec{F}_{\tau(n-1)} + \vec{F}_{ext(n)}; \quad n = 1, 2, \dots, N; \quad (2)$$

$$\vec{F}_{ext(n)} = \vec{F}_{g(n)} + \vec{F}_{b(n)} + \vec{F}_{f(n)}.$$

To designate the controlling forces, let us introduce the index  $m=1, 2, \dots, N-1$ . Thus, the forces  $\vec{F}_{\tau(m)}$  are the controlling ones.

The control objective resides in providing the correspondence of the distances between the adjacent FT elements  $l_m$  to their given values  $l_{g(m)}$ :

$$l_m = l_{g(m)};$$

$$l_m = |\vec{p}_{(m+1)} - \vec{p}_m|. \quad (3)$$

At the uniform distribution of the FT elements, the value of  $l_{g(m)}$  can be found from the following equation:

$$l_{g(m)} = \frac{L}{N-1}, \quad (4)$$

where  $L$  – the FT length.

The controlling forces  $\vec{F}_{\tau(m)}$  are directed along the axial curvilinear FT coordinate  $s$ . They can affect the FT element motion only in the axial coordinate unlike the external forces  $\vec{F}_{ext(n)}$ , which affect the FT element motion in the three-dimensional space. At that, the problem of control is also formulated in the one-dimensional statement. In this

regard, the principal equation of the FT motion can be given in the one-dimensional form:

$$\mu_n a_{s(n)} = -F_{\tau(n-1)} + F_{\tau(n)} + F_{\text{ext}(n)}, \tag{5}$$

where  $a_{s(n)}$  – the FT element acceleration along the axial coordinate  $s$ ,  $F_{\text{ext}(n)}$  – the resultant of the external forces affecting the FT element along the axial coordinate  $s$ ,  $F_{\tau(n-1)}$  – the tension forces from the previous FT element,  $F_{\tau(n)}$  – the tension forces from the next FT element. Similarly to equation (1), at  $n=1$  and at  $n=N$  the forces  $-F_{\tau(0)}$  and  $F_{\tau(N)}$  are the axial components of the propulsive forces of the FT at its inboard and running ends respectively:

$$F_{\text{root}} = -F_{\tau(0)}, \quad F_{\text{fast}} = F_{\tau(N)}.$$

The values  $F_{\text{ext}(n)}$ ,  $-F_{\tau(0)}$  and  $F_{\tau(N)}$  are determined respectively as the projections of the vectors  $\vec{F}_{\text{ext}(n)}$ ,  $-\vec{F}_{\tau(0)}$  and  $\vec{F}_{\tau(N)}$  on the vectors  $\vec{i}_n$ ,  $\vec{i}_1$  and  $\vec{i}_N$ .

Therefore, we obtain a multi-dimensional automatic control system (ACS) with complete initial information (Fig. 3).

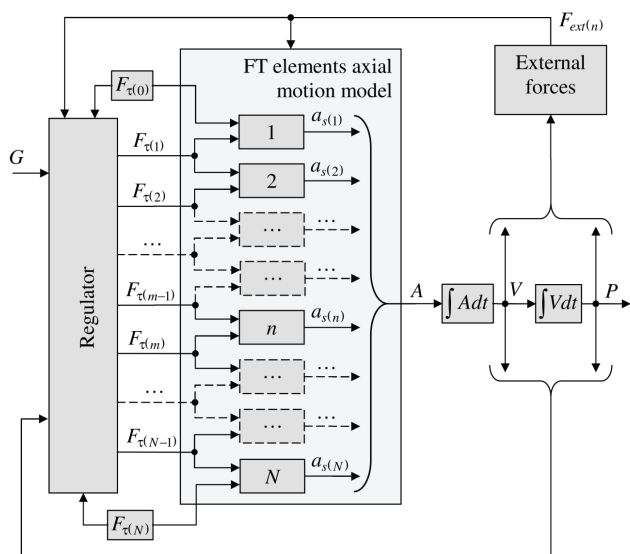


Fig. 3. Generalized structure of the automatic control system for the flexible tether elements motion along the axial coordinate  $s$

The set of given distances between the elements  $G$  and the sets of the kinematic parameters of the elements  $V$  and  $P$  come at the input of the regulator; they are calculated by means of numerical methods on the basis of the set of acceleration values  $A$ :

$$G = \{l_{g(1)}, l_{g(2)}, \dots, l_{g(m)}, \dots, l_{g(N-1)}\};$$

$$A = \{a_{s(1)}, a_{s(2)}, \dots, a_{s(n)}, \dots, a_{s(N)}\};$$

$$V = \{v_{s(1)}, v_{s(2)}, \dots, v_{s(n)}, \dots, v_{s(N)}\};$$

$$P = \{p_{s(1)}, p_{s(2)}, \dots, p_{s(n)}, \dots, p_{s(N)}\},$$

where  $v_{s(n)}$  – the FT element velocity in its motion along the axial line  $s$ ,  $p_{s(n)}$  – the FT element coordinate along the axial line  $s$ .

The distances between the adjacent FT elements are determined on the basis of the elements of the set  $P$  and form the set  $\Lambda$ :

$$\Lambda = \{l_1, l_2, \dots, l_m, \dots, l_{N-1}\}; \quad l_m = p_{s(m+1)} - p_{s(m)}.$$

On the basis of the kinematic parameters of the FT, the ACS regulator is to calculate such values of  $F_{\tau(m)}$  that the condition  $l_m = l_{g(m)}$  is fulfilled, in other words, to fulfill the control task. On the basis of the calculated  $F_{\tau(m)}$ , the vectors of the constraint reaction forces  $\vec{F}_{\tau(m)} = \{F_{\tau x(m)}, F_{\tau y(m)}, F_{\tau z(m)}\}$  are obtained through transferring from the natural coordinate system to the basic one:

$$\begin{pmatrix} F_{\tau x(m)} \\ F_{\tau y(m)} \\ F_{\tau z(m)} \end{pmatrix} = K_n \begin{pmatrix} F_{\tau(m)} \\ 0 \\ 0 \end{pmatrix}.$$

Afterwards, the FT motion is modeled with the use of the governing equation of the FT elements motion (2). In fact, there is no need to model equation (5) to obtain the sets of the axial kinematic parameters of the FT elements  $V$  and  $P$ . Instead, the FT model with automatic control of its elements axial motion (ACEAM) consists of the regulator, the external forces calculation unit and the governing motion equation (Fig. 4).

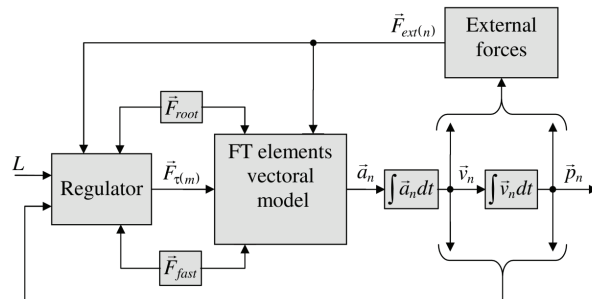


Fig. 4. Mathematical model of the flexible tether dynamics as an automatic control system

The required transformations of the components of the vectors from the basic coordinate system to the natural and back to the basic one are performed by the regulator. The elements  $l_{g(m)}$  of the set  $G$  are determined from equation (4), the elements  $l_m$  of the set  $\Lambda$  are determined from equation (3), the elements  $v_{s(m)}$  of the set  $V$  are determined as projections of the vectors  $\vec{v}_n$  on the axis  $O_n x_n$  (that is, on the vector  $\vec{i}_n$ ). The FT length  $L$  is specified by the constant, and the general FT motion is controlled by the forces  $\vec{F}_{\text{root}}$  and  $\vec{F}_{\text{fast}}$ .

Let us consider a variant of the proportional control law with the coefficient of proportionality  $k$ :

$$F_{\tau(m)} = -k \frac{(l_{g(m)} - l_m)}{l_{g(m)}}, \quad m = 1, 2, \dots, N-1.$$

It is easy to notice that the proportional control law transforms the FT mathematical model into the linear LMS model. In such a model, the constraint reaction forces appear at  $|l_{g(m)} - l_m| > 0$ . Hence, there will always be a static error at the FT motion in the flow of liquid. To study inextensible FTs, more effective automatic control methods should be applied.

#### 4. 4. Synthesis of the flexible tether model regulator using the inverse dynamics method

The inverse dynamics method implements the concept of inverse control and with the complete initial information provides highly precise control of complex nonlinear objects including UCFT [21]. Let us apply it for synthesis of the FT model regulator.

The regulator should provide control of the distances between the FT elements  $l_m$ . According to the inverse dynamics method, let us select a reference model for controlling the values  $l_m$ :

$$\alpha_m = \frac{\frac{1}{T_0}(l_{g(m)} - l_m) - \gamma_m}{T_1};$$

$$\gamma_m = v_{s(m+1)} - v_{s(m)},$$

where  $\alpha_m$  – the desired acceleration of the value  $l_m$ ,  $\gamma_m$  – the actual rate of change in the value  $l_m$ .

Let us form the inverse model of the controlled object on the basis of the equation of the FT elements axial motion (5) and replace the acceleration values  $a_{s(m)}$  with their desired values  $a_{d(m)}$ :

$$-F_{\tau(n-1)} + F_{\tau(n)} = \mu_n a_{d(n)} - F_{\text{ext}(n)},$$

where  $\mu_n$  – the FT element mass with added masses of water.

This system includes the unknowns  $F_{\tau(m)}$ ,  $m=1, 2, \dots, N-1$ , and  $a_{d(n)}$ ,  $n=1, 2, \dots, N$ . The total number of the unknowns makes up  $2N-1$ , while the total number of the equations is equal to the value of  $N$ . Let us supplement the inverse model with the equations which couple the FT elements accelerations  $a_{d(n)}$  to the accelerations of the distances between them  $\alpha_m$ :

$$a_{d(m+1)} - a_{d(m)} = \alpha_m.$$

Thus, we obtain a system of linear algebraic equations with the number of unknowns equal to the number of equations in the system:

$$-F_{\tau(n-1)} + F_{\tau(n)} = \mu_n a_{d(n)} - F_{\text{ext}(n)}; \quad n = 1, 3, \dots, N;$$

$$a_{d(m+1)} - a_{d(m)} = \alpha_m; \quad m = 2, 3, \dots, N-1.$$

If this system of equations is supplemented by elements with zero coefficients, it can be presented in the matrix form:

$$B \cdot H = R;$$

$$H = [F_{\tau(1)}, \dots, F_{\tau(N-1)}, a_{d(1)}, \dots, a_{d(N)}]^T;$$

$$R =$$

$$= [F_{\text{ext}(1)} - F_{\tau(0)}, F_{\text{ext}(2)}, \dots, F_{\text{ext}(N-1)}, F_{\text{ext}(N)} + F_{\tau(N)}, \alpha_1 \dots \alpha_{N-1}]^T,$$

where  $H$  – the matrix of unknown forces and accelerations,  $R$  – the matrix of given forces and accelerations,  $B$  – the matrix of coefficients.

For example, if we accept the value of  $N$  to be 4, the matrix  $B$  takes the following form:

$$B = \begin{bmatrix} -1 & 0 & 0 & \mu_1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & \mu_2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \mu_4 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Calculation of the constraint reaction forces  $F_{\tau(m)}$ , as well as the desired axial accelerations of the FT elements  $a_{d(n)}$ , in other words, the elements of the matrix  $H$ , is performed by solving the following matrix equation:

$$H = B^{-1}R. \tag{6}$$

At that, it is sufficient to calculate the matrix  $B^{-1}$  once, before the modeling starts, since its elements are constant. Afterwards, the constraint reaction forces are singled out from the matrix  $H$  and transformed into the vector form  $\vec{F}_{\tau(m)}$  to be used in the governing equation of the FT elements motion (2).

The controlling forces  $\vec{F}_{\text{root}}$  and  $\vec{F}_{\text{fast}}$ , are given to the input of the developed FT mathematical model, and the kinematic parameters of the FT elements are obtained at its output. However, the practical value is assigned to the FT model which obtains the kinematic parameters of its inboard and running ends  $\vec{p}_{\text{root}}$  and  $\vec{p}_{\text{fast}}$  at the input and the tension forces at the output [1]. It is due to the fact that the mathematical models of MMOs (surface vessels, underwater vehicles, etc.) are also obtained on the basis of the Newton's second law, and their interaction with FT is modeled by giving the vectors of the tension forces to the input.

To implement such MMO models in conjunction with the developed FT model, the points  $\vec{p}_1$  and  $\vec{p}_N$  should be separated from the points  $\vec{p}_{\text{root}}$  and  $\vec{p}_{\text{fast}}$  where the inboard and running FT ends respectively are mounted. Then, using the proportional control law, we obtain the tension forces on the inboard and running FT ends:

$$F_{\text{root}} = k \frac{(l_{g(0)} - l_0)}{l_{g(0)}}; \quad l_0 = |\vec{p}_1 - \vec{p}_{\text{root}}|;$$

$$F_{\text{fast}} = -k \frac{(l_{g(N)} - l_N)}{l_{g(N)}}; \quad l_N = |\vec{p}_{\text{fast}} - \vec{p}_N|,$$

where  $l_0$  – the actual distance between the first FT element and the FT mounting point on the MMO,  $l_N$  – the actual distance between the last FT element and the FT mounting point on the MMO.

The given values of these distances  $l_{g(0)}$  and  $l_{g(N)}$  can be taken as equal to zero or calculated on the basis of equation (4), but in this case two additional links should be considered for the formation of the matrix  $G$ :

$$l_{g(q)} = \frac{L}{N+1}, \quad q = 0, 1, \dots, N.$$

It should be noted that the FT can be mounted not only on MMOs, but also on immobile objects, for example, on bottom.

The vectors of the forces affecting the first and the last FT elements are determined from the following equations:

$$\vec{F}_{\tau(0)} = -\vec{F}_{\text{root}} = -F_{\text{root}} \vec{i}_1;$$

$$\vec{F}_{\tau(N)} = \vec{F}_{\text{fast}} = F_{\text{fast}} \vec{i}_N.$$

The vectors of the forces affecting the MMO as a result of its interaction with the FT will be equal to  $\vec{F}_{\text{root}}$  and  $-\vec{F}_{\text{fast}}$  in the points of mounting of the inboard and running ends respectively.

**4. 5. Dynamic change of the flexible tether length**

If the FT length is changed during the UCFT operation, an additional load appears; it is transferred to MMOs and affects their movement. In this regard, the FT mathematical model should include appropriate algorithms to take these loads into account.

The easiest way to change the FT length in the process of its simulation is changing the parameter L and, as a result, the set of the given distances between the FT elements G. Yet, instead of the synchronous FT elements motion along the axial coordinate, this method will move them non-uniformly, which does not correspond to the real process.

The consecutive change of the length of the elements  $l_{g(m)}$  of the set G is more accurate in representing the reality. Let us assume that the FT length change is carried out from its inboard end:

$$l_{g(q)} = \begin{cases} 0, & \text{if } q < q_r; \\ L_{\text{link}}(q_r + 1) - L_{\text{coiled}}, & \text{if } q = q_r; \\ L_{\text{link}}, & \text{else;} \end{cases} \quad (7)$$

$$L_{\text{link}} = \frac{L_{\text{full}}}{N + 1};$$

$$L_{\text{coiled}} = L_{\text{full}} - L_r;$$

$$q_r = \text{floor}\left(\frac{L_{\text{coiled}}}{L_{\text{link}}}\right);$$

$$q = 0, 1, \dots, N,$$

where  $L_{\text{full}}$  – the full FT length,  $L_r$  – the released FT length,  $L_{\text{coiled}}$  – the coiled FT length,  $L_{\text{link}}$  – the length of one link,  $q_r$  – ordinal number of the link next to the inboard end of the released part of the FT, floor – rounding function to the lowest integer value.

Changing the length of the released part of the FT, i. e. the parameter  $L_r$ , will affect the length of only those links which get into the coiled part of the FT. Thus, the even coiling (hauling) or release (veering) of the FT is ensured.

However, in this case the FT elements with the distance between them being equal to zero will concentrate in the area of the inboard FT end and continue interacting with liquid, which also does not correspond to the actual process.

In reality, the FT elements stop interacting with liquid at the FT coiling, and come into interaction with liquid at the FT release. This effect should be achieved in the simulating of the FT length change. For this purpose, it is suggested to exclude from calculation those FT elements which completely belong to the coiled part of the FT, i. e. elements with the numbers  $q < q_r$ . This can be done without the need to re-calculate the matrix of coefficients  $B^{-1}$  in equation (6) with

each change in the FT length. It is sufficient to set to zero the external forces with the indices  $n \leq q_r$ ,  $n = 1, 2, \dots, N$ , and the desired accelerations with the indices  $m \leq q_r$ ,  $m = 1, 2, \dots, N - 1$  in the matrix of given forces and accelerations R. Meanwhile, the propulsive force  $F_{\text{root}} = -F_{\tau(0)}$  should be added to  $F_{\text{ext}(n)}$  with the index  $n = q_r$ .

**5. Flexible tether simulation**

**5. 1. Simulation of the flexible tether motion dynamics with the use of the developed method**

Let us perform the FT simulation with the help of the mathematical model on the basis of the developed ACEAM method. It is assumed that the flexible tether has a cylindrical shape and zero buoyancy, and the stream velocity is uniformly distributed in space. For the period of simulation, the inboard end is fastened in the origin of the basic coordinate system Oxyz. At the moment of the simulation start  $t_0$ , the running end is located at the depth of 50 m under the inboard one and is movable. The main parameters of the mathematical model are summarized in Table 1.

Table 1

Parameters of the flexible tether mathematical model

Physical parameters		Regulator parameters	
Parameter	Value	Parameter	Value
$L_{\text{full}}$ , m	100	$T_0$ , s	0.1
N	101	$T_1$ , s	0.025
$\rho$ , kg/m <sup>3</sup>	1024	k, N	10000
$d_{FT}$ , mm	10	–	–
$c_n$	1	–	–
$c_t$	0.1	–	–
$c_l$	0.0	–	–

Let us perform the simulation under the following conditions:

$$\vec{F}_{g(n)} = -\vec{F}_{b(n)}, \quad \vec{v}_{c(n)} = \{-1, 0, 0\} \text{ m/s},$$

$$\vec{p}_{\text{root}}(t) = \{0, 0, 0\} \text{ m}, \quad \vec{p}_{\text{fast}}(t_0) = \{0, 0, -50\} \text{ m}.$$

The numerical solution of the nonlinear differential equations of the FT model will be carried out using the Dormand-Prince method with a variable step. The simulation results are shown in Fig. 5.

The moments of time specified in Fig. 5 have the following values:  $t_0 = 0$  s,  $t_1 = 10$  s,  $t_2 = 22.5$  s,  $t_3 = 35$  s,  $t_4 = 45$  s,  $t_5 = 57.5$  s and  $t_6 = 70$  s.

Over the time period from  $t_0$  to  $t_1$ , the running end does not move, and the FT line takes over a steady position. At the moment of time  $t_1$ , the running end starts moving upwards at the velocity of 1 m/s. In such a way, it moves for 25 s and stops at the moment of time  $t_3$ . Afterwards, the running end does not move for 10 seconds, then at the moment of time  $t_4$  it starts moving forward at the velocity of 1 m/s. In such a way, it moves for 25 s and stops at the moment of time  $t_6$ .

Fig. 5, a shows the FT lines depending on the time and position of the running end. In particular, there is a change

in the line over the period of time from  $t_3$  to  $t_4$ , when the running end does not move. The difference between the length of the released part of the FT  $L_r$  and the sum of distances between its elements  $L_a$  does not exceed 0.03 % of  $L_r$  during the whole time of simulation (Fig. 5, *b*). At that, this difference is caused by the inboard (root) and running (fast) links, whose lengths are controlled on the basis of the proportional law. This indicates a highly precise operation of the inverse regulator of the FT mathematical model.

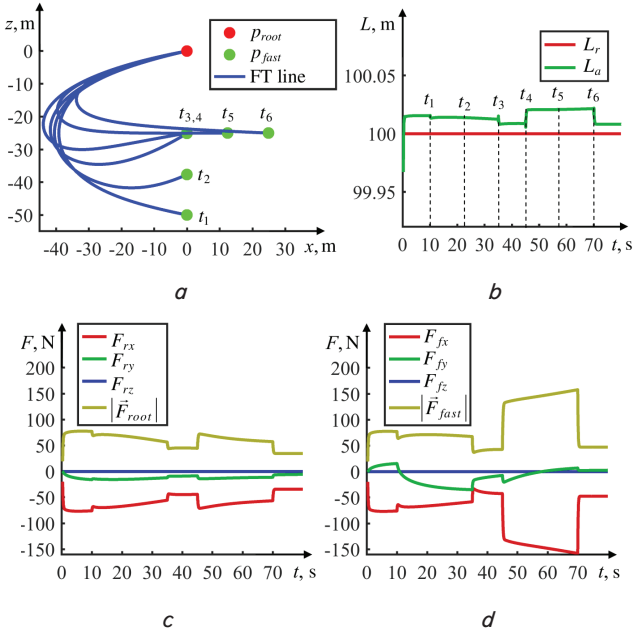


Fig. 5. Simulation of the flexible tether dynamics at  $L_r(t)=const$ : *a* – the flexible tether configurations at the running end motion, *b* – the length of the released part of the flexible tether  $L_r$  and the sum of distances between the centers of its elements  $L_a$ , *c* – the tension force on the inboard (root) FT end, *d* – the tension force on the running (fast) FT end

Fig. 5, *c, d* show the components of the vectors of the tension forces on the inboard and running ends respectively:

$$\vec{F}_{root} = \{F_{rx}, F_{ry}, F_{rz}\};$$

$$\vec{F}_{fast} = \{F_{fx}, F_{fy}, F_{fz}\}.$$

The change in the running end velocity affects the components of the vectors  $\vec{F}_{root}$  and  $\vec{F}_{fast}$ . In addition, there is a change in the vectors of the tension forces over the period of time from  $t_3$  up to  $t_4$ , when the running end does not move. It is caused by the FT line changing its configuration.

Let us perform the FT simulation under the same conditions, but with the change of the length of the released part  $L_r$  during the running end motion. The simulation results are shown in Fig. 6.

When the running end moves upward over the period of time from  $t_1$  to  $t_3$ , the length of the released part of the FT decreases, that is, the FT is being coiled. Over the period of time from  $t_4$  up to  $t_6$ , the FT length increases at the rate of 0.5 m/s, in other words, the FT is being released. With the scale presented in Fig. 6, *b*, the difference between the lengths  $L_r$  and  $L_a$  is not visible and does not exceed 0.07 % of  $L_r$ . The FT length change affects the FT line configuration

(Fig. 6, *a*) and the tension forces on its ends (Fig. 6, *c, d*). At that, the FT coiling considerably increases the tensions on its ends, while the release decreases them.

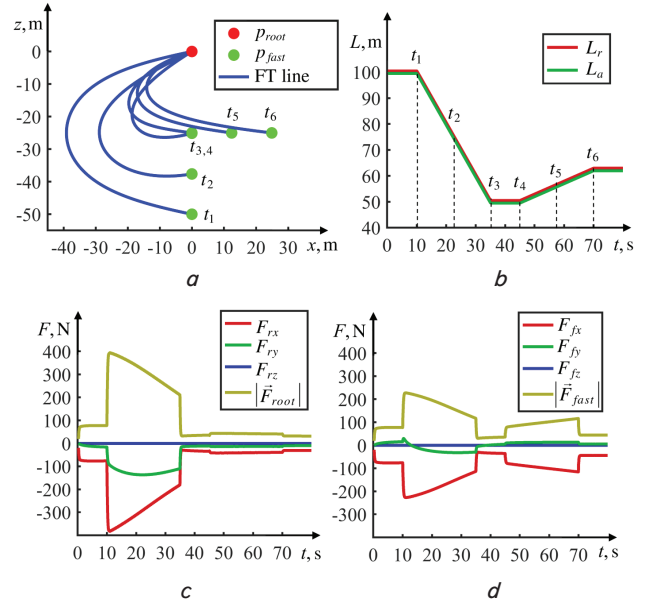


Fig. 6. Modeling of the flexible tether dynamics at  $L_r(t) \neq const$ : *a* – the flexible tether configurations at the running end motion, *b* – the length of the released part of the flexible tether  $L_r$  and the sum of distances between the centers of its elements  $L_a$ , *c* – the tension force on the inboard (root) FT end, *d* – the tension force on the running (fast) FT end

## 5. 2. Comparison of the developed modeling method to the linear method of lumped masses and springs

Let us introduce the following variables:  $L_{acm}$  – the sum of the link lengths between the FT elements in modeling with the ACEAM method,  $L_{lms}$  – the sum of the link lengths between the FT elements in modeling with the LMS method. For the magnitude  $L_{lms}$  to be of the same order of magnitude  $L_{acm}$ , it is necessary to select a proper coefficient of proportionality  $k_{lms}$  for the LMS method. Let it be  $k_{lms} = 40k_{acm}$ , where  $k_{acm}$  – the coefficient of proportionality in controlling with the ACEAM method. This will ensure the equality (non-strict) of the tension forces on the FT ends while simulating with different methods.

Let us perform the modeling at  $L_r(t)=const$  with the linear LMS method and compare the obtained results with the ACEAM method (Fig. 7, *a*).

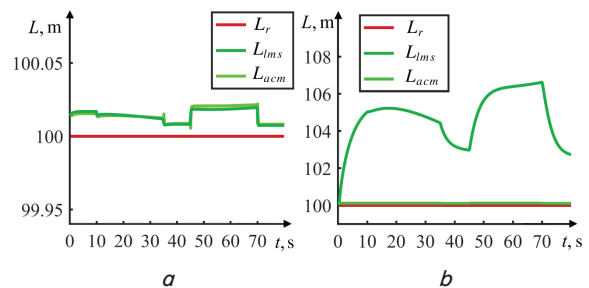


Fig. 7. Comparison of the flexible tether lengths: *a* – modeling with the coefficient of proportionality  $k_{lms} = 40k_{acm}$ , *b* – modeling with the coefficient of proportionality  $k_{lms} = 0.11k_{acm}$



The graphs of the tension forces obtained with the LMS method are not presented, since at  $k_{lms}=40k_{acm}$  they coincide with the graphs obtained with the ACEAM method, which are shown in Fig. 5, c, d.

The simulation results have shown that simulation of the given mode of motion with the LMS method requires approximately 25 times more computer time than that with the ACEAM method.

Let us set such value of the coefficient of proportionality that the time of modeling with LMS and ACEAM methods is approximately the same:  $k_{lms}=0.11k_{acm}$ . Fig. 7, b shows the lengths, and Fig. 8 – the tension forces on the inboard and running FT ends in modeling with different methods.

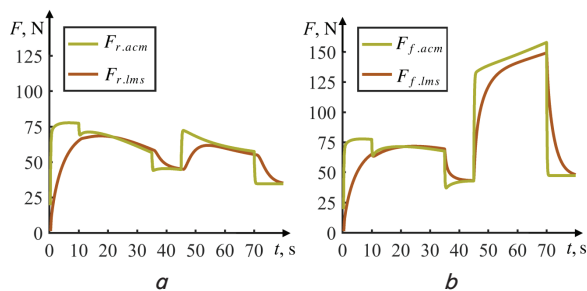


Fig. 8. Comparison of the tension forces on the FT ends in simulation with the LMS and ACEAM methods: a – the tension forces on the inboard (root) end, b – the tension forces on the running (fast) end

The designations in Fig. 8 are as follows:  $F_{r,acm}$ ,  $F_{f,acm}$  – the absolute values of the vectors of the tension forces on the inboard and running FT ends respectively in simulation with the ACEAM method,  $F_{r,lms}$ ,  $F_{f,lms}$  – the absolute values of the vectors of the tension forces on the inboard and running FT ends respectively in simulation with the LMS method.

The length  $L_{lms}$  is considerably different from its given value  $L_r$ . In addition, there is a substantial delay in the FT response to the running end motion, which is caused by the small coefficient of proportionality (extensible stiffness) of the LMS method.

Analysis of the modeling results shows that lowering the coefficient of proportionality when using the LMS method leads to obtaining inaccurate results for inextensible FTs. Increasing the coefficient of proportionality reduces the integration step and makes the time of calculation substantially longer.

The FT modeling method developed in this paper allows obtaining reliable results with less time spent on computations than that of the classical methods.

**6. Discussion of the flexible tether dynamics modeling method with automatic control of its elements axial motion**

The pure ACEAM method allows obtaining the FT model, the inputs of which are the vectors of the force on the FT ends, and the outputs are the coordinates of the FT ends and other elements. Separation of the FT end (boundary) elements and their mounting points on MMOs or immobile objects requires using the proportion-

al control law, which is similar to the linear LMS method. This introduces an inessential error in the simulation of inextensible FTs. However, here only the boundary links undergo elastic deformation, which is not the case with the classical LMS method. Generally, the ACEAM method provides an almost instantaneous force response of the model for changes in the motion parameters in comparison with the classical LMS method. At that, the actual FT length is almost the same as the given one. In addition, the presence of only two elastic links enables choosing rather large coefficients of proportionality  $k$  in comparison with the classical LMS method without losing the stability of the model.

The suggested method makes it possible to model UCFTs at the periodic nature of the FT ends motion, such as at the vertical moving of the surface vessel on waves. When the LMS method with a low coefficient  $k$  is applied, the amplitude of the tension force on the running end will be reduced due to elastic deformation of the FT elements, which is not valid for inextensible FTs. The suggested ACEAM method is not characterized by this disadvantage.

At the FT modeling with the use of the segment method, differential equations of the FT are presented in the form of finite differences not only by the axial FT coordinate, but also by time. At that, the systems of equations are to be integrated implicitly, which makes it necessary to use iterative methods for finding the solution. The presence of the iterative component in the simulating algorithm considerably increases the simulation time. The problem of the segment method is made even more complex by the fact that the UCFT mathematical model should include equations for one or more solid bodies. In this case, the iterative component of the model may even lead to the impossibility of finding the numerical solution. Thus, the modeling of multilinked UCFTs appears hardly probable. Moreover, the study of the ACS for the spatial motion of the multilinked UCFT with the help of the computer modeling method presents a most difficult computation problem.

The ACEAM method does not contain the iterative component. The FT equation is reduced to the Cauchy normal form, which allows using effective methods of numerical solution of ordinary differential equations. The inverse control has shown to be effective and highly precise as a part of the ACEAM method, since the FT model as an automatic control system forms a system with complete initial information.

Concerning the change in the length of the released part of the FT, it can be properly carried out when the lengths of the links between the elements  $l_m$  correspond to their given values  $l_{g(m)}$  accurately enough. In this case, excluding the element from calculation at the FT coiling or including the element into calculation at the FT release does not result in substantial force surges at the FT ends. This effect is hard to achieve with the LMS method, since its application always involves a static error in the lengths.

The disadvantage of the suggested method is the reduction in the regulator operation accuracy at the FT overbending, i.e. at the decrease of the angles between the links. This is caused by the fact that the axial components of the vectors of the external forces  $\vec{F}_{ext(n)}$  and the FT elements velocity in relation to the flow of liquid  $\vec{v}_{f(n)}$  are determined as their pro-

jections on the unit vector  $\vec{i}_n$  of the natural coordinate system of the  $n$ -th element. This disadvantage shows up when the angles between the links are in the range of  $<90^\circ \dots 110^\circ$ . However, the more elements  $N$  there are in the FT modeling, the smaller the risk of such overbending is.

The suggested ACEAM method combines the advantages of both classical FT modeling methods: the LMS method and the segment method. Like the segment method, it provides a nearly constant value of the link lengths. At the same time, its application engages non-iterative methods of solving differential equations like that of the LMS method.

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## 7. Conclusions

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1. For the mathematical modeling of the flexible tether dynamics in the flow of liquid, it is represented as a set of series-connected elements – solid bodies, affected by internal and external forces. The external forces include gravity force, buoyancy force and hydrodynamic drag force, while the internal ones are the axial constraint reaction forces between the flexible tether elements.

2. The fundamentals of the flexible tether dynamics mathematical modeling method with automatic control of its elements axial motion are formulated. The nature of the

method resides in presenting the flexible tether as a multi-dimensional automatic control system. It consists of a set of the flexible tether elements, the adjacent elements interact via the constraint reaction forces, and the regulator, which is designed to provide the given distances between the adjacent flexible tether elements.

3. With the inverse dynamics method, the regulator is synthesized as a part of the flexible tether mathematical model. It provides a highly precise control of the distances between the elements and, therefore, accurate modeling of the inextensible flexible tether dynamics.

4. There is suggested the method of the flexible tether simulation considering that its length varies during its operation on the basis of the dynamic change in the number of its elements being involved in the calculation process. This allows considering the impact of the motion dynamics and the change of the flexible tether length on marine moving objects of the underwater complex.

5. Simulation of the flexible tether motion dynamics was carried out by means of the method with automatic control of its elements axial motion and the classical method of lumped-masses-springs. Comparing the simulation results showed that the numerical calculation of inextensible flexible tethers carried out with the developed method takes 25 less time due to ensuring the same accuracy of the methods at the increased integration step.

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