Отримано рівняння траєкторії руху чотирьохколісної машини в параметричній формі у функції кута повороту остова машини. Запропоновані рівняння дозволяють будувати траєкторії з урахуванням інтенсивності повороту передніх керованих коліс. Розроблено спосіб урахування явища відведення коліс обох осей під дією бічних сил. Наведена програма побудови складної траєкторії і спосіб спряження окремих її ділянок

Ключові слова: чотирьохколісна машина, криволінійний рух, рівняння траєкторії, курсовий кут, бічне відведення

Получены уравнения траектории движения четырёхколёсной машины в параметрической форме в функции угла поворота остова машины. Предложенные уравнения позволяют строить траектории с учётом интенсивности поворота передних управляемых колёс. Разработан способ учёта явления увода колёс обеих осей под действием боковых сил. Приведена программа построения сложной траектории и способ сопряжения отдельных её участков

Ключевые слова: четырёхколёсная машина, криволинейное движение, уравнение траектории, курсовой угол, боковой увод

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ANALYTICAL METHOD OF EXAMINING THE CURVILINEAR MOTION OF A FOUR-WHEELED VEHICLE

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1. Introduction

Theoretical research into curvilinear motion of wheeled vehicles aims, above all, at receiving the mathematical equations of a motion trajectory. This issue is relevant in view of the trend of implementing into practice of conducting field operations an automated control over machine-tractor units (MTU). As is known, these attempts so far have been limited to running the unit along the trajectory close to a curvilinear edge of the field, or at circumventing the hindrances. The second reason that necessitates studying the curvilinear motion of MTU is related to the swerving motion of tractors on the field. They should be performed in the most rational and economic way, to limit unproductive power consumption and prevent damaging the area on which the turns are executed. That is why the curvilinear motion still remains an important topic of scientific research.

2. Literature review and problem statement

Authors of paper [1], based on Lagrange equations of the second kind, constructed differential equations of the motion of a mounted tractor unit, and received a system of inhomogeneous second order differential equations, which describes the longitudinal motion of an arable tractor unit. However, the equation of the trajectory is missing in the paper. Article [2] developed a mathematical model of the curvilinear motion of MTA in the form of differential equations in partial derivatives, but their solutions are lacking. Paper [3] presented a model of the curvilinear motion of a wheeled vehicle in the form of the Appel's equations, but, again, without solving them. Some studies attempted to obtain the trajectory of a turn. Thus, article [4] constructed the trajectory of an articulated lorry entering the turn based on numerical modeling, that is, without an analytical expression. Almost all of these studies set the aim of receiving mathematical models of motion in the form of differential equations that prove too complicated when using them in practice. In order to reduce working time and energy costs, special algorithms are developed, which ensure minimum distances when operating the machines [5]. The tractor-robots are also designed that work without a driver using the remote control [6, 7]. In order to accurately determine position of a tractor on the field, a technique is proposed to eliminate errors in the motion via a GPS-receiver [8]. A model is devised to control a tractor during the rectilinear motion [9]. A technique is proposed, which, by using a GPS-system, can control the motion along circular trajectories [10]. Employing a satellite guidance system provides higher stability of the trajectory compared with an autopilot system [11]. The shortcoming of all these papers is the lack of analytical equations for the trajectory of motion that would make it possible to improve the existing methods of mapping the trajectories.

The importance of solving this problem is first of all associated with inadequate use of engines' power of modern tractors, which makes up from 40 to 88 %, which leads to the losses of energy resources, increases the labor-intensity of repair and maintenance. Significant deficiencies in control over tractor units can be eliminated largely by partial or complete automation. Under such circumstances, during field work, it is necessary to determine position of MTU relative to the selected coordinate system and compare them with those coordinates, which should be in place when executing the program of motion, which exists in the memory of the given coordinate system. In other words, one must have the equations of the trajectory of motion in the form accessible for use. Such equations are typically constructed by the D'Alembert's principle, or based on the Lagrange equations. Nevertheless, there no solutions to them as yet.

Trajectory of the curvilinear motion of a wheeled vehicle depends largely on the intensity of turning a steering wheel. To a certain extent, it depends also on the wheel slip angles. The intensity of wheel slipping is also affected by the speed of motion and distribution of driving power between leading axes in the case of a four-wheel drive. These are the basic factors related directly to a machine, which determine the curvature of a trajectory. In addition, there are many other factors that depend on the properties of soil, relief, nature of loading, type of trailed or mounted tools. Turnability of a machine and, accordingly, the steepness of a trajectory to a large extent depend on the kinematic scheme of the chassis: with controllable wheels of a single axis, with all controllable wheels, or with articulated frame, which has the capability to fold or break in the plane of motion. The turning angle ϕ of a machine body is the purpose of present study; moreover, this particular angle is necessary to ensure when controlling the motion of a machine or a unit.

3. The aim and tasks of research

The aim of present research is to obtain and study universal equations of the curvilinear motion of a wheeled vehicle, which would take into account the intensity of turning a steering wheel and the slip of wheels under the action of lateral forces.

To accomplish the set aim, the following tasks are to be solved:

– to receive analytical equations of the motion trajectories of a four-wheeled vehicle when entering a turn, exiting a turn, as well as when turning with a fixed position of the steering wheel at arbitrary values of the initial parameters of motion;

– to develop a technique for the conjugation of trajectories when entering a turn, when turning with a fixed position of the steering wheel and when exiting a turn, to construct complex trajectories;

– to consider the phenomenon of wheel slip caused by the action of lateral forces.

4. Materials and methods of examining the curvilinear motion of machines

We shall adopt the following designations of sections of the curvilinear motion:

– entering the left (right) turn – rotation of the machine body counterclockwise (clockwise) at alternating position of a steering wheel and at a change in the angle ϕ from 0 to final value of ϕ_{final};

– exiting the left (right) turn – rotation of the machine body clockwise (counterclockwise) due to the alignment of controlled wheels with the starting position;

– the left (right) turn – motion of the machine counterclockwise (clockwise) at fixed position of a steering wheel, respectively, after entering the left (right) turning.

To determine theoretical functions of trajectory of the curvilinear motion of the center of weight C of a four-wheeled machine with front controlled wheels in the section of entering a turn, we shall write the projections of velocity υ on the x and y axis and in a stationary coordinate system x–y (Fig. 1):

$$\upsilon_x = \upsilon\cos(\varphi+\alpha), \quad \upsilon_y = \upsilon\sin(\varphi+\alpha), \tag{1}$$

where υ is the absolute velocity of the center of weight C, tangent to the trajectory of motion; ϕ is the turning angle of machine frame relative to the Ox axis; α is the angle of deviation of velocity vector υ of weight center C from the axis of a machine body (course angle), which in our case can be determined by formula:

$$\mathrm{tg}\alpha = \frac{\ell}{L}\mathrm{tg}\alpha_1, \tag{2}$$

where α_1 is the angle of rotation of the front controlled wheels; ℓ is the distance between the center of mass C and the rear axis; L is the base of machine.

Based on the differential relationship between velocity υ and time t, it is possible to record the coordinates of center C at arbitrary point of the stationary coordinate system x–y:

$$x = \int \upsilon_x dt = \int \upsilon\cos(\varphi+\alpha)dt;$$

$$y = \int \upsilon_y dt = \int \upsilon\sin(\varphi+\alpha)dt. \tag{3}$$

The function of angle α depends entirely on the will of the driver and can be expressed depending on the angle ϕ. We shall adopt a linear dependence in the form $\alpha=\alpha_o+k\phi$, where k is the coefficient of proportionality. In a general case, this coefficient depends on the intensity of turn and the boundaries of change in the angle ϕ, within which the turning is performed; α_o is the initial value of angle α at $\phi=0$. In each particular case, the solution will hold within the predetermined limits.

There are four independent variables in integral equations (3): t, α, ϕ and velocity υ. In order to reduce integrand functions to one variable magnitude, we shall change the differential dt to $d\phi$. For this purpose, we shall express the elementary arc – displacement of point B relative to point C (Fig. 2) – through the velocity $\upsilon=\upsilon_C$ and distance ℓ from the center of machine weight to the rear axis: $dS=\ell d\phi=\upsilon_{BC}dt$, where υ_{BC} is the velocity of point B relative to point C. From the diagram of velocities in Fig. 2: $\upsilon_{BC}=\upsilon\sin\alpha\approx\upsilon\alpha=\upsilon(\alpha_o+k\phi)$. Here it is taken into account that the maximum value of angle α, which depends on the angle of turning the controlled wheels α_1, may vary in the limits, for which $\sin\alpha \approx \alpha$. Under these conditions, we receive:

$$dt = \frac{\ell}{\upsilon} \cdot \frac{1}{\alpha} d\varphi = \frac{\ell}{\upsilon} \cdot \frac{d\varphi}{\alpha_o + k\varphi}. \quad (4)$$

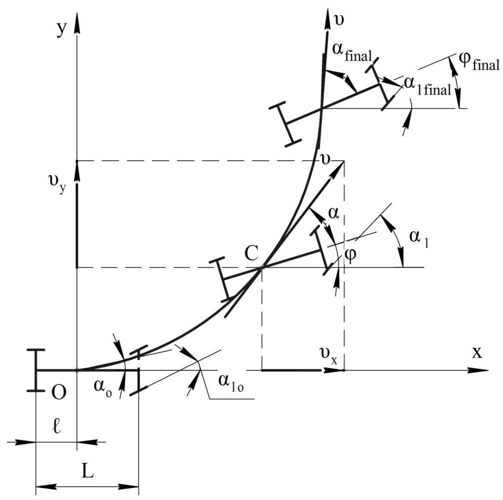

Fig. 1. Schematic of motion of a machine along curvilinear trajectory at variable values of angle α

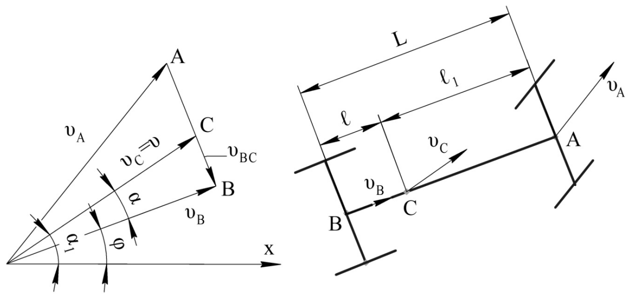

Fig. 2. Diagram of velocities for the characteristic points of tractor frame

Velocity υ can be expressed through its projections onto coordinate axe (Fig. 1):

$$\upsilon = \frac{\upsilon_x}{\cos(\alpha+\varphi)} = \frac{\upsilon_x}{\cos\left[\alpha_o + (1+k)\varphi\right]};$$

$$\upsilon = \frac{\upsilon_y}{\sin(\alpha+\varphi)} = \frac{\upsilon_y}{\sin\left[\alpha_o + (1+k)\varphi\right]}. \quad (5)$$

Then the resulting equations of curvilinear motion in the integral form are:

$$x = \ell \int \frac{\cos(\alpha+\varphi)}{\alpha} d\varphi = \ell \int \frac{\cos\left[\alpha_o + (1+k)\varphi\right]}{\alpha_o + k\varphi};$$

$$y = \ell \int \frac{\sin(\alpha+\varphi)}{\alpha} d\varphi = \ell \int \frac{\sin\left[\alpha_o + (1+k)\varphi\right]}{\alpha_o + k\varphi}. \quad (6)$$

A necessity to introduce the initial value of angle $\alpha_o \neq 0$ is predetermined by an uncertainty in finding integration constants if we assume $\alpha_o = 0$. However, the resulting integrals in this form do not have a solution. Therefore, we shall apply the approximated primary functions. For this purpose, we decompose $\cos[\alpha_o + (1+k)\phi]$ and $\sin[\alpha_o + (1+k)\phi]$ into the Maclaurin series and take for the cosine the first two members of the series, and one – for the sinus. The final desired integrals take the form:

$$x = \ell \left[\int \frac{d\varphi}{\alpha_o + k\varphi} - \int \frac{\left[\alpha_o + (1+k)\varphi\right]^2}{2(\alpha_o + k\varphi)} d\varphi \right];$$

$$y = \ell \int \frac{\alpha_o + (1+k)\varphi}{\alpha_o + k\varphi} d\varphi. \quad (7)$$

$$x = \frac{\ell}{k} \left[-\frac{(1+k)^2}{4}\varphi^2 + \frac{\alpha_o(1-k^2)}{2k}\varphi - \frac{\alpha_o^2 - 2k^2}{2k^2} \ell n \left|\alpha_o + k\varphi\right| \right] + C;$$

$$y = \frac{\ell}{k} \left[(1+k)\varphi - \frac{\alpha_o}{k} \ell n \left|\alpha_o + k\varphi\right| \right] + D. \quad (8)$$

After defining constants C and D under conditions ($x=0$; $\phi=0$) and ($y=0$; $\phi=0$), we shall obtain parametric equations of the trajectory of motion of a machine with the front controlled wheels in the section of entering the turn with a varying angle of the steering wheel turning:

$$x = \frac{\ell}{k} \left[-\frac{(1+k)^2}{4}\varphi^2 + \frac{\alpha_o(1-k^2)}{2k}\varphi - \frac{\alpha_o^2 - 2k^2}{2k^2} \ell n \left| \frac{\alpha_o + k\varphi}{\alpha_o} \right| \right];$$

$$y = \frac{\ell}{k} \left[(1+k)\varphi - \frac{\alpha_o}{k} \ell n \left| \frac{\alpha_o + k\varphi}{\alpha_o} \right| \right]. \quad (9)$$

A technique, proposed here, of integrating the equations of curvilinear motion (6) with substitution (4) can be used for the integration of the D'Alembert's differential equations. This was performed in article [12] in order to receive general equations of a machine velocity in a turn. At a repeated integration of velocity equations using the substitution (4), we receive the same result – equation (9).

The equations obtained are suitable both for the sections of entering the left and the right turns and for the sections of exiting the left and the right turns. In the latter case, the turning angle ϕ, as is adopted in the right coordinate system, must be taken as negative. In the sections of exiting the left or the right turns, coefficient of intensity of change in the course angle – coefficient k – has a negative value.

In order to build a trajectory that consists of conjugated curvilinear sections, we shall introduce the following designations (Fig. 1): α_{1o} is the original mean value of the turning angle of controlled wheels; $k = (\alpha_{final} - \alpha_o)/(\phi_{final} - \phi_o)$ is the coefficient of intensity of change in the course angle α when turning; ϕ_o and ϕ_{final} are the values of angle ϕ at the beginning and at the end of the turning section; α_o and α_{final} are the values of angle α at the beginning and at the end of the section of entering a turn or exiting it.

The initial value of course angle α_o in the section of entering a turn can be assumed as arbitrary, except zero. The latter corresponds to a real situation when at the beginning of entering a turn, the driver fulfills an instantaneous turn of controlled wheels. Then angle α_1 is evenly increasing over the entire period of entering the turn.

The final equations (9) cannot be used for calculating a trajectory of the motion of machine with a fixed position of the steering wheel, as in this case it is necessary to assume $k=0$. Then it is possible to use the integral forms of equations (6). After their integration and determining the constants, from conditions (x=y=0; ϕ=0) we obtain the equation of circular trajectory:

$$x=\frac{\ell}{\alpha_o}\left[\sin(\alpha_o+\varphi)-\sin\alpha_o\right];$$

$$y=\frac{\ell}{\alpha_o}\left[\cos(\alpha_o+\varphi)-\cos\alpha_o\right]. \qquad (10)$$

In order to construct a complex trajectory of the motion of a machine in a unified coordinate system x_1y_1 (Fig. 3), it is necessary to make the conjugation of its individual sections. Formulas for converting the coordinates of the x_i and y_i coordinate system into the x_1y_1 coordinate system in this case are:

$$x_{i1}=x_{(i-1)n}+x_i\cos\theta_{i1}-y_i\sin\theta_{i1};$$

$$y_{i1}=y_{(i-1)n}+y_i\cos\theta_{i1}+y_i\sin\theta_{i1}, \qquad (11)$$

where θ_{i1} is the turning angle of the i-th coordinate system relative to the x_1y_1 coordinate system; $x_{(i-1)n}$ and $y_{(i-1)n}$ are the coordinates of a point at the end of trajectory of the previous section.

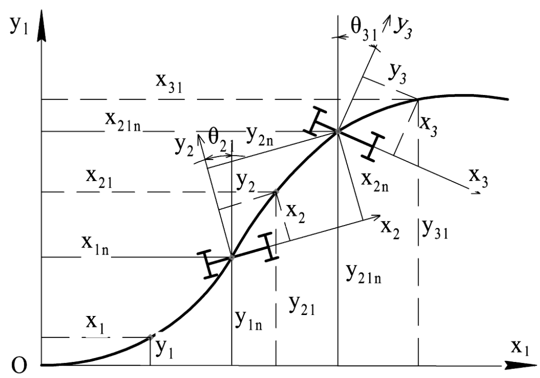

Fig. 3. To convert coordinates of individual sections of the trajectory into a unified x_1y_1 coordinate system

Equations (9) can be used as well for obtaining a trajectory of the curvilinear motion with regard to the phenomena of wheels slip. For this purpose, the function of course angle in the section of entering a turn should be represented in dependence on the turning angle of a machine body ϕ in the following way:

$$\alpha=\alpha'-\alpha''=(\alpha_o'+k_1\varphi)-(\alpha_o''+k_2\varphi)=\alpha_o+k\varphi, \qquad (12)$$

where α' and α'' are the angles of deviation of velocity vector υ from the axis of the tractor caused, respectively, by the turning of the front wheels or by the turning a steering wheel and front wheel side offtake; α_o' and α_o'' are the initial values of angles α_o' and α_o'', which depend on the turning a steering wheel at the moment of the beginning of entering a turn or at the moment of the early exiting the turn; k_1 and k_2 are the coefficients that determine the intensity of change in the angles α' and α'' and which are accepted depending on the limits of change in the angle ϕ, within which the turn is fulfilled.

Slip of front wheels reduces curvature of the trajectory, and, therefore, the magnitude of course angle that reduces the turnability of a machine. Slip of rear wheels gives rise to the reverse effect. One should consider this when taking the plus sign or minus sign for the angle α'' and the coefficient of intensity of its change k_2. Thus, for the section of entering a turn with regard to the impact of retraction of the front wheels, we obtain obvious interrelations:

$$\alpha_o=\alpha_o'-\alpha_o'',\ \ k=k_1-k_2. \qquad (13)$$

Equations (9) can be considered universal trajectory equations of the curvilinear motion of a four-wheeled machine with front steered wheels. They allow us to build trajectories both excluding and taking into account the phenomenon of retraction of both rear and front wheels, caused by the action of lateral forces. In a general case, the coefficient $k=k_1+k_2+k_3$, where components k_1, k_2, k_3 are the coefficients of intensity of change in the angle α. They consider, respectively, the angle of the front wheels turning from the turn of a steering wheel, the slip angle of front wheels and the slip angle of rear wheels. Coefficient k_1 in the section of entering a turn is positive, while it is negative in the section of exiting a turn. Coefficient k_2 always have a sign opposite to the sign of coefficient k_1. Front wheel slip reduces the course angle predetermined by the turn of a steering wheel. Coefficient k_3 always matches the sign of coefficient k_1.

The value of course angle at the end of entering a turn taking into account the wheels slip can be determined by known formula:

$$\mathrm{tg}\alpha_{\text{final}}=\left[\ell\,\mathrm{tg}(\alpha_{1\text{final}}-\alpha_{1\text{final}}')+\ell_1\mathrm{tg}\alpha_{2\text{final}}'\right]/L, \qquad (14)$$

where ℓ_1 is the distance from the center of the weight of machine to the front axis; $\alpha_{1\text{final}}'$ and $\alpha_{2\text{final}}'$ are the slip angles at the end of the section of entering a turn caused by the slip of, respectively, the front and rear wheels: $\alpha_{1\text{final}}'=\alpha_{1o}'+k_2\varphi_{\text{final}}$; $\alpha_{2\text{final}}'=\alpha_{2o}'+k_3\varphi_{\text{final}}$.

Here, the initial values of the slip angles of front and rear wheels α_{1o}' and α_{2o}' are accepted depending on the angle α_o. In the section of exiting a turn, the initial value of course angle is equal to its value at the end of the section of entering a turn.

5. Results of examining the trajectory of motion of the tractor MTZ-82

Fig. 4 shows the built sections of trajectory of turning the tractor MTZ-82 (ℓ=0.93 m; L=2.45 m) for the following initial data: k=0,2; ϕ=0...90° and α_o=5° – in the section of entering a turn; ϕ=0...90° and k=–0,2, α_o=23° – in the section of exiting a turn. Simplified equations of the trajectory of motion for this case take the form:

Entering a turn:

$$x=4{,}65(-0{,}36\varphi^2+0{,}209\varphi+0{,}905\ell n|1+2{,}29\varphi|);$$

$$y=4{,}65(1{,}2\varphi-0{,}436\ell n|1+2{,}29\varphi|). \qquad (15)$$

Exiting a turn:

$$x=4{,}65(0{,}16\varphi^2+0{,}96\varphi+\ell n|1-0{,}5\varphi|);$$

$$y=4{,}65(-0{,}8\varphi-2\ell n|1-0{,}5\varphi|). \qquad (16)$$

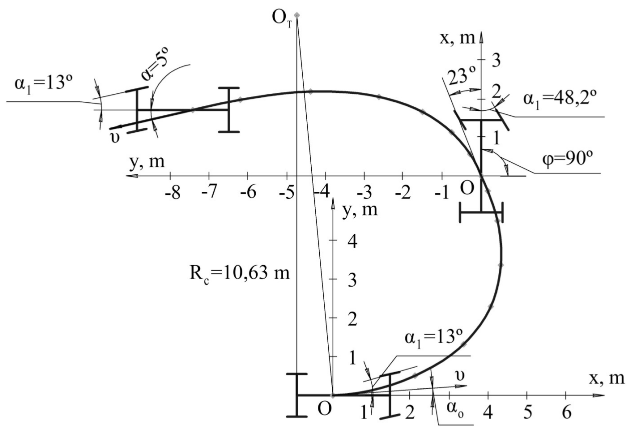

Fig. 4. Trajectories of entering a turn and exiting a turn at 90° for the tractor MTZ-82

The methods of the theory of mechanisms and machines can be employed to build the graphs of change in the radius of curvature R_C, angles α and α_1, the angular velocity of turning a body of the tractor $\omega=\upsilon/R_C$ and other parameters of motion. Fig. 5–7 show the graphs of change in the angles α and α_1, radius R_C and the angular velocity at constant speed of the tractor MTZ-82 $\upsilon\approx 2$ m/s when it enters the left turn. Fig. 8 shows the trajectories of sections of entering the left and the right tirns of the tractor, as well as the sections of exiting the turns.

The time of entering a turn and the time of exiting a turn at constant velocity υ can be determined by using formula (4) for dt, given that $R_C=\ell/\alpha$ and $d\phi=d\alpha/k$. Then:

$$t=\int_{0}^{\varphi}\frac{R_c}{\upsilon}d\varphi=\frac{\ell}{\upsilon}\cdot\frac{1}{k}\int_{\alpha_o}^{\alpha_{final}}\frac{d\alpha}{\alpha}=\frac{\ell}{\upsilon}\cdot\frac{1}{k}\ell n\left|\frac{\alpha_{max}}{\alpha_o}\right|. \quad (17)$$

For the above example (υ=7 km/h≈2 m/s), $t_{entry}=t_{exit}$=3.55 s. The lengths of sections of trajectories of entering a turn and exiting a turn are, respectively: $S_{entry}=\upsilon t_{entry}=S_{exit}$=2·3.55=7.1 m.

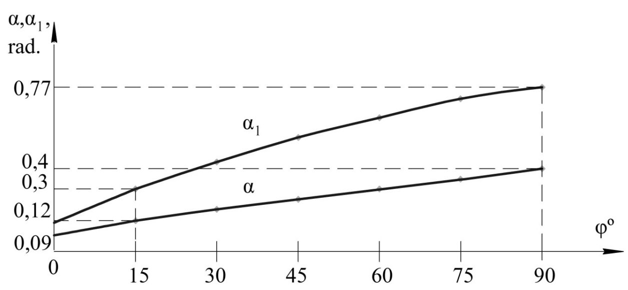

Fig. 5. Graphs of change in the turning angle of steered wheels and in the course angle

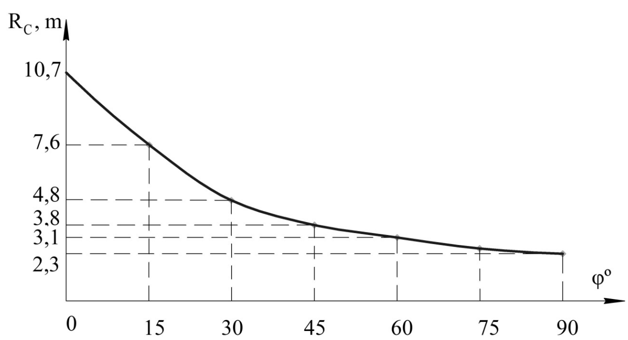

Fig. 6. Graph of change in the radius of curvature

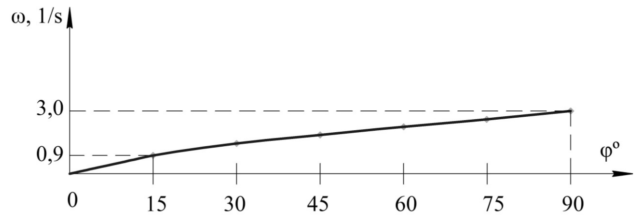

Fig. 7. Graph of change in the angular speed

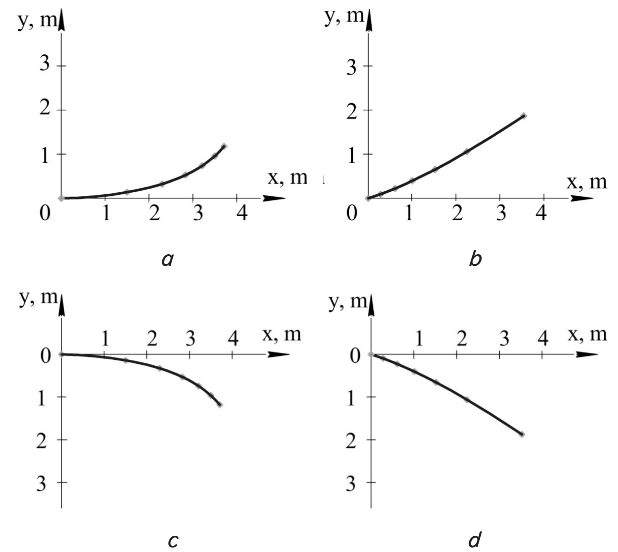

Fig. 8. Separate sections of trajectory of the curvilinear motion of tractor MTZ-82: *a* – entering the left turn (α_o=2°; α_{final}=17°; ϕ_{final}=30°; k=0.5); *b* – exiting the left turn (α_o=17°; α_{final}=2°; ϕ_{final}=30°; k=–0.5); *c* – entering the right turn (α_o=–2°; α_{final}=–17°; ϕ_{final}=–30°; k=0.5); *d* – exiting the right turn (α_o=–17°; α_{final}=–2°; ϕ_{final}=–30°; k=–0.5)

Fig. 9 shows a circular trajectory of the right turn of a four-wheeled tractor at angle ϕ=240°, which matches the course angle α=17°.

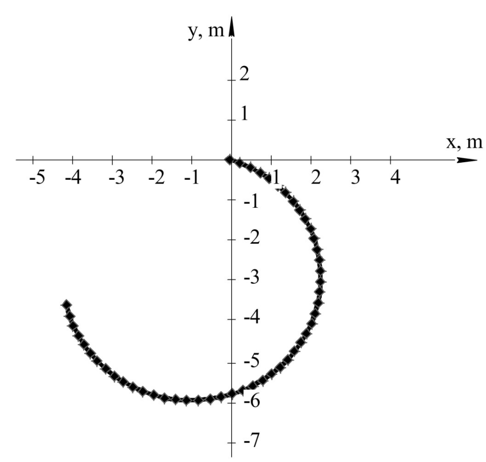

Fig. 9. Circular trajectory of the right turning of a four-wheeled tractor (α_o=–17°; ϕ_{final}=–240°, ℓ=0.93 m)

By using equations (9) for constructing a trajectory with regard to the front wheels slip, it is necessary to assign coefficients k_1 and k_2, initial angles α'_o and α''_o, as well as the values of angle ϕ, which define the limits of the section of entering a turn or the section of exiting a turn.

We shall consider a particular case on the example of tractor MTZ-82. Assume it is required to calculate the basic parameters of the sections of entering a turn at 90° and subsequent exiting a turn at 90°. As a result, the tractor must turn by 180°. We accept the following initial data in the section of entering a turn: $\alpha\ \alpha'_o=5°$, $\alpha''_o=0.05°$, $k_1=0.2$ and $k_2=-0.05$. Find the value of the course angle caused by the turn of the steered wheels at the end of the section: $\alpha'_{final}=\alpha'_o+$ $+k_1\phi=5+0.2\cdot 90=23°$, which corresponds to the mean turning angle of the steered wheels $\alpha'_к=48.2°$, and the value of course angle caused by the wheels slip is $\alpha''_{final}=\alpha''_o+$ $+k_2\phi=0.5+0.05\cdot 90=5°$. The course angle at the end of the section of entering a turn is $\alpha_{final}=\alpha'_{final}-\alpha''_{final}=23-5=18°$.

In order to consider the phenomenon of wheels slip, it is necessary to take into account particular conditions, applying the existing information about the type of wheels, condition of the soil and the predetermined motion speed at turning. In the given example, $k=0.2-0.05=0.15$; $\alpha_o=5-0.5=4.5°$. The simplified equations of the motion trajectory in the section of entering a turn for examined example will take the form:

$$x=6,2(-0,33\varphi^2+0,226\varphi+0,863\ell n\left|1+1,91\varphi\right|);$$

$$y=6,2(1,15\varphi-1,523\ell n\left|1+1,91\varphi\right|). \tag{18}$$

In the section of exiting a turn, the origin of coordinate system should be aligned with the end point of trajectory of entering a turn and one should accept the following initial data: $\phi=90°...0°$, $\alpha'_o=28°$, $\alpha''_o=5°$. At the beginning of this section, the front wheels are turned by angle $\alpha_{1o}=48.2°$. The lateral forces both at entering a turn and when exiting the turn retain the direction and the intensity of change, which is why $\alpha_o=\alpha'_o+\alpha''_o=23-5=18°$ and $k_2=-0.05$. Assume that at the end of exiting a turn it is required to satisfy the condition: $\alpha_{final}=\alpha'_{final}+\alpha''_{final}=5-0.5=4.5°$, where angle $\alpha''_{final}=\alpha''_o-k_2\phi=5-0.05\cdot 90=0.5°$. Then $k_1=(\alpha'_{final}-\alpha'_o)/\phi=$ $=(5-23)/90=-0.2$ and the total intensity coefficient of change in the course angle in the section of exiting a turn is $k=k_1-k_2=-0.2-(-0.05)=-0.15$, or $k=(\alpha_{final}-\alpha_o)/\phi=$ $=(4.5-18)/90=-0.15$. The equations of trajectory in the section of exiting a turn in this case will take the form:

$$x=6,2(0,18\varphi^2+1,023\varphi+1,191\ell n\left|1-0,478\varphi\right|);$$

$$y=6,2(-0,85\varphi-2,093\ell n\left|1-0,478\varphi\right|). \tag{19}$$

Fig. 10 shows a built trajectory, which consists of the sections shown in Fig. 9, conjugated using formulas (11). We employed the Microsoft Excel software to plot all sections of the trajectory.

Fig. 11 shows the trajectories of entering a turn and exiting a turn by the tractor MTZ-82 for the above conditions both with (curve 1) and without (curve 2) taking into account the impact of the front wheels slip. It is obvious that with the accepted average values of coefficient k_2 there is a substantial mismatch between the trajectories that are built with and without consideration of influence of the front wheels slip, both in the section of entering a turn and in the section of exiting a turn. The slip of front wheels always reduces the intensity of entering a turn and exiting a turn.

Taking into account the influence of the front wheels slip, the time of motion in the sections of entering a turn and exiting a turn according to (17) is: $t_{entry}=t_{exit}=4.3$ s. The lengths of these sections are $S_{entry}=S_{exit}=8.6$ m

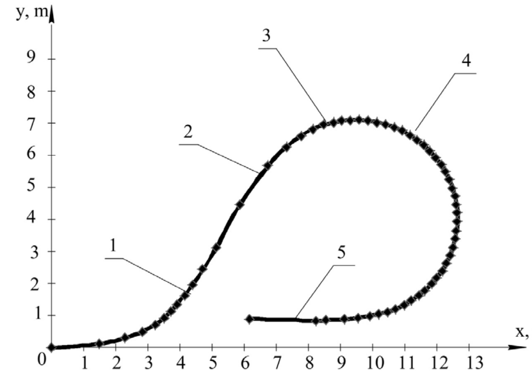

Fig. 10. Trajectory of turning a four-wheeled machine: 1 — entering the left turn; 2 — exiting the left turn; 3 — entering the right turn; 4 — the right turn; 5 — exiting the right turn

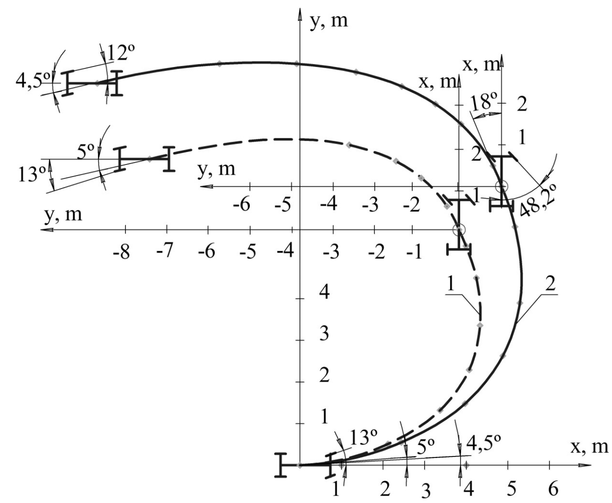

Fig. 11. Trajectories of entering a turn and exiting a turn by the tractor MTZ-82: 1 — without regard to the wheels slip; 2 — with regard to the wheels slip

6. Discussion of results of studyin the equations of curvilinear motion

The received equations of the curvilinear motion of a machine make it possible to accelerate the solution of many tasks in agroindustrial production, in particular:

– to run analysis of MTU turning when processing a field and to choose the best parameters for trajectories;

– to compile the programs of fully automated control over the motion of machines without a driver;

– to reduce unproductive power consumption for turning;

– to decrease additional cost in bringing the land on which the u-turns of MTU are performed into proper condition.

It is advisable to continue this work. It is necessary to study the impact on the accuracy of motion of such factors as the condition of soil, the relief of fields, weather conditions, sliding, skidding and the wheels slip in the curvilinear motion. Maintaining a stability of machines on sloping fields will make it possible to cultivate the land plots, which are considered unsuitable for farming.

The equations obtained in the present work and the examples of calculations provided demonstrate the possibility to analyze various variants of turning the units in a theoretical way. Optimal results may be recommended for practical application. Presented analysis of the scientific literature data reveals that there are no identified analogues to the proposed solution. The result of the study should be considered as a possible way to solve the problem on receiving the analytical formulas to describe the curvilinear motion of a wheeled vehicle. The equations derived are simple and might be applied for all possible sections of motion. We did not find any variants for comparisons.

7. Conclusions

1. The equations of the curvilinear trajectory a wheeled vehicle were obtained by integrating the projections of velocity of the center of mass onto the axes of inertial coordinate system. In order to reduce integrand functions to one variable, we found a special substitution that replaces the differential of time to the differential of turning angle of the body of a machine. The course angle is also represented as a function of the turning angle. A linear dependence will be the simplest. The proportionality factor (coefficient of intensity of change in the course angle, predetermined by the rotation velocity of steered wheels) depends on the turning of a steering wheel. Trigonometric functions under integrals are decomposed into the Maclaurin series. This made it possible to calculate coordinates of the center of mass of a machine depending on the turning angle and the intensity of change in the course angle. The equations are suitable for all possible sections of entering a turn and exiting a turn.

2. We proposed separate equations for the circular motion of a machine. They are also the functions of turning angle of the machine frame. Together with the equations for entering a turn and exiting a turn, they provide the possibility to build the trajectories of complicated turns in a unified coordinate system. For the conjugation of separate sections of the trajectory, we used formulas of change in the coordinates at parallel carry and turn of the coordinate axes. This allows us to use in order to compute coordinates of points along the trajectory such software tools as, for example, the Microsoft Excel.

3. In order to consider the impact of the wheels slip on the shape of a trajectory, we employed the coefficients of intensity of change in the course angle caused by the slip of front and rear wheels. In total with the intensity coefficient that considers the turning of steered wheel, they define the total intensity coefficient of change in the course angle.

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