Система нелінійних диференціальних рівнянь руху автономного підводного апарату (АПА) з маніпулятором на борту у інерціальній системі координат (ІСК) трансформована до рекурентної послідовності. Аналітичні вирази лінійних та кутових швидкостей, кутових положень та координат АПА у ІСК отримані для повної кінематичної матриці. Представлено вирази похибки кутових положень, що виникають внаслідок спрощень кінематичної матриці

Ключові слова: автономний підводний апарат, інерціальна система координат, рекурентна послідовність, кінематична матриця

Система нелинейных дифференциальных уравнений движения автономного подводного аппарата (АПА) с манипулятором на борту в инерциальной системе координат (ИСК) преобразована к рекуррентной последовательности. Аналитические выражения линейных и угловых скоростей, угловых положений и координат АПА в ИСК получены для полной кинематической матрицы. Представлены выражения ошибки угловых положений, которые возникают вследствие упрощений кинематической матрицы

Ключевые слова: автономный подводный аппарат, инерциальная система координат, рекуррентная последовность, кинематическая матрица

UDC 519.876.2:517.962.27]:[629.584:681.516.54
DOI: 10.15587/1729-4061.2017.95783

RECURRENT TRANSFORMATION OF THE DYNAMICS MODEL FOR AUTONOMOUS UNDERWATER VEHICLE IN THE INERTIAL COORDINATE SYSTEM

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1. Introduction

The experience in managing the process of motion of unmanned underwater vehicles (UUV) acquired when conducting emergency repair work or technological survey indicates many false assumptions that are accepted in the formation of the model [1]. As demonstrated by experimental studies, for example, in article [2], control over spatial motion needs to take into account the interrelation between translational and rotational motion. Not less important are the conclusions of analysis of the development of modern and perspective unmanned underwater vehicles at the service of US Navy [3], which predict a need for designing the means to realize complex trajectories. An analysis of trends in the development of unmanned underwater vehicles reveals a predominant attribute – autonomy [4]. Attempts at constructing the models for spatial motion of marine movable objects [5], and autonomous underwater vehicles (AUV) in particular, are based on a series of assumptions and the simplification of kinematic matrix [6]. Thus, a special role belongs to the assumption about the smallness of angles of heel and trim [7], used to substantiate the simplification of kinematic matrix [8]. Their immediate manifestation is observed when the model is formed and identified, as well as for the compilation of knowledge base for the automated control systems (ACS) of AUV [9]. In addition, despite the application of methods for the identification of the model, such assumptions and simplifications lead to considerable errors [10]. Their magnitudes can be assessed by the methodology of hydrodynamic tests [11] or as reference magnitudes through re-calculating by the prototype [12]. However, in most implementations of the systems and projects [13], one may observe a lack of simultaneous representation of the AUV angular position and the position of movable equipment aboard it. Thus, partially available information on the displacement of movable objects aboard it [14] relative to the connected, semi-connected and ICS cannot be combined because of false assumptions about the smallness of the angles [5, 6]. The same reason prevents processing, comparing and combining the information obtained for different coordinate systems and stored in special libraries [15]. Such fragmentation and incomparability also leads to non-effective solutions [16] and additional reasons for the formation of errors [17]. The latter is caused by the inability of the models, which are formed by means of simplified equations for the dynamics of movable interacting elements, for predictive assessment of the full set of kinematic parameters [18]. These manifestations are especially vivid under conditions of additional disturbances that are caused by drift and rough water or additional moments of forces that occur in the presence of several vehicles, which oscillate simultaneously. The main reason for these results is that the existing models [1, 5, 6] simplify an impact from the values of angles of heel and trim and employ assumptions on that their magnitudes are lower than the tenth of radian. Analytical solution for the system of motion equations of AUV and the manipulator, which was constructed in the form of recurrent sequence [10, 18], allowed us to model and simultaneously explore changes in eighteen kinematic parameters. Results of the motion modeling for the problems on search indicate that such assumptions practically do not affect results in the modeling of AUV kinematic parameters. However, despite these new opportunities, the drawbacks in the simplifications of kinematic matrices [17, 18] for the problems in dynamics in the course of repair and assembly essentially affect the AUV motion and prevent the work of the decision maker. Thus, in the absence of results from instantaneous forecasting of relative positions obtained based on the analytical solutions for the problems on dynamics [16–18], taking into account the actual angular positions of AUV, of the manipulator and other elements, the error in determining the positions exceeds the permissible one [17]. The latter considerably complicates construction of the analytical solution while the problem on accounting for the actual angular positions for the prediction and evaluation of action alternatives becomes particularly relevant, rendering it the main unresolved task.

2. Literature review and problem statement

Modern literature [3] demonstrates a growing trend towards improving efficiency in the promising projects of AUV through the introduction of ACS. Such a conclusion has been drawn lately for the three types of devices: technological, survey and monitoring, AUV and equipment carriers [7]. The design of these types of AUV projects is complemented by additional technological equipment and tools, that change in their operation the shape of hulls and their spatial location on them [8]. In addition, the relative orientation of parts of the body [10] or of the onboard technological equipment changes [12]. As a result of such displacements, there arise, on one hand, additional forces and moments whose compensation becomes impossible [13]. On the other hand, accounting for the forces of friction, heel, trim and inertial properties [18] is complicated for the control systems of technological operations [16]. It should be specifically noted that under these circumstances, the variable asymmetry and change in geometric properties of the body are the main causes for the unmanaged dynamics of AUV [14]. Such manifestations complicate the process of preparing personnel to effectively manage the AUV automated control systems. It should be noted that the success rate observed recently in determining individual learning trajectory and the formation of controlling rules is achieved only based on the individual personnel testing [19]. Such a comprehensive approach, despite its complexity, along with the modern teaching methods opens up new opportunities [20]. Thus, the lack of methods that allow the mathematical models to account for peculiarities in the change of geometry and inertia moments of the hull, angular position and kinematic matrix, predetermines the emergence of error in the process of control over mechanical motions of AUV [18]. The latter is due to the qualitative estimates in the prediction of states, which, along with inadequate level of personnel qualification, leads to an emergency situation, or a complete failure, or partial damage to separate components and equipment [5, 10]. Current publications showcase the methods for building up systems to make decisions based on the principles of theory of fuzzy sets [21]. For example, applying for ground-based mobile complexes [21], equipped with manipulators of developed sensor systems [22], the principles of fuzzy logic partially eliminates the drawbacks in the models of dynamics. Other approaches that are implemented for AUV are the use of feedback navigation systems [23], or a recurrent network to control the tethered UV [24]. However, they are based either on the deterministic data from a feedback sensor system or on data of the deterministic model [21, 22], which is represented in ICS [24]. This is especially true for AUV, which lack wireless navigation systems and which do not have external factors for determining their spatial position [23]. The main contradictions between two forms of recording the motion equations in the inertial and connected coordinate systems for AUV, which moves in the fluid, are asymmetrical. Thus, the ease of determining the angular position and coordinates and the complexity of recording the forces and moments are typical of ICS [23]. And vice versa, the connected coordinate system is characterized by the ease of determining the forces and moments and the complexity in determining the angular position and coordinates [24]. The latter makes the main unresolved problem the construction of model that would take into account the impact of angular position of AUV in ICS and would enable determining its kinematical parameters, selecting it by several alternatives [25]. Analyticity of the expressions that are capable by their accuracy and simplicity to perform rapid predictive calculations will make it possible to apply them in contemporary systems for decision-making [17] and to compare the alternatives by several criteria [26].

3. Aim and tasks of the study

The aim of present study is to formulate a mathematical model that would account for the impact of angular position and kinematic parameters in the modeling of AUV motion in ICS.

To accomplish the research aim, we shall state and resolve the following tasks:

– to build up a solution for the direct problem of AUV dynamics in ICS as a recurrent sequence, with regard to the angular position and kinematic parameters;

– to synthesize an algorithm, that provides for the representation of recurrent sequence and calculations for the analytical representation of solution for the direct problem of AUV dynamics;

– to put forward an assessment for the error that arises as a consequence of angular deviations and simplifications of kinematic matrix as a function of the UUV parameters and kinematic parameters in the modeling of AUV motion in ICS.

4. Statement of the problem on AUV dynamics in ICS with regard to angular position and kinematic parameters

Let us consider an equation of the AUV motion with the hull that contains openings. An influence of infiltration effects and hull's dissymmetry will be taken into account in the form of magnitudes in the coefficients of added masses, which were derived in papers [17, 27]. We shall choose point O, which is located in the AUV hull and is rigidly connected to it, as the reference origin in the OXYZ coordinate system. We assume that the AUV hull is an absolutely solid body, and the origin of coordinates O moves translationally at speed $\overline{v}$ and rotates at angular velocity $\overline{\Omega}$. Next, we consider as known time functions the main vector of external forces $\overline{R}$ and the main moment of external forces $\overline{M}$. According to the laws of mechanics, by analogy with the description of motion of an absolutely solid body in boundless incompressible fluid, presented, for example, in articles [5, 6], we write down the motion equation of AUV in the classical setting, and in the associated coordinate system that are reduced by [18, 27] to the system of equations:

$$\frac{dv_j}{dt}=\frac{1}{a_{jj}}\left\{R_j-G_{rj}(t)\right\},\ \ \frac{d\omega_j}{dt}=\frac{1}{a_{j+3,j+3}}\left\{M_j-G_{mj}(t)\right\}, \qquad (1)$$

where index j runs over values 1, 2, 3 and corresponds to projections onto the x, y, z axes. In order to simplify the appearance, we indicated functions in the equations of system (1):

$$\begin{aligned}G_{rx}(t)=&\frac{1}{2}\frac{d}{dt}\left[(a_{15}+a_{51})\omega_y+(a_{16}+a_{61})\omega_z\right]+\\&+\omega_y\left\{a_{33}v_z+\frac{1}{2}\left[(a_{34}+a_{43})\omega_x+(a_{35}+a_{53})\omega_y\right]\right\}-\\&-\omega_z\left\{a_{22}v_y+0.5\left[(a_{24}+a_{42})\omega_x+(a_{26}+a_{62})\omega_z\right]\right\};\end{aligned}$$

$$\begin{aligned}G_{ry}(t)=&\frac{1}{2}\frac{d}{dt}\left[(a_{24}+a_{42})\omega_x+(a_{26}+a_{62})\omega_z\right]+\\&+\omega_z\left\{a_{11}v_x+\frac{1}{2}\left[(a_{15}+a_{51})\omega_y+(a_{16}+a_{61})\omega_z\right]\right\}-\\&-\omega_x\left\{a_{33}v_z+0.5\left[(a_{34}+a_{43})\omega_x+(a_{35}+a_{53})\omega_y\right]\right\};\end{aligned}$$

$$\begin{aligned}G_{rz}(t)=&\frac{1}{2}\frac{d}{dt}\left[(a_{34}+a_{43})\omega_x+(a_{35}+a_{53})\omega_y\right]+\\&+\omega_x\left\{a_{22}v_y+\frac{1}{2}\left[(a_{24}+a_{42})\omega_x+(a_{26}+a_{62})\omega_z\right]\right\}-\\&-\omega_y\left\{a_{11}v_x+0.5\left[(a_{15}+a_{51})\omega_y+(a_{16}+a_{61})\omega_z\right]\right\}; \qquad (2)\end{aligned}$$

$$G_{mx}(t)=\left\{\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left[\begin{aligned}&(a_{24}+a_{42})v_y+(a_{34}+a_{43})v_z+\\&+(a_{45}+a_{54})\omega_y+(a_{46}+a_{64})\omega_z\end{aligned}\right]+\\&+\omega_y\left[a_{66}\omega_z+\frac{1}{2}\left[\begin{aligned}&(a_{16}+a_{61})v_x+(a_{26}+a_{62})v_y+\\&+(a_{46}+a_{64})\omega_x+(a_{56}+a_{65})\omega_y\end{aligned}\right]\right]-\\&-\omega_z\left[a_{55}\omega_y+\frac{1}{2}\left[\begin{aligned}&(a_{15}+a_{51})v_x+(a_{35}+a_{53})v_z+\\&+(a_{45}+a_{54})\omega_x+(a_{56}+a_{65})\omega_z\end{aligned}\right]\right]+\\&+v_y\left[a_{33}v_z+\frac{1}{2}\left[(a_{34}+a_{43})\omega_x+(a_{35}+a_{53})\omega_y\right]\right]-\\&-v_z\left[a_{22}v_y+\frac{1}{2}\left[(a_{24}+a_{42})\omega_x+(a_{26}+a_{62})\omega_z\right]\right]\end{aligned}\right\},$$

$$G_{my}(t)=\left\{\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left[\begin{aligned}&(a_{15}+a_{51})v_x+(a_{35}+a_{53})v_z+\\&+(a_{45}+a_{54})\omega_x+(a_{56}+a_{65})\omega_z\end{aligned}\right]+\\&\omega_z\left[a_{44}\omega_x+\frac{1}{2}\left[\begin{aligned}&(a_{24}+a_{42})v_y+(a_{34}+a_{43})v_z+\\&+(a_{45}+a_{54})\omega_y+(a_{46}+a_{64})\omega_z\end{aligned}\right]\right]-\\&-\omega_x\left[a_{66}\omega_z+\frac{1}{2}\left[\begin{aligned}&(a_{16}+a_{61})v_x+(a_{26}+a_{62})v_y+\\&+(a_{46}+a_{64})\omega_x+(a_{56}+a_{65})\omega_y\end{aligned}\right]\right]+\\&+v_z\left[a_{11}v_x+\frac{1}{2}\left[(a_{15}+a_{51})\omega_y+(a_{16}+a_{61})\omega_z\right]\right]-\\&-v_x\left[a_{33}v_z+\frac{1}{2}\left[(a_{34}+a_{43})\omega_x+(a_{35}+a_{53})\omega_y\right]\right]\end{aligned}\right\},$$

$$G_{mz}(t)=\left\{\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left[\begin{aligned}&(a_{16}+a_{61})v_x+(a_{26}+a_{62})v_y+\\&+(a_{46}+a_{64})\omega_x+(a_{56}+a_{65})\omega_y\end{aligned}\right]+\\&\omega_x\left[a_{55}\omega_y+\frac{1}{2}\left[\begin{aligned}&(a_{15}+a_{51})v_x+(a_{35}+a_{53})v_z+\\&+(a_{45}+a_{54})\omega_x+(a_{56}+a_{65})\omega_z\end{aligned}\right]\right]-\\&-\omega_y\left[a_{44}\omega_x+\frac{1}{2}\left[\begin{aligned}&(a_{24}+a_{42})v_y+(a_{34}+a_{43})v_z+\\&+(a_{45}+a_{54})\omega_y+(a_{46}+a_{64})\omega_z\end{aligned}\right]\right]+\\&+v_x\left[a_{22}v_y+\frac{1}{2}\left[(a_{24}+a_{42})\omega_x+(a_{26}+a_{62})\omega_z\right]\right]-\\&-v_y\left[a_{11}v_x+\frac{1}{2}\left[(a_{15}+a_{51})\omega_y+(a_{16}+a_{61})\omega_z\right]\right]\end{aligned}\right\}. \qquad (3)$$

The system (1) received in [17, 27] is nonlinear because the projections of main vector of external forces include friction forces, which are proportional to the square of projections of linear and angular velocity vectors. General expressions of external forces and moments are given in [18], which is why they will not be presented in the current article, though we shall define the forces and moments that are caused by the work of elements that move.

4. 1. Forces and moments that occur as result of work of the manipulator and other additional elements

When performing technological operations, the manipulator and equipment to fix AUV: gripper, fixing element, hydraulic suction unit, are protruded outside the lightweight body.

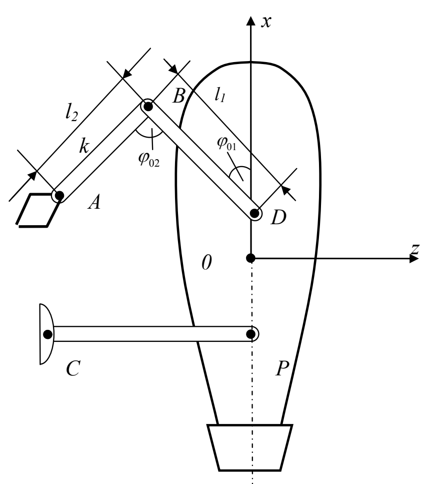

Fig. 1. Kinematic schematic of the manipulator on board and the element for fixing AUV

Fig. 1 schematically shows the manipulator, Fig. 2 – fixing element with grippers in working position. Their action is replaced with the forces of support reaction and moments that are concentrated in the fixing points D and P. The magnitude of the force will be written through the mass and volume of gripper and fixing element:

$$\begin{aligned}&R_{3x}=\left[\rho_w(V_{feg}+V_3)-m_{feg}-m_3\right]g\sin\psi;\\&R_{3y}=\left[\rho_w(V_{feg}+V_3)-m_{feg}-m_3\right]g\cos\psi\cos\theta; \qquad (4)\\&R_{3z}=\left[\rho_w(V_{feg}+V_3)-m_{feg}-m_3\right]g\cos\psi\sin\theta,\end{aligned}$$

where ρ_w ,V_{feg}, m_{feg}, l_{feg} are the density of water, volume, weight and length of fixing element; V_3, m_3, l_{cgg}, l_g are the volume, mass and coordinate of the center of weight and length of the gripper, respectively.

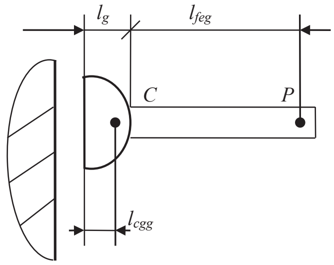

Fig. 2. Constructive scheme of fixing element

Coordinates of the points of centers of gripper weight and fixing element are determined by the coordinates of point P, provided the OZ and element's axes are parallel:

$$X_{cgg} = x_{feg} = x_p,\; Y_{cgg} = y_{feg} = y_p,$$

$$Z_{cgg} = Z^0_{feg} + z\ ;\ Z_{feg} = Z^0_{feg} + z_p, \tag{5}$$

where

$$Z^0_{ogg} = l_3 - l_{cgg} + l_{feg} + \frac{S_z}{2\pi}\int_0^t\int_0^t \frac{M_r}{J_z}dt^2,$$

$$Z^0_{feg} = \frac{1}{2}l_{fego} + \frac{S_z}{2\pi}\int_0^t\int_0^t \frac{M_r}{J_z}dt^2.$$

The action of the manipulator at AUV is substituted by forces:

$$R_x = \left[\rho_w(V_1+V_2) - m_1 - m_2\right]g\sin\psi;$$

$$R_y = \left[\rho_w(V_1+V_2) - m_1 - m_2\right]g\cos\psi\cos\theta;$$

$$R_z = \left[\rho_w(V_1+V_2) - m_1 - m_2\right]g\cos\psi\sin\theta, \tag{6}$$

where V_1, V_2 and m_1, m_2 are the volume and masses of the first and second elements of the manipulator. Coordinates of the centers of weight of the manipulator elements are determined by the coordinates of points D, respectively:

$$X_{cg1} = X_D + \frac{l_1}{2}\cos\varphi_1;\ \varphi_1 = (\omega_1 t + \varphi_{01});$$

$$X_{cg2} = X_D + l_1\cos\varphi_1 + \frac{l_2}{2}\cos(\omega_2 t + \pi - \varphi_{02} + \varphi_1);$$

$$Y_{cg1} = Y_{cg2} = Y_D;$$

$$Z_{cg1} = Z_D - \frac{l_1}{2}\sin\varphi_1;$$

$$Z_{cg2} = Z_D - l_1\sin\varphi_1 - \frac{l_2}{2}\sin(\omega_2 t + \pi - \varphi_{02} + \varphi_1). \tag{7}$$

Projections of the moment of forces, created by the reaction of support, are calculated by the coordinates of point in the basis of fixing element of the gripper:

$$M_{3x} = R_{3z}y_p - R_{3y}z_p;$$

$$M_{3y} = R_{3x}z_p - R_{3z}x_p;$$

$$M_{3z} = R_{3y}x_p - R_{3x}y_p, \tag{8}$$

or the manipulator:

$$M_{Dx} = R_z y_D - R_y z_D,$$

$$M_{Dy} = R_x z_D - R_z x_D,$$

$$M_{Dz} = R_y x_D - R_x y_D. \tag{9}$$

There is no doubt that the reduced expressions for the magnitudes of forces and moments in a general case are the time functions and are defined by the types of drives for the mechanisms of kinematic elements of both the manipulator and fixing elements.

4. 2. Solving the systems of motion equations of AUV

In order to form a solution for the system of nonlinear differential equations, in view of its nonlinearity, we shall apply the expansion by the method of recurrent approximation [17]. An explanation for the idea of one possible approach, for simplicity and transparency, will be performed on the example of the first equation from system (1). According to this approach, confined by linear approximation scheme [17] of polynomials of order K, we write down:

$$\frac{dv_{x,n+1}}{dt} = \frac{1}{a_{11}}\left\{\begin{array}{l} R_{xn} + \left(v_{x,n+1} - v_{x,n}\right)\times \\ \times\sum_{k=1}^{K}\left[\left.\frac{\partial^k R_x}{\partial v_x^k}\right|_{v_x=v_{xn}} \frac{\left(v_{x,n} - v_{x,n-1}\right)^{k-1}}{k!}\right] - G_{rxn}(t)\end{array}\right\}. \tag{10}$$

Derivatives from the projection of force with regard to the expression and direction of the friction force, upon differentiation and algebraic transforms, will be written down as:

$$\frac{\partial R_j}{\partial v_j} = -\frac{1}{2}\rho_w S C_j v\left(\frac{v_j^2}{v^2} + 1\right). \tag{11}$$

We shall introduce an integrating multiplier:

$$F(R_j, v_{jn}, s) = \exp\left(-\frac{1}{a_{jj}}\int_0^s\sum_{k=1}^{K}\left[\left.\frac{\partial^k R_j}{\partial v_j^k}\right|_{v_j=v_{jn}} \frac{\left(v_{j,n} - v_{j,n-1}\right)^{k-1}}{k!}\right]dr\right), \tag{12}$$

$$F(M_j, \omega_{jn}, s) = \exp\left(\frac{1}{a_{j+3,j+3}}\int_0^s\sum_{k=1}^{K}\left[\left.\frac{\partial M_j}{\partial \omega_j}\right|_{\omega_j=\omega_{jn}} \frac{\left(\omega_{j,n} - \omega_{j,n-1}\right)^{k-1}}{k!}\right]dr\right), \tag{13}$$

where j runs over values 1, 2, 3 and corresponds to the x, y, z coordinates. Assuming in the first approximation that the values of velocities and angles that are included in the right part of the equation are equal to the initial values, taking into account the initial conditions, we shall record recurrent solution (10) in the form:

$$v_{x,n+1} = \left[F(R_x, v_{xn}, s)\right]^{-1}\times$$

$$\times\left\langle v_{xo} + \frac{1}{a_{11}}\int_0^t\left\{\begin{array}{l}\left[\begin{array}{l} R_{xn} - G_{rxn}(t) - \\ -v_{x,n}\sum_{k=1}^{K}\left[\left.\frac{\partial^k R_x}{\partial v_x^k}\right|_{v_x=v_{xn}} \frac{\left(v_{x,n} - v_{x,n-1}\right)^{k-1}}{k!}\right]\end{array}\right]\times \\ \times F(R_x, v_{xn}, s)\end{array}\right\}ds\right\rangle; \tag{14}$$

$$v_{y,n+1}=\left[F(R_y,v_{yn},s)\right]^{-1}\times \left\langle v_{yo}+\frac{1}{a_{22}}\int_0^t\left\{\left[\begin{array}{l}R_{yn}-G_{ryn}(t)-\\ -v_{y,n}\sum_{k=1}^{K}\left[\left.\frac{\partial^k R_y}{\partial v_y^k}\right|_{v_y=v_{yn}}\frac{(v_{y,n}-v_{y,n-1})^{k-1}}{k!}\right]\\ \times F(R_y,v_{yn},s)\end{array}\right]\times\right\}ds\right\rangle; \quad (15)$$

$$v_{z,n+1}=\left[F(R_z,v_{zn},s)\right]^{-1}\times \left\langle v_{zo}+\frac{1}{a_{33}}\int_0^t\left\{\left[\begin{array}{l}R_{zn}-G_{rzn}(t)-\\ -v_{z,n}\sum_{k=1}^{K}\left[\left.\frac{\partial^k R_z}{\partial v_z^k}\right|_{v_z=v_{zn}}\frac{(v_{z,n}-v_{z,n-1})^{k-1}}{k!}\right]\\ \times F(R_z,v_{zn},s)\end{array}\right]\times\right\}ds\right\rangle; \quad (16)$$

$$\omega_{x,n+1}=\left[F(M_x,\omega_{xn},s)\right]^{-1}\times \left\langle \omega_{xo}+\frac{1}{a_{44}}\int_0^t\left\{\left[\begin{array}{l}M_{xn}-G_{mxn}(t)-\\ -\omega_{x,n}\sum_{k=1}^{K}\left[\left.\frac{\partial M_x}{\partial \omega_x}\right|_{\omega_x=\omega_{xn}}\frac{(\omega_{x,n}-\omega_{x,n-1})^{k-1}}{k!}\right]\\ \times F(M_x,\omega_{xn},s)\end{array}\right]\times\right\}ds\right\rangle; \quad (17)$$

$$\omega_{y,n+1}=\left[F(M_y,\omega_{yn},s)\right]^{-1}\times \left\langle \omega_{yo}+\frac{1}{a_{55}}\int_0^t\left\{\left[\begin{array}{l}M_{yn}-G_{myn}(t)-\\ -\omega_{y,n}\sum_{k=1}^{K}\left[\left.\frac{\partial M_y}{\partial \omega_y}\right|_{\omega_y=\omega_{yn}}\frac{(\omega_{y,n}-\omega_{y,n-1})^{k-1}}{k!}\right]\\ \times F(M_y,\omega_{yn},s)\end{array}\right]\times\right\}ds\right\rangle; \quad (18)$$

$$\omega_{z,n+1}=\left[F(M_z,\omega_{zn},s)\right]^{-1}\times \left\langle \omega_{zo}+\frac{1}{a_{66}}\int_0^t\left\{\left[\begin{array}{l}M_{zn}-G_{mzn}(t)-\\ -\omega_{z,n}\sum_{k=1}^{K}\left[\left.\frac{\partial M_z}{\partial \omega_z}\right|_{\omega_z=\omega_{zn}}\frac{(\omega_{z,n}-\omega_{z,n-1})^{k-1}}{k!}\right]\\ \times F(M_z,\omega_{zn},s)\end{array}\right]\times\right\}ds\right\rangle. \quad (19)$$

We denote by number in the lower index the number of component approximation defined in the corresponding approximation of velocity. Assume that index zero denotes initial value of the generalized coordinates and velocities in the initial time moment, and then we shall write down for the first approximation:

$$v_{x1}=v_{x0}+\frac{1}{a_{11}}\int_0^t\left[R_{x0}(\tau)-G_{rx0}(\tau)\right]d\tau; \quad (20)$$

$$v_{y1}=v_{y0}+\frac{1}{a_{22}}\int_0^t\left[R_{y0}(\tau)-G_{ry0}(\tau)\right]d\tau; \quad (21)$$

$$v_{z1}=v_{z0}+\frac{1}{a_{33}}\int_0^t\left[R_{z0}(\tau)-G_{zy0}(\tau)\right]d\tau; \quad (22)$$

$$v_1=\sqrt{v_{x1}^2+v_{y1}^2+v_{z1}^2}. \quad (23)$$

The current position of velocity vector in the associated coordinate system will be defined also by the first approximation and by angles β – of drift and α – of attack whose trigonometric functions are calculated by the projections of vector of instantaneous velocity:

$$\sin\beta=v_{z1}/v_1;$$

$$\cos\beta=\sqrt{1-(v_{z1}/v_1)^2};$$

$$\sin\alpha=v_{y1}/v_1\cos\beta;$$

$$\cos\alpha=\sqrt{1-(v_{y1}/v_1\cos\beta)^2}. \quad (24)$$

Determining these angles and the values of component of the velocity vector in the first approximation (20)–(22) allows us to recalculate the projections of friction force and angles of heel, course and trim, respectively:

$$\omega_{x1}=\omega_{x0}+\frac{1}{a_{44}}\int_0^t\left[M_{x0}(\tau)-G_{mx0}(\tau)\right]d\tau;$$

$$\omega_{y1}=\omega_{y0}+\frac{1}{a_{55}}\int_0^t\left[M_{y0}(\tau)-G_{my0}(\tau)\right]d\tau;$$

$$\omega_{z1}=\omega_{z0}+\frac{1}{a_{66}}\int_0^t\left[M_{z0}(\tau)-G_{mz0}(\tau)\right]d\tau;$$

$$\omega_1=\sqrt{\omega_{x1}^2+\omega_{y1}^2+\omega_{z1}^2}. \quad (25)$$

By using connection equations of the rotational motion, we shall associate projections of the angular velocity vector with the associated axes and the Euler angles derivatives – of heel, course and trim [5]:

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}=\begin{bmatrix}\dot\theta+\dot\varphi\sin\psi\\ \dot\varphi\cos\theta\cos\psi+\dot\psi\sin\theta\\ -\dot\varphi\sin\theta\cos\psi+\dot\psi\cos\theta\end{bmatrix}. \quad (26)$$

The latter will enable recording the kinematic matrix B_ω:

$$B_\omega=\begin{bmatrix}1,\sin\psi,0\\ 0,\cos\psi\cos\theta,\sin\theta\\ 0,-\cos\psi\sin\theta,\cos\theta\end{bmatrix}$$

and, upon solving the system relative to the Euler angles derivatives, to find B_ω^{-1} inverse to it:

$$B_\omega^{-1}=\begin{bmatrix}1,-\sin\psi,\sin\theta\,\mathrm{tg}\psi\\ 0, \quad 1, \quad -\dfrac{\sin\theta}{\cos\psi}\\ 0, \quad \sin\theta, \quad \cos\theta\end{bmatrix}. \quad (27)$$

In most of the problems on modeling, they are typically simplified to the form [5]:

$$B_\omega=\begin{bmatrix}1,\sin\psi,\ 0\\ 0,\cos\psi,0\\ 0,\ 0,\ 1\end{bmatrix};\quad B_\omega^{-1}=\begin{bmatrix}1,-\sin\psi,0\\ 0,\ 1,\ 0\\ 0,\ 0,\ 1\end{bmatrix}, \quad (28)$$

which is exactly one of the sources of error that occurs in the process of modeling the dynamics. Received expressions of linear and angular velocities are presented in the associated coordinate system. Based on the consistent rotation by directions of heel, course and trim, we shall represent the angular velocity vector in the associated coordinate system as the product of kinematic matrix of rotational motion by the matrix column-angular velocity vector in ICS [5]:

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}=B_\omega\begin{bmatrix}\dot\theta\\ \dot\varphi\\ \dot\psi\end{bmatrix}. \qquad (29)$$

A transition to ICS is performed by using the inverse kinematic matrix of rotational motion by the ratios:

$$\begin{bmatrix}\dot\theta\\ \dot\varphi\\ \dot\psi\end{bmatrix}=B_\omega^{-1}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix},$$

or, in integral form:

$$\begin{bmatrix}\theta\\ \varphi\\ \psi\end{bmatrix}=\int_0^t B_\omega^{-1}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}d\tau,$$

where the inverse matrix takes the form (27). Under conditions of the first approximation of components of the angular velocity vector, the angles of heel, course and trim in the first approximation (denoted by lower index 1) will be represented through integrals:

$$\theta_1=\theta_0+\int_0^t\left[\omega_{x1}-\omega_{y1}\sin\psi+\omega_{z1}\sin\theta\,\mathrm{tg}\psi\right]d\tau;$$

$$\varphi_1=\varphi_0+\int_0^t\left[\omega_{y1}-\frac{\omega_{z1}\sin\theta}{\cos\psi}\right]d\tau;$$

$$\psi_1=\psi_0+\int_0^t\left[\omega_{y1}\sin\theta+\omega_{z1}\cos\theta\right]d\tau. \qquad (30)$$

The coordinates of point of the center of mass in the inertial coordinate system, as shown in paper [5], will be represented through the components of its velocity vector in the associated coordinate system by using the inverse kinematic matrix of translational motion

$$B_v^{-1}=\begin{bmatrix}\cos\varphi\cos\psi, & (\sin\varphi\sin\theta-\cos\varphi\sin\psi\cos\theta), & (\cos\varphi\sin\psi\sin\theta+\sin\varphi\cos\theta)\\ \sin\psi, & \cos\psi\cos\theta, & -\cos\psi\sin\theta\\ -\sin\varphi\cos\psi, & (\sin\varphi\sin\psi\cos\theta+\cos\varphi\sin\theta), & (\cos\varphi\cos\theta-\sin\varphi\sin\psi\sin\theta)\end{bmatrix} \qquad (31)$$

in the following fashion: $\frac{d}{dt}\begin{bmatrix}x_g\\ y_g\\ z_g\end{bmatrix}=B_v^{-1}\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix},$

or, in integral form: $\begin{bmatrix}x_g\\ y_g\\ z_g\end{bmatrix}=\int_0^t B_v^{-1}\begin{bmatrix}v_x\\ v_y\\ v_z\end{bmatrix}d\tau.$

Coordinates of the center of masses in the first approximation (denoted by lower index 1), upon substitution of expressions for the components of linear velocity vector, angle of heel, course, trim, will be represented through integrals:

$$\begin{bmatrix}x_{g1}\\ y_{g1}\\ z_{g1}\end{bmatrix}=\begin{bmatrix}x_{g0}\\ y_{g0}\\ z_{g0}\end{bmatrix}+\int_0^t\left\{\begin{bmatrix}\begin{bmatrix}v_{x1}\cos\varphi_1\cos\psi_1+\\ +v_{y1}(\sin\varphi_1\sin\theta_1-\cos\varphi_1\sin\psi_1\cos\theta_1)+\\ +v_{z1}(\cos\varphi_1\sin\psi_1\sin\theta_1+\sin\varphi_1\cos\theta_1)\end{bmatrix}\\ \left[v_{x1}\sin\psi_1+v_{y1}\cos\psi_1\cos\theta_1-v_{z1}\cos\psi_1\sin\theta_1\right]\\ \begin{bmatrix}-v_{x1}\sin\varphi_1\cos\psi_1+\\ +v_{y1}(\sin\varphi_1\sin\psi_1\cos\theta_1+\cos\varphi_1\sin\theta_1)+\\ +v_{z1}(\cos\varphi_1\cos\theta_1-\sin\varphi_1\sin\psi_1\sin\theta_1)\end{bmatrix}\end{bmatrix}\right\}d\tau. \qquad (32)$$

The second approximation will be received with regard to the first, and so on by the recurrent algorithm:

$$v_{x,2}=\exp\left(-\frac{\rho SC_x}{2a_{11}}\int_0^t v_1\left(\frac{v_{x1}^2}{v_1^2}+1\right)dq\right)\times$$
$$\times\left\langle v_{xo}+\frac{1}{a_{11}}\int_0^t\left\{\begin{matrix}\left[R_{x1}-G_{rx1}(q)+v_{x,1}\frac{\rho SC_x}{2}v_1\left(\frac{v_{x1}^2}{v_1^2}+1\right)\right]\times\\ \times\exp\left(\frac{\rho SC_x}{2a_{11}}\int_0^q v_1\left(\frac{v_{x1}^2}{v_1^2}+1\right)dr\right)\end{matrix}\right\}dq\right\rangle; \qquad (33)$$

$$v_{y,2}=\exp\left(-\frac{\rho SC_y}{2a_{22}}\int_0^t v_1\left(\frac{v_{y1}^2}{v_1^2}+1\right)dq\right)\times$$
$$\times\left\langle v_{yo}+\frac{1}{a_{22}}\int_0^t\left\{\begin{matrix}\left[R_{y1}-G_{ry1}(q)+v_{y,1}\frac{\rho SC_y}{2}v_1\left(\frac{v_{y1}^2}{v_1^2}+1\right)\right]\times\\ \times\exp\left(\frac{\rho SC_y}{2a_{22}}\int_0^q v_1\left(\frac{v_{y1}^2}{v_1^2}+1\right)dr\right)\end{matrix}\right\}dq\right\rangle; \qquad (34)$$

$$v_{z,2}=\exp\left(\frac{\rho SC_z}{2a_{33}}\int_0^t v_1\left(\frac{v_{z1}^2}{v_1^2}+1\right)dq\right)\times$$
$$\times\left\langle v_{zo}+\frac{1}{a_{33}}\int_0^t\left\{\begin{matrix}\left[R_{z1}-G_{rz1}(q)-v_{z,1}\frac{\rho SC_z}{2}v_1\left(\frac{v_{z1}^2}{v_1^2}+1\right)\right]\times\\ \times\exp\left(-\frac{\rho SC_z}{2a_{33}}\int_0^s v_1\left(\frac{v_{z1}^2}{v_1^2}+1\right)dr\right)\end{matrix}\right\}dq\right\rangle; \qquad (35)$$

$$\omega_{x,2}=\left[F(M_x,\omega_{x1},s)\right]^{-1}\times$$
$$\times\left\langle \omega_{xo}+\frac{1}{a_{44}}\int_0^t\left\{\begin{matrix}\begin{bmatrix}M_{x1}-G_{mx1}(q)-\\ -\omega_{x,1}\sum_{k=1}^{K}\left[\left.\frac{\partial M_x}{\partial\omega_x}\right|_{\omega_x=\omega_{x1}}\frac{(\omega_{x,1}-\omega_{x,0})^{k-1}}{k!}\right]\end{bmatrix}\times\\ \times F(M_x,\omega_{x1},s)\end{matrix}\right\}ds\right\rangle; \qquad (36)$$

$$\omega_{y,2}=\left[F(M_y,\omega_{y1},s)\right]^{-1}\left\langle \omega_{yo}+\frac{1}{a_{55}}\int_0^t\left\{\left[M_{y1}-G_{my1}(q)-\omega_{y,1}\sum_{k=1}^{K}\left[\left.\frac{\partial M_y}{\partial\omega_y}\right|_{\omega_y=\omega_{y1}}\frac{(\omega_{y,1}-\omega_{y,0})^{k-1}}{k!}\right]\right]F(M_y,\omega_{y1},s)\right\}ds\right\rangle; \quad (37)$$

$$\omega_{z,2}=\left[F(M_z,\omega_{z1},s)\right]^{-1}\left\langle \omega_{zo}+\frac{1}{a_{66}}\int_0^t\left\{\left[M_{z1}-G_{mz1}(q)-\omega_{z,1}\sum_{k=1}^{K}\left[\left.\frac{\partial M_z}{\partial\omega_z}\right|_{\omega_z=\omega_{z1}}\frac{(\omega_{z,1}-\omega_{z,0})^{k-1}}{k!}\right]\right]F(M_z,\omega_{z1},s)\right\}ds\right\rangle. \quad (38)$$

A transition from the associated coordinate system to the inertial will be carried out using the first and the second approximation of angles after finding them by the second approximation of velocity, respectively:

– for the angles:

$$\theta_2=\theta_0+\int_0^t\left[\omega_{x2}-\omega_{y2}\sin\psi_1+\omega_{z2}\sin\theta_1\operatorname{tg}\psi_1\right]d\tau;$$

$$\varphi_2=\varphi_0+\int_0^t\left[\omega_{y2}-\frac{\omega_{z2}\sin\theta_1}{\cos\psi_1}\right]d\tau; \quad \psi_2=\psi_0+\int_0^t\left[\omega_{y2}\sin\theta_1+\omega_{z2}\cos\theta_1\right]d\tau. \quad (39)$$

– for the coordinates:

$$\begin{bmatrix}x_{g2}\\ y_{g2}\\ z_{g2}\end{bmatrix}=\begin{bmatrix}x_{g0}\\ y_{g0}\\ z_{g0}\end{bmatrix}+$$

$$+\int_0^t\begin{bmatrix}v_{x2}\cos\varphi_2\cos\psi_2+v_{y2}(\sin\varphi_2\sin\theta_2-\cos\varphi_2\sin\psi_2\cos\theta_2)+v_{z2}(\cos\varphi_2\sin\psi_2\sin\theta_2+\sin\varphi_2\cos\theta_2)\\ v_{x2}\sin\psi_2+v_{y2}\cos\psi_2\cos\theta_2-v_{z2}\cos\psi_2\sin\theta_2\\ -v_{x2}\sin\varphi_2\cos\psi_2+v_{y2}(\sin\varphi_2\sin\psi_2\cos\theta_2+\cos\varphi_2\sin\theta_2)+v_{z2}(\cos\varphi_2\cos\theta_2-\sin\varphi_2\sin\psi_2\sin\theta_2)\end{bmatrix}d\tau. \quad (40)$$

The second approach to solving the system of differential motion equations takes into account projections of the linear and angular velocities on other axes. Thus, for example, after substituting derivatives and velocities in the first of the six equations of system (1) with regard to denotations (2), (3), after algebraic transforms, we shall obtain:

$$\frac{dv_{x,n+1}}{dt}=\frac{1}{a_{11}}\left\{\begin{array}{l}R_{xn}+(v_{x,n+1}-v_{x,n})\left.\dfrac{\partial R_X}{\partial v_X}\right|_{v_x=v_{xn}}-\\ -\left\{\begin{array}{l}\dfrac{a_{15}}{a_{55}}\left\{M_y-\left\{\begin{array}{l}\dfrac{d}{dt}\left[a_{15}v_x+a_{35}v_z+a_{45}\omega_x+a_{56}\omega_z\right]+\\ +\omega_z\left[a_{44}\omega_x+\left[\begin{array}{l}a_{24}v_y+a_{34}v_z+\\ +a_{45}\omega_y+a_{46}\omega_z\end{array}\right]\right]-\omega_x\left[a_{66}\omega_z+\left[\begin{array}{l}a_{16}v_x+a_{26}v_y+\\ +a_{46}\omega_x+a_{56}\omega_y\end{array}\right]\right]+\\ +v_z\left[a_{11}v_x+\left[a_{15}\omega_y+a_{16}\omega_z\right]\right]-v_x\left[a_{33}v_z+\left[a_{34}\omega_x+a_{35}\omega_y\right]\right]\end{array}\right.\right\}\right\}+\\ +\dfrac{a_{16}}{a_{66}}\left\{M_z-\left\{\begin{array}{l}\dfrac{d}{dt}\left[a_{16}v_x+a_{26}v_y+a_{46}\omega_x+a_{56}\omega_y\right]+\\ +\omega_x\left[a_{55}\omega_y+\left[a_{15}v_x+a_{35}v_z+a_{45}\omega_x+a_{56}\omega_z\right]\right]-\\ -\omega_y\left[a_{44}\omega_x+\left[a_{24}v_y+a_{34}v_z+a_{45}\omega_y+a_{46}\omega_z\right]\right]+\\ +v_x\left[a_{22}v_y+\left[a_{24}\omega_x+a_{26}\omega_z\right]\right]-v_y\left[a_{11}v_x+\left[a_{15}\omega_y+a_{16}\omega_z\right]\right]\end{array}\right.\right\}\right\}+\\ +\omega_y\left\{a_{33}v_z+\left[a_{34}\omega_x+a_{35}\omega_y\right]\right\}-\\ -\omega_z\left\{a_{22}v_y+\left[a_{24}\omega_x+a_{26}\omega_z\right]\right\}\end{array}\right.\end{array}\right\}. \quad (41)$$

The last differential equation as one of the equations in system (1) contains derivatives from the components of linear and angular velocity vector. It is by its volume and form is more cumbersome. Such a transform will only complicate the system, but will not convert it into the quasilinear. In this case, however, to solve the system (1), it is necessary to employ the method of recurrent approximation whose convergence will improve insignificantly. Anyway, analytical form (41) allows us to conclude that the impact of angular velocities on the longitudinal speed of AUV is similar to the increase in the resistance coefficients.

5. Discussion of results: modeling the dynamics of AUV motion in ICS with regard to angular position and kinematic parameters

In order to modeling the dynamics of AUV motion in ICS considering angular position and kinematic parameters, we have chosen the "Skarus" project [10, 17, 28]. The device is additionally equipped with a manipulator and an element for fixing the device to underwater structures. Characteristic parameters that are applied at modeling were taken unchanged from articles [10, 28]. The obtained expressions for the elements of sequence of the first (30), (32) and the second (39), (40) approximations include angles of heel and trim, as well as employ refined kinematic matrices (27) and (31). The presence of such expressions allows us to determine the error that is caused both by the error of angles and the error in the approximation of kinematic matrix:

$$\begin{bmatrix}\Delta\theta\\ \Delta\varphi\\ \Delta\psi\end{bmatrix}=$$

$$=\int_0^t\begin{bmatrix}0,0,\sin\theta\operatorname{tg}\psi\\ 0,0,\dfrac{\sin\theta}{\cos\psi}\\ 0,\sin\theta,\cos\theta-1\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}d\tau=$$

$$=\int_0^t\begin{bmatrix}\omega_z\sin\theta\operatorname{tg}\psi\\ \omega_z\dfrac{\sin\theta}{\cos\psi}\\ \omega_y\sin\theta+\omega_z(\cos\theta-1)\end{bmatrix}d\tau. \quad (42)$$

Results of modeling the impact of the manipulator's work aboard the AUV, which creates a moment of forces up to

100 Nm, on the magnitude of error when calculating by the simplified kinematic matrices are given in Table 1.

Table 1

Analysis of impact from the angles of heel, course, trim and simplifications of kinematic matrices on the magnitude of error when modeling the motion of AUV

Time, t, s	Δθ/θ	Δφ, rad	Δψ, rad	Δθ/θ	Δφ, rad	Δψ, rad
	θ=0,1; ω_z=0,01; ω_y=0,005			θ=0,01; ω_z=0,01; ω_y=0,005		
1	0	0.00099833	0.000449209	0	9.99983E-05	4.94992E-05
2	9.9836E-05	0.01098217	0.004941296	0.000100002	0.010100332	0.004999416
3	0.00029853	0.02096749	0.009433383	0.000299025	0.020102145	0.009949333
4	0.00059613	0.03095527	0.013925471	0.000597118	0.030106421	0.01489925
5	0.0009927	0.04094648	0.018417558	0.00099434	0.04011414	0.019849168
6	0.00148831	0.05094212	0.022909646	0.001490769	0.050126288	0.024799085
7	0.00208306	0.06094316	0.027401733	0.002086502	0.060143851	0.029749002
8	0.00277706	0.07095059	0.03189382	0.002781656	0.070167818	0.034698919
9	0.00357047	0.08096541	0.036385908	0.003576369	0.08019918	0.039648836
10	0.00446342	0.09098861	0.040877995	0.004470798	0.090238935	0.044598753

In addition, it should be noted, that heel error Δθ is affected by both its magnitude θ and the magnitude of trim angle ψ, angular velocity ω_x, and the magnitude of time period t. A similar pattern is characteristic of the error in course angle Δφ. However, as far as the magnitude of error in trim angle Δψ, then it is not explicitly affected by the magnitude of trim ψ, rather it is defined by two angular velocities ω_y and ω_x and heel angle θ. Thus, expressions (42) allow us to analyze the impact of angles of heel, course trim and simplifications of the kinematic matrices and determine the magnitudes of error when modeling the AUV motion in analytical form. The predicted magnitudes of absolute error allow the operator to form an action strategy when operating the manipulator. A decision to use or not to use elements for fixing the body of device is also made based on the magnitude of the predicted error. The latter allows us to reduce errors in determining the position of the working tool.

Due to applying the recurrent approximation method, we received solutions for the direct problem of dynamics (30), (32), (39), (40) for the devised model of AUV with a manipulator aboard. The modeling performed testifies that these solutions eliminate contradictions between the simplicity in recording the external forces and the complexity in determining the spatial position of the AUV body. The latter is the main advantage of the resulting outcome of the study. Despite the fact that the forces and moments are recorded only for one of the possible implementations of kinematic schemes for manipulator and fastener, they will not lose generality for other types of designs.

Relative simplicity of analytical expressions for determining the angular position and coordinates of the centre of weight of the AUV hull and their capability to rapidly calculate position of the elements of manipulator and fastener in the same ICS will not change this benefit. The latter will make it possible to use them for constructing a decision support system for the operator of ACS. Along with this, the proposed algorithm upon verification confirms simplicity of its realization to calculate an error in the angles of angular position vector that occurs as a consequence of simplifications and assumptions. The obtained modeling data allow us to track the character of error dependence and to characterize its changes and impacts; they will also be useful in the systems of information support for control systems designers.

6. Conclusions

1. We constructed and verified the model as a recurrent sequence that describes the dynamics of AUV in ICS and takes into account the angular position and kinematic parameters without simplifications in the kinematic matrix.

2. The algorithm that is synthesized based on this model provides for the sequence of actions and calculations for analytical representation of solution for the direct problem of the AUV dynamics. The calculation of kinematic parameters and simultaneous consideration of angular position when modeling the AUV motion is represented in one inertial coordinate system.

3. Analytical approximations of the model allow us to represent an error that occurs due to angular deviations and simplifications of the kinematic matrices as a function of characteristics and kinematic parameters of AUV. The existence of such expressions will make it possible for the operator of ACS to apply them to predict and select the model when modeling the motion of AUV in the inertial coordinate system.

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