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**Запропоновано сукупність методів передпрогнозного фрактального аналізу часових рядів для визначення рівнів персистентності хаотичних інформаційних потоків в системі управління бурінням свердловини. Для шести часових рядів отримані значення показників Херста  $H$  фрактальної розмірності  $D$ , просторової розмірності  $n$  і міри кореляції  $C$ . Запропоновано отримані результати використовувати для прогнозування і раннього виявлення відхилень технологічного процесу буріння від норми**

**Ключові слова:** персистентність, показник Херста, часовий ряд, фрактальна розмірність, послідовний  $R/S$ -аналіз

**Предложена совокупность методов передпрогнозного фрактального анализа временных рядов для определения уровней персистентности хаотических информационных потоков в системе управления бурением скважины. Для шести временных рядов получены значения показателей Херста  $H$  фрактальной размерности  $D$ , пространственной размерности  $n$  и степени корреляции  $C$ . Предложено полученные результаты использовать для прогнозирования и раннего выявления отклонений технологического процесса бурения от нормы**

**Ключевые слова:** персистентность, показатель Херста, временной ряд, фрактальная размерность, последовательный  $R/S$ -анализ

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# DEVELOPMENT OF A SET OF METHODS FOR PREFORECASTING FRACTAL TIME SERIES ANALYSIS TO DETERMINE THE LEVEL OF PERSISTENCE

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## 1. Introduction

The process of oil and gas well drilling is a non-reproducible, non-stationary, stochastic-chaotic, time-evolving object. Such process is described by nonlinear equations with unknown parameters and proceeds in conditions of a priori and current uncertainty under the influence of immeasurable disturbances. The main task of the drilling control system is to ensure well-construction in the shortest time with the least losses. However, cost minimization in each bit run does not minimize the total cost of well-drilling. The reason for this is the significant effect of time and energy consumption on round-trip and auxiliary operations, elimination of consequences of complications and emergencies, well flushing process control.

Forecasting of energy expenditure for drilling for one or two days or the next few hours, as well as pre-emergency situations and complications should be carried out without stoppage of drilling equipment in real time.

Forecasting difficulties are associated with the presence of different types of accidents and complications that are es-

entially different and usually not amenable to precise mathematical description. Most accidents and complications that arise during drilling can be divided into two main groups. These are accidents and complications caused by geological or technological reasons.

Forecasting process, in this case, can be viewed as a certain sequence of diagnostic checks, the efficiency of which depends on the strategy of diagnosis search in many possible causes based on time series analysis.

Various data on the drilling process come in the form of an ensemble of time series  $x(t) = \{x_j(t)\}$ ,  $j=1,2,\dots,n$ ;  $t \in T$ ;  $T$  is the time span of observation.

Since their main characteristics change over time, they are usually non-stationary. Time series forecasting is fraught with certain difficulties.

Today there are a great many forecasting methods, techniques and methodologies based on two approaches: heuristic or mathematical.

Heuristic methods are subjective, rather complicated and laborious due to the need to combine forecasts obtained from

different experts. Mathematical methods require an explicit mathematical model that describes the behavior of a forecast object correctly. Only in this case, acceptable results of forecasting would be expected.

Therefore, an important scientific and applied problem is the use of methods for the pre-forecasting fractal analysis of chaotic information flows in the system of real-time oil and gas well drilling control.

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## 2. Literature review and problem statement

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The scientific problem of determining the levels of persistence of chaotic time series for forecasting problems based on fractal analysis has been studied in the works of many researchers [1, 2]. Note that the method of fractal time series analysis [1] as part of the methods of discrete nonlinear dynamics is designed to study nonlinearities in the dynamics of time series. The main objective of fractal analysis is to determine the Hausdorff fractal dimension  $D$  and Hurst exponent  $H$  [2] for the discrete time sequence  $y(1), y(2), \dots, y(i), \dots, y(k)$ , where  $k$  is discrete current time.

According to the principles of fractal analysis [3], time series have the fractal dimension  $1 < D < 2$  and are endowed with the properties of scale self-similarity and memory of their initial conditions.

A straight line has the fractal dimension  $D=1$ . If  $D=1$ , the distribution of the fractal time series is a Gaussian. In practical calculations, many authors replace the fractal dimension  $D$  with the Hurst exponent  $H$  based on implementation of the procedure of sequential R/S analysis, where  $R(t)$  is the range of a sequence of the accumulated deviations,  $S(k)$  is standard deviation [4]. Thus, the Hurst exponent  $H$  is the number  $H \in [0;1]$ , which describes the ratio of the trend function component to white noise and can be used as a measure of persistence, i.e. the tendency of processes to trends.

In [5, 6], based on cognitive process modeling, materials management scenarios have been developed, identification of mathematical models has been carried out, process optimization under uncertainty has been performed. However, the applicability of the fractal time series analysis to solve this problem has been substantiated insufficiently.

Current problems in well-drilling automation have been considered in detail [7]. Conclusions have been made about the expediency of developing specialized automation means [8] to compute nonlinear dependencies in real time [9], but such tools have not yet been developed.

The researchers have been focused mainly on the time series analysis based on stochastic fractal models [10, 11], as well as methods for the pre-forecasting fractal time series analysis [12, 13]. Most attention has been paid to the definition of the Hurst exponent  $H$  using the fractal dimension computed only by the box-counting method on the example of short time series [14]. The issue of developing the identification algorithm of chaotic sequences of the well-drilling process [15] without taking into account the capabilities of the current time series analysis has also been considered.

However, these studies have examined only some theoretical principles of the fractal time series analysis. The implementation features of pre-forecasting analysis to determine the types of persistence of time series of such a complex process as oil and gas well drilling have been unexplored.

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## 3. The aim and objectives of the study

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The aim of the research is to develop a set of methods for the pre-forecasting fractal analysis of chaotic information flows in the automated control system of the oil and gas well drilling process.

To achieve this aim, it is necessary to accomplish the following main objectives:

- to combine a set of methods for the pre-forecasting fractal time series analysis in a single methodology;
- to analyze the fractal dimension and the Hurst exponent based on implementation of the procedure of sequential R/S analysis;
- to assess the autocorrelation impact of the previous values of time series on the following values and determine future trends;
- to determine spatial dimensions of the achieved dynamic processes.

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## 4. Materials and methods of research of chaotic information flows

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The methodological basis of this work are the following methods, approaches and techniques for the study of complex dynamic control objects, namely:

- the method of the fractal time series analysis to calculate the fractal dimension;
- the method of sequential R/S analysis to identify chaotic sequences using the Hurst exponent  $H$ ;
- the theory of random processes and mathematical statistics to study the ergodicity of dynamic processes.

The methodological apparatus is the energy-information approach and the theory of random processes, which are the basis for determining the levels of persistence of chaotic information flows in the oil and gas well drilling control system.

To study the information flows, a set of methods and techniques is used:

- Mandelbrot-Hudson method – for the fractal time series analysis;
- autocorrelation analysis – to determine the degree of statistical dependence between different values of the random sequence, created by the dataset field;
- MatLab R2014a software – for data processing of the studied time series;
- graphic method – for a visual representation of the theoretical material.

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## 5. A set of methods for determining the levels of persistence of time series

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Each well (prospecting, exploratory, development, etc.) is drilled under substantial uncertainty and is unique in geological terms. Therefore, drilling process control optimization should be based on accumulation and processing of measurement data during the drill bit operation, i. e. on-line.

Note that the issue of forming homogeneous time series of parameters and indicators of the drilling process is fundamental to solving the control optimization problems [16, 17].

The analysis of methods for determining the optimum drilling parameters [18] has shown that the known methods of forming homogeneous time series [19, 20] are either unus-

able during the drill bit operation or can be used only for one drilling indicator.

In addition, the results of the research [21, 22] contain the values that cannot be quantified in real time, or are insufficiently developed methodically. Therefore, this issue requires elaboration, further development and fundamental improvement.

Homogeneous time series of drilling parameters and indicators are needed to determine the optimization problem of the combined operating parameters within their framework. Violations of homogeneity of time series of controlled parameters and indicators of drilling in the main operation period of the drill bit are related to the definition of emergencies and changes in drilling conditions.

However, the problem is under-investigated. There is no clarity in the information base for early detection of emergencies, as well as various complications and normal situations, and also, emergencies in the main and final operation periods of the drill bit. The methods of determining the operating range for each controlled parameter and indicator of the drilling process are not developed.

Early detection of various complications of the drilling process creates conditions for the safe introduction of advanced drilling technology with the adopted minimum excess pressure in the well over the formation pressure or at equilibrium pressures.

Homogeneous groups of time series in the bit runs are created under the influence of two groups of factors – changes in natural geological drilling conditions and changes in the combination of drilling operating parameters, i.e. control actions.

To determine the level of trend characteristics of time series, the method of the pre-forecasting fractal analysis based on sequential R/S analysis to determine the Hurst exponent  $H$  is proposed.

A random process can be studied using one implementation if the conditions of its stationary and ergodicity are met.

However, the class of stationary and ergodic random processes is very large. So, it is necessary to use additional assumptions about the specific type of distribution. Most often, the distribution is believed to be Gaussian. Note that the class of random processes with this type of distribution is also very wide. Therefore, a further process specification according to the type of the autocorrelation function is needed.

So, the first step of the study is to define the distribution laws, stationarity conditions and determine compliance with the ergodicity conditions.

A necessary and sufficient condition for ergodicity relative to average  $m_z$  is defined by [23]:

$$\lim_{T \rightarrow \infty} M \left[ \frac{1}{T} \int_0^T Z(t) dt - m_z \right]^2 =$$

$$= \lim_{T \rightarrow \infty} \frac{2}{T} \int_0^T \left( 1 - \frac{\tau}{T} \right) R_{zz}(\tau) d\tau = 0, \quad (1)$$

where

$$\hat{m}_z = \frac{1}{T} \int_0^T Z(t) dt$$

is sample average;

$$Z(t) = Z[x(t)]$$

is some stationary function of the stationary random process  $x(t)$  with average  $m_z$ ;

$$M \left[ \frac{1}{T} \int_0^T Z(t) dt - m_z \right]$$

is the mean;  $T$  is the length of implementation;  $t_1, t_2, \dots, t_n \in T$ ;  $\tau$  is lag;  $R_{zz}(\tau)$  is the autocorrelation function.

If the condition (1) is met, the average can be calculated based on one implementation of infinite length or for finite  $T$  – estimation of the mean by the formula

$$m_z = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T Z(t) dt. \quad (2)$$

Since the necessary and sufficient condition for ergodicity is impossible to verify due to the lack of ensemble of implementations, we use a meaningful argument regarding the definition of sufficient conditions for ergodicity

$$\lim_{\tau \rightarrow \infty} R_{zz}(\tau) = 0. \quad (3)$$

That is, the studied implementation meets the conditions of ergodicity if the autocorrelation function decreases with increasing distance between sections (lag). However, the results of this analysis can be applied only to the investigated implementation. The autocorrelation function is relatively easy to determine only for Gaussian random processes using a single implementation

$$R_{xx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \tilde{x}(t) \tilde{x}(t + \tau) dt, \quad (4)$$

where  $\tilde{x}(t) = x(t) - M[x(t)]$  is a digital random process.

If the condition of stationarity and ergodicity is met, the time series can be studied using one implementation. However, the class of stationary time series is very wide: deterministic, random and chaotic. Therefore, a particular type of distribution will be taken into account. Most often, the distribution is believed to be Gaussian. The advantage of ergodic dynamic systems is that with enough observation time, such systems can be described by statistical methods, but the system ergodicity must be proved first.

However, classical statistical methods used to achieve time series are inadequate in most cases [24]. This is because the classical mathematical statistics is based on the law of large numbers, which states that with the increasing number of tests, the limiting distribution of random values will be the Gaussian distribution. This means that events should be independent and equiprobable, and the chaotic behavior of time series behavior is caused by the growth and decline of levels.

In addition, since pre-emergency situations and complications during drilling occur much more frequently than is assumed by the theory, the distributions describing the state of the drilling process differ from normal distribution. This necessitates the application of the methods based on the use of fuzzy logic, artificial neural networks, genetic algorithms, etc. [25].

It is known that none of these methods cannot consider such a property of the drilling process as fractality of the drill string, self-regulation, self-organization, self-adjustment. Therefore, to solve this problem, it is advisable to apply the theory of fractals of forecasting and estimating

the degree of dynamic stability of the well-drilling process to determine the trend characteristics of drilling parameters and indicators.

During the construction of fractals, the principles of non-linearity in the system development selection process are implemented.

That is, fractals are generally applied in such cases where the control object has several development options and its status is determined by the current fractal position, namely when it is needed to simulate the chaotic development.

The main objective of the fractal analysis is to determine the Hausdorff fractal dimension  $D$  and Hurst exponent  $H$  of the Gaussian series represented by a sequence of values of the indicator  $y(k)$ , where  $k$  is discrete current time [14]. Fractal dimension characterizes the occupation of the embedding space of the studied object. For example, for a one-dimensional histogram, the embedding space is a surface – two-dimensional Cartesian coordinate system. If the Hausdorff dimension  $D(x)$  is not an integer, the set  $X$  is fractal.

Fractal dimension shows what number  $N$  of “balls” with the diameter  $d$  is needed for each such ball to have at least one element of the set  $X$ . Thus, the number of the balls corresponds to the ratio

$$N(d) \approx 1/d^D, \text{ when } d \rightarrow 0: N \cdot d^D = 1. \quad (5)$$

Then the value of the fractal dimension is calculated as

$$D = \lim_{d \rightarrow 0} \frac{\log N(d)}{\log \left( \frac{1}{d} \right)}, \quad N \cdot d^D = 1. \quad (6)$$

That is, the fractal dimension  $D$  of the object is an exponent in the ratio between the number of equal sub-objects  $N$  and the similarity factor  $d$ .

Note that the drilling process is a non-reproducible, stochastic-chaotic and time-evolving process. To identify the opportunities for the well-drilling process control, the value of the Hausdorff fractal dimension  $D$  and spatial dimension  $n$  are calculated [10].

Fractal dimension  $D$  is recognized as one of the major factors characterizing the complexity of such a dynamic process as well-deepening. If the value of the Hausdorff dimension  $D$  is not an integer, this indicates a chaotic nature of the process, but if in this case  $D \in [1.0; 2.0]$ , the chaos is controlled. If  $D > 2$ , the system loses stability, the values of the parameters quickly either grow or fall depending on current trends, the system will experience unpredictable chaotic fluctuations.

The value of the spatial dimension  $n$  is the number of factors involved in a dynamic process that is implemented. The relationship between the fractal dimension  $D$  and spatial dimension  $n$  of a dynamic process is shown in Fig. 1 [10].

Fig. 1 shows that the fractal dimension  $D$  of the dynamic process of oil and gas well deepening does not exceed  $D \leq 1.55$ . So, this chaos is controlled.

To characterize the dynamic properties of the drilling process, we will consider such a property as persistence (the tendency of a process to trends).

There are [14] three methods of determining the fractal dimension  $D$  – a quantitative index of the degree of fragmentation and frequency of time series of drilling indicators:

– the classic box-counting method when the histogram is covered with a series of grids and the fractal dimension  $D$  is determined in the same way as for geometric fractals. The value of  $D$ , determined by this method, is substantially free of empiricism and has a well-defined sequence of operations;

– the B. Mandelbrot method, based on N. E. Hurst's studies and called R/S method. This method is based on the analysis of a relationship between the range  $R$  of the parameter and standard deviation  $S$ ;

– the method based on the curve length variation depending on a scale. If the studied curve is close to fractal, the curve length will increase in a stepped manner with decreasing scale. Based on the analysis of alternation of areas with different fractal dimension, one can predict system behavior, diagnose and forecast unstable states.

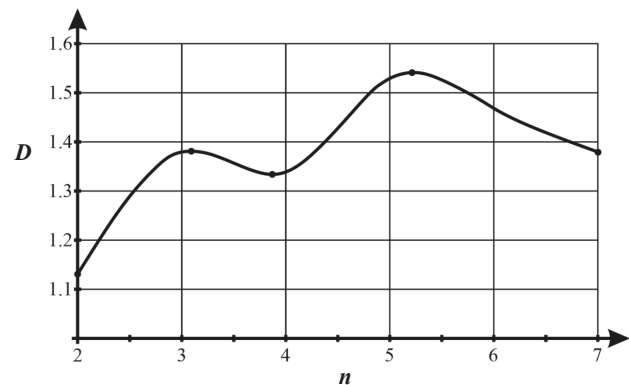


Fig. 1. Fractal and spatial dimensions of the dynamic process of well-deepening

The persistence (the tendency of the drilling process to trends) definition method based on the algorithm of R/S time series analysis, i.e. the algorithm of calculation of the Hurst exponent  $H$  is applied. This indicator for random processes with the Gaussian distribution is associated with the traditional box-counting fractal dimension  $D$  by the ratio  $H = 2 - D$ , which is true for the majority of dynamic processes.

However, the chaotic series that contain thousands of values are used in practice for determining the Hurst exponent according to this formula [24]. Dataset reduction is possible using the factor of normalized range  $R/S$ .

For the discrete time sequence  $y(1), y(2), \dots, y(i), \dots, y(k)$ , the Hurst exponent  $H$  can be written in the form [25]:

$$\frac{R(k)}{S(k)} = (\alpha k)^H, \quad (7)$$

where  $k$  is discrete current time;  $S(k)$  is standard deviation;  $\alpha$  is an unknown parameter, selected generally from purely empirical assumptions for each particular process;  $R(k)$  is the range of the sequence of the accumulated deviations  $x(i, k)$ , estimated according to the equations:

$$R(k) = \max_{1 \leq i \leq k} x(i, k) - \min_{1 \leq i \leq k} x(i, k), \quad (8)$$

$$x(i, k) = \sum_{j=1}^k (y(j) - \bar{y}(i)), \quad (9)$$

$$\bar{y}(i) = \frac{1}{i} \sum_{j=1}^i y(j), \quad (10)$$

$$\bar{y}(k) = \frac{1}{k} \sum_{i=1}^k y(i), \quad (11)$$

$$S(k) = \sqrt{D(k)} = \sqrt{\frac{1}{k} \sum_{i=1}^k (y(i) - \bar{y}(k))^2}, \quad (12)$$

where  $D(k)$  is the variance of the sequence  $y(i)$ .

For non-stationary objects, a method is proposed [25] for calculating the Hurst exponent in the sliding window as follows:

$$\frac{\bar{R}(k)}{\bar{S}(k)} = w^H, \quad (13)$$

where  $w$  is the length of the sliding window;  $\bar{R}(k)$  is the average value  $R(k)$  in the sliding window.

So,

$$H(k) = \log_w \frac{\bar{R}(k)}{\bar{S}(k)}. \quad (14)$$

The Hurst exponent is the number  $H \in [0; 1]$ , which describes the ratio of a component of the trend function to white noise and can be used to classify time series [11]: determine nonrandom time series with a steady trend and random series (including non-Gaussian).

The Hurst exponent value is determined by different methods [14]: the general Hausdorff algorithm, with the exact number of boxes, with the minimum coverage in boxes, using the equation  $H=2-D$ , using the modified method of E. Peters, etc.

Fractal Hurst  $H$  statistics is also used for assessing the autocorrelation impact of the previous values of a dynamic series on its next values and determining future trends based on the ratio [26]:

$$C = 2^{2H} - 1, \quad (15)$$

where  $C$  is the correlation measure.

Thus, depending on the ranges, which include the values of the Hurst exponent, fractal dimension  $D$  and correlation measure  $C$ , there are three main characteristics of a time series:

– if  $0.5 \leq H < 1 \wedge 1 < D < 1.5 \wedge 0 < C < 1$ , the analyzed time series is persistent or trend-resistant. The dynamics, demonstrated by a time series in the past will be continued in the same direction for a certain period of time with a high probability of preservation;

– if  $H=0.5 \wedge D=1.5 \wedge C=0$ , the values of a time series are random, completely independent, without any correlation of all the series values. This means that the uncertainty, i.e. the current state of an indicator, is in no way connected with its future state (“white noise”). In this case, there will be no correlation between the retrospective and forecast data;

– if  $0 < H < 0.5 \wedge D > 1.5 \wedge -0.5 < C < 0$ , the time sequence belongs to “anti-persistent series”, i.e. the process will change direction to the opposite one that was characteristic of the previous period. If the system indicates growth in the previous period, the decline is likely to start in the next period and vice versa.

Therefore, the main objective of the Hurst exponent is to distinguish random time series from non-random if the ran-

dom series is not Gaussian, i.e. the probability distribution is not normal [24, 26].

Since the ordered fractals, as a rule, do not exist, we can only talk about the fractal phenomena that should be considered only as models, which are approximately fractals in a statistical sense. However, these fractal models provide fairly accurate and adequate forecasts [11, 12].

The fractal theory provides a new approach to modeling the oil and gas well drilling process. One of the major limiting factors is a chaotic nature of the fractal model due to the exclusive interdependence of input and output parameters of well-drilling. It is noted [2] that even the slightest change in the input parameter or a small mistake in setting it can lead to totally unpredictable behavior of the model. However, given the underdeveloped mathematical apparatus of the theory, it is impossible to verify or estimate the results obtained by fractal modeling. Moreover, it is indeed the most promising modern direction of mathematics in terms of applied research in drilling automation.

Note that for many natural phenomena, the Hurst exponent often takes values in the range  $H \in [0.6; 0.8]$ , so  $H \approx 0.7$  is considered a characteristic value.

## 6. Practical application of methods for determining the parameters and indicators of the drilling process

Trend characteristics of the time series  $F(t)$ ,  $I(t)$ ,  $N(t)$ ,  $V(t)$ ,  $E(t)$ ,  $w(t)$  were investigated, where  $F(t)$  is the axial load on a drill bit,  $I(t)$  is the load current of the electric drill motor,  $N(t)$  is the power consumption of an electric drill,  $V(t)$  is the penetration rate,  $E(t)$  the time of drilling of 1m of the well,  $w(t)$  is the specific energy consumption.

Initial data were obtained during drilling the exploratory well in the Carpathian region. Drilling was performed with the Uralmash-6E-61 installation (Russia) using the EP 240-8 B5 electric drill (Ukraine), 295.3S GW bit (Ukraine), drilling interval was 1,317–1,327 m.

Table 1

Characteristics of the EP 240-8 B5 electric drill

| No. | Characteristics of the electric drill | Value     |
|-----|---------------------------------------|-----------|
| 1   | Diameter                              | 240 mm    |
| 2   | Length                                | 13,690 mm |
| 3   | Rated power                           | 210 kW    |
| 4   | Rated voltage                         | 1,700 W   |
| 5   | Rated current                         | 144 A     |
| 6   | Rated speed                           | 690 rpm   |
| 7   | Rated torque                          | 2.8 kNm   |
| 8   | Maximum torque                        | 7.0 kNm   |
| 9   | Efficiency                            | 75 %      |
| 10  | $\cos(\varphi)$                       | 0.66      |
| 11  | Weight                                | 3,650 kg  |

Implementations of the studied time series  $F(t)$ ,  $I(t)$ ,  $N(t)$ ,  $V(t)$ ,  $E(t)$ ,  $w(t)$  are graphically presented in Fig. 2.

Sampling of the time series was conducted with a sampling step  $\Delta t = 0.84$  s, determined on the basis of the Shannon-Kotelnikov theorem.

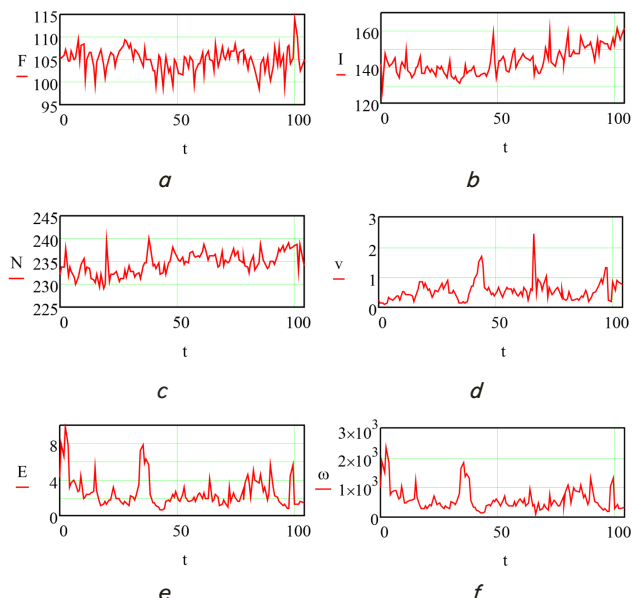


Fig. 2. Histograms of the studied time series:  
a – F(t); b – I(t); c – N(t); d – V(t); e – E(t); f – w(t)

For processing the obtained arrays, the programs developed in the MatLab R2014a software environment were used. To search for the mean, variance, standard deviation, the MatLab standard features were used, namely:

$M = \text{mean}(T)$  – mean;

$S = \text{std}(T)$  – standard deviation;

$Ds = (S)^2$  – variance.

The following results are obtained:

– mean for 6 signals:

$M1 = 104.912$ ;

$M2 = 143.031$ ;

$M3 = 234.601$ ;

$M4 = 0.547$ ;

$M5 = 2.590$ ;

$M6 = 602.25$ ;

– variance:

$Ds1 = 6.665$ ;

$Ds2 = 52.405$ ;

$Ds3 = 5.919$ ;

$Ds4 = 0.114$ ;

$Ds5 = 2.889$ ;

$Ds6 = 159097.27$ ;

– standard deviation:

$S1 = 2.582$ ;

$S2 = 7.228$ ;

$S3 = 2.423$ ;

$S4 = 0.338$ ;

$S5 = 1.699$ ;

$S6 = 398.89$ .

For the studied time series, the values of the Hurst exponent  $H$  were found using the developed program:

$H1 = 0.646$ ;

$H2 = 0.868$ ;

$H3 = 0.757$ ;

$H4 = 0.742$ ;

$H5 = 0.825$ ;

$H6 = 0.827$ .

To determine the fractal dimension  $D$ , spatial dimension  $n$  and correlation measure  $C$  of the dynamic process, we use the formulas (6), (15) and Fig. 2.

The results of the calculation are shown in Table 2.

Table 2

The results of calculations of the chaotic parameters of time series

| Time series | Hurst exponent $H$ | Fractal dimension $D$ | Spatial dimension $n$ | Correlation measure $C$ |
|-------------|--------------------|-----------------------|-----------------------|-------------------------|
| F(t)        | 0.646              | 1.354                 | 2                     | 0.224                   |
| I(t)        | 0.868              | 1.132                 | 2                     | 0.666                   |
| N(t)        | 0.757              | 1.243                 | 2                     | 0.428                   |
| V(t)        | 0.742              | 1.258                 | 2                     | 0.399                   |
| E(t)        | 0.825              | 1.175                 | 2                     | 0.569                   |
| $\omega(t)$ | 0.827              | 1.173                 | 2                     | 0.574                   |

According to Table 2, the Hurst exponent  $H$  for all the signals is greater than 0.5. Consequently, the time series have some chaos, but the dynamics of these signals will not change and will develop in the same direction as in the past. This allows solving the problems of forecasting and early detection of possible changes in drilling conditions (breakdowns) through calculating the Hurst exponent  $H$  in real time taking into account the features of the drilling process.

A chaotic nature of the drilling process is indicated by the fact that the fractal dimension  $D$  for all investigated parameters is not an integer. However, since  $D \in [1.0; 2.0]$  and  $D < 1.55$ , the chaos is controlled.

The value of the spatial dimension  $n=2$  indicates the number of factors involved in this dynamic process.

## 7. Discussion of the persistence phenomenon research results

The advantage of the obtained results of the study of the persistence levels of parameters and indicators of the well-drilling process is that this allowed determining the fractal and spatial dimension of each time series. The value of the spatial dimension is the number of factors involved in the studied process. The Hurst exponent allows determining such an important property for the well-deepening process as the tendency to trends. This indicator is versatile and can be applied to any time series, even with unknown distributions.

On the basis of the factor-targeted analysis, a set of methods for the pre-forecasting fractal time series analysis to determine the levels of persistence, allowing the forecasting of pre-emergency situations and complications under uncertainty were combined.

The results of the study are used in automated control systems of drilling processes for:

- modeling of trend characteristics of Gaussian series;
- recognition of violations of the drilling process dynamics;
- improvement of the forecasting reliability;
- early detection of deviations from normal operation.

These studies require further development to determine the persistence level of time series regarding drilling with other means – rotary and turbine drills.

## 8. Conclusions

1. Based on the implementation of the procedure of sequential R/S analysis, the fractal dimensions and Hurst

exponents for the following parameters and indicators of the well-drilling process were analyzed:

- axial load on a drill bit  $D=1.354$ ;  $H=0.646$ ;
- load current of the electric drill motor  $D=1.132$ ;  $H=0.868$ ;
- power consumption of the electric drill motor  $D=1.243$ ;  $H=0.757$ ;
- penetration rate  $D=1.258$ ;  $H=1.742$ ;
- time of drilling of 1m of the well  $D=1.175$ ;  $H=0.825$ ;
- specific energy consumption  $D=1.173$ ;  $H=0.827$ .

2. Specific energy consumption, current and time of drilling of 1m of the well have the greatest strength of trend.

Since  $0.5 < H < 1$ ;  $1 < D < 1.5$ , it can be concluded that the studied time series are persistent and the tendency to trends is peculiar to the drilling process. This phenomenon can be

used for solving problems of forecasting and early detection of process deviations the norm in real time.

3. Estimation of autocorrelation impacts of the previous values of the studied time series on their next values is performed. It is shown that the correlation ratios are in the range  $0 < C < 1$ , confirming the trend-resistance of the series. This trend will be continued in the future for a certain period of time. This helps to solve forecasting problems in the support system of decision-making regarding control of stochastic-chaotic processes.

4. To identify the opportunities to control such stochastic-chaotic processes, the spatial dimension  $n$  can be determined based on fractal dimension  $D$ . The value  $n=2$  of the spatial dimension is the number of factors involved in process control.

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*Приведений аналіз розв'язків звичайних диференціальних рівнянь з класифікацією фазових траєкторій. З використанням матриці синхронізації проведений аналіз процесів синхронізації систем Реслера. Встановленні комбінації елементів матриці, при яких реалізується повна, фазова та топологічна синхронізації систем. Показано, що для систем з нелінійною динамікою може мати місце топологічна синхронізація навіть у випадку відсутності зв'язку між ними*

*Ключові слова: система Реслера, повна та фазова синхронізація, матриця синхронізації, топологічна синхронізація*

*Приведен анализ решений обыкновенных дифференциальных уравнений с классификацией фазовых траекторий. С использованием матрицы синхронизации проведен анализ процессов синхронизации систем Реслера. Установлены комбинации элементов матрицы, при которых реализуется полная, фазовая и топологическая синхронизации систем. Показано, что для систем с нелинейной динамикой может иметь место топологическая синхронизация даже в случае отсутствия связи между ними*

*Ключевые слова: система Реслера, полная и фазовая синхронизация, матрица синхронизации, топологическая синхронизация*

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# A STUDY OF SYNCHRONIZATION PROCESSES OF NONLINEAR SYSTEMS IN THE DIFFERENCE SPACE OF PHASE VARIABLES

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## 1. Introduction

Due to the prospects of using systems with nonlinear dynamics in information and communication networks, the study of synchronous work of systems with nonlinear dynamics is important and promising in various fields of modern science [1], despite the fact that the methods of solving differential equations with nonlinear functions are already known. It should be noted that the known methods for solving differential equations by linearization technique

do not exclude the possibility of having incorrect solutions in the process of system buckling analysis. Lyapunov and Sylvester stability criterion is one of the best known criteria.

In addition, the use of linearization techniques provides the study of only complete synchronization of the two systems, the essence of which is that over time solutions of the main and controlled systems are identical. Herewith, the phase space will have a fixed point.

Phase synchronization will occur in case of solutions to differential equations that describe the behaviour of the